EXPLOITING THE PACKING-FIELD ROUTE TO CRAFT CUSTOM TIME CRYSTALS

Pablo I. Hurtado

Institute Carlos I for Theoretical and Computational Physics
Departamento de Electromagnetismo y Física de la Materia
Universidad de Granada (Spain)

Condensed Matter and Material Physics (CMMP) Seminar
University College London, April 9 (2024)

CAN WE DO SOMETHING (USEFUL?) WITH A RARE FLUCTUATION?

Pablo I. Hurtado

Institute Carlos I for Theoretical and Computational Physics
Departamento de Electromagnetismo y Física de la Materia
Universidad de Granada (Spain)

INSTITUTO CARLOS I
DE FÍSICA TEÓRICA Y COMPUTACIONAL

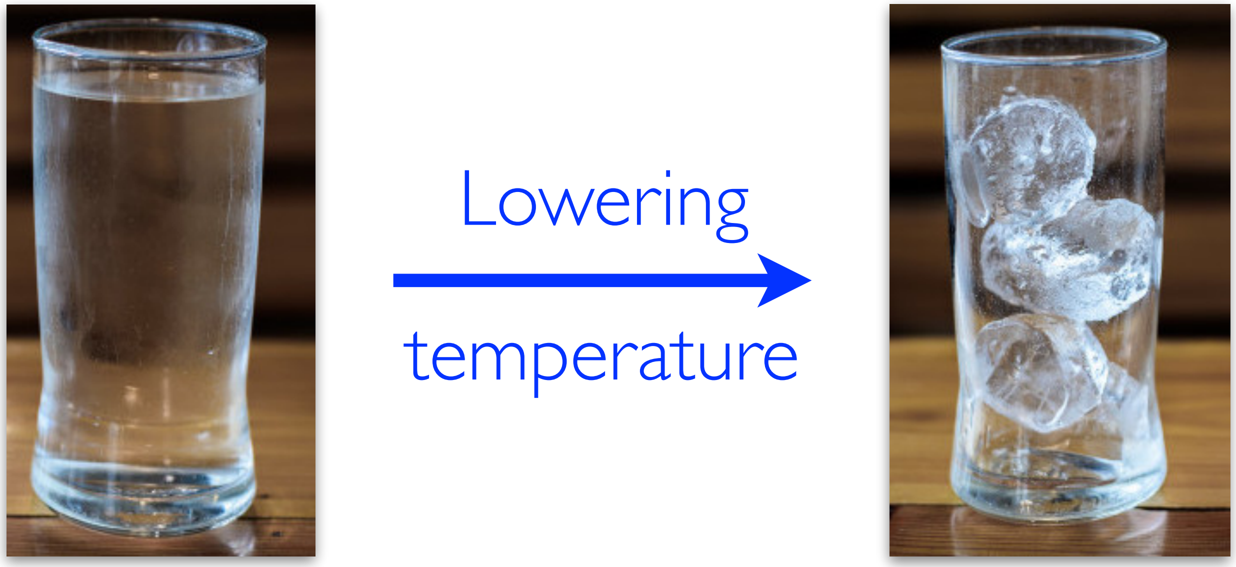
Condensed Matter and Material Physics (CMMP) Seminar
University College London, April 9 (2024)

Sketch of the talk

1) A glimpse at time crystals
2) Dynamical phase transitions and time crystals
3) Time crystal phase in WASEP
4) Packing field mechanism
5) Exploiting the packing field route

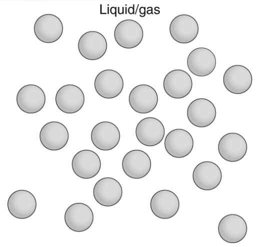
SYMMETRY BREAKING AND TIME CRYSTALS

- Most **symmetries** in nature can be **spontaneously broken**
- **Example**: spatial-translation symmetry and crystals

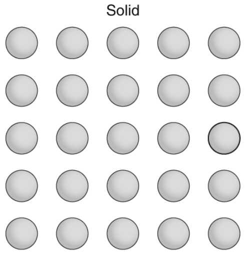


SYMMETRY BREAKING AND TIME CRYSTALS

- Most **symmetries** in nature can be **spontaneously broken**
- **Example**: spatial-translation symmetry and crystals



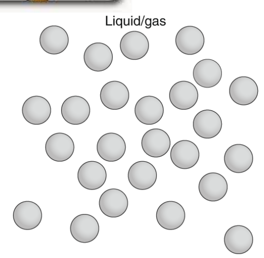
Lowering temperature



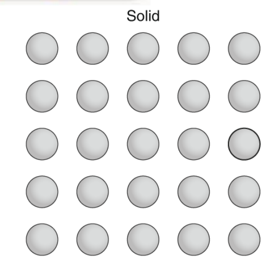
SYMMETRY BREAKING AND TIME CRYSTALS

- Most **symmetries** in nature can be **spontaneously broken**

- **Example**: spatial-translation symmetry and crystals



Lowering temperature



- But **time-translation symmetry** seems to be special and **fundamentally unbreakable**

- **Time crystal**: systems whose **ground state spontaneously breaks time-translation symmetry** and thus exhibits **long-range temporal order & robust periodic motion** [Wilczek, Shapere PRL 2012]

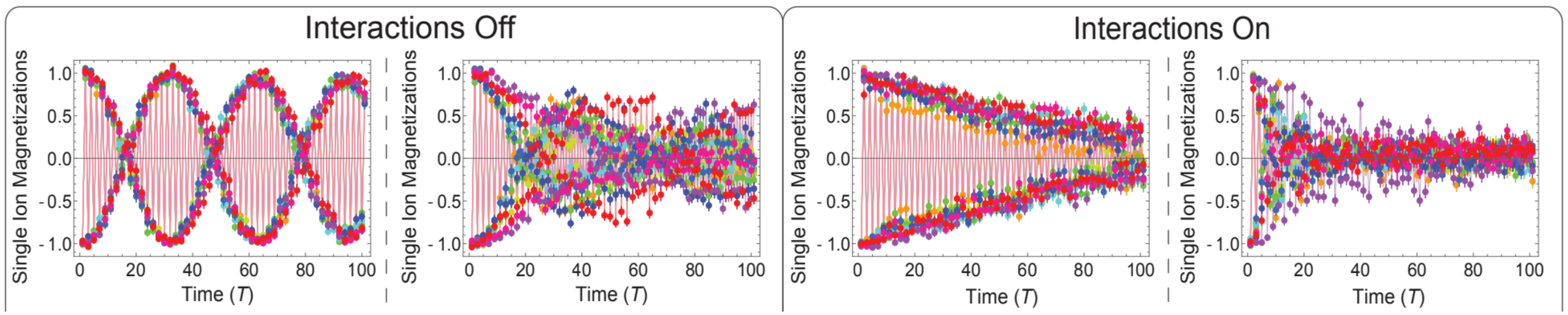
- **Caveats**: no-go theorems in equilibrium, but **possible out of equilibrium** [Bruno (2013), Watanabe et al (2015), Sacha et al (2018), Moessner et al (2017), Yao&Nayak (2018)]

- **Periodically driven (Floquet) systems**: discrete time-translation symmetry breaking via subharmonic entrainment → **discrete time crystals**

TIME CRYSTALS IN EXPERIMENTS

[Zhang et al, Nature **543**, 217 (2017)]

- First observation of **discrete time crystals** in a **interacting spin chain of trapped atomic ions.** Also observed in **disordered ensemble of spin impurities in diamond**
[Choi et al, Nature **543**, 221 (2017))]

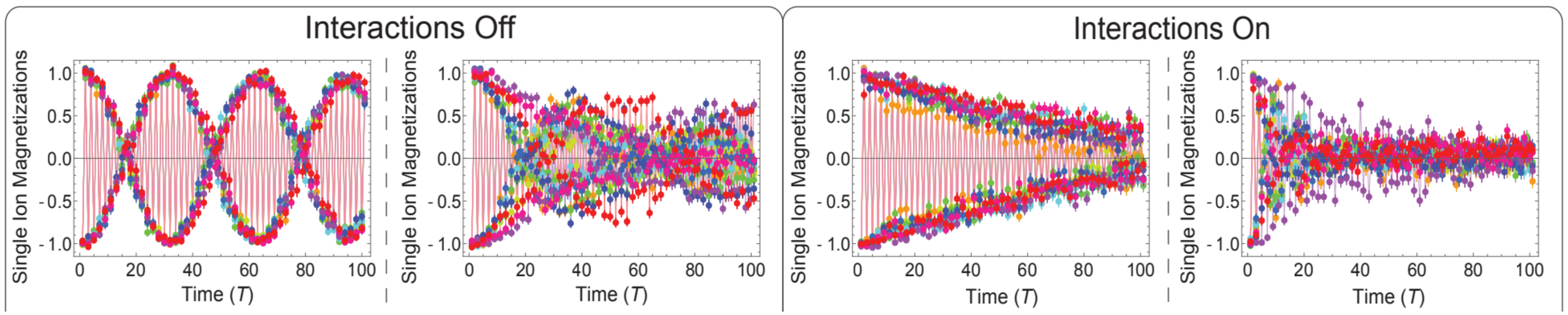


- **Since then, multiple observations** of **quantum discrete time crystals** reported
[Rovny et al, PRL (2018); Smits et al, PRL (2018); Autti et al, PRL (2018); O'Sullivan et al, NJP (2020); Kyprianidis et al, Science (2021); Randall et al, Science (2021), Keßler et al, PRL (2021); Kongkhambut et al. PRL (2021); Xiao et al, Nature (2022); etc.]

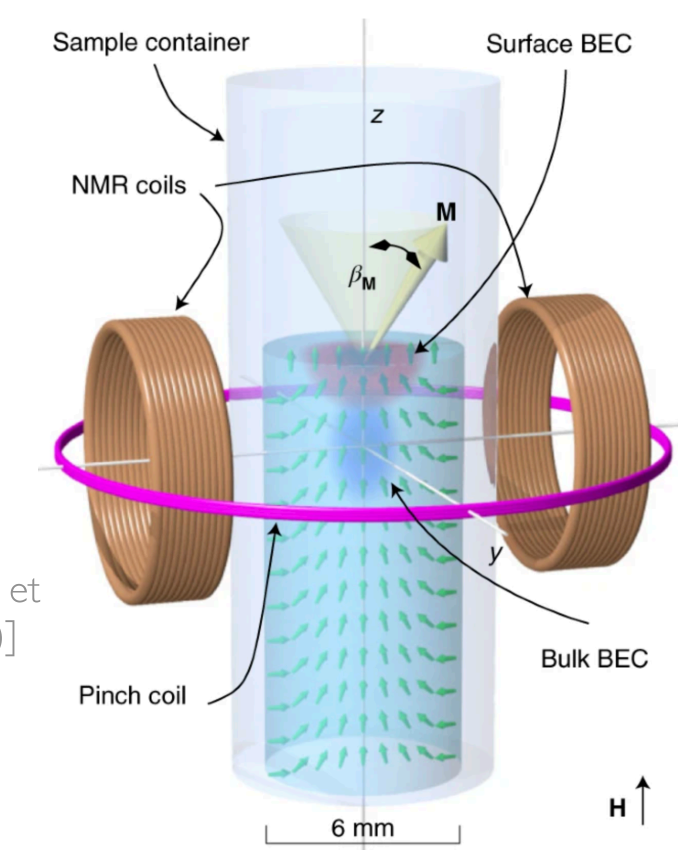
TIME CRYSTALS IN EXPERIMENTS

[Zhang et al, Nature **543**, 217 (2017)]

- First observation of **discrete time crystals** in a **interacting spin chain of trapped atomic ions.** Also observed in **disordered ensemble of spin impurities in diamond** [Choi et al, Nature **543**, 221 (2017))]



- **Since then, multiple observations** of **quantum discrete time crystals** reported [Rovny et al, PRL (2018); Smits et al, PRL (2018); Autti et al, PRL (2018); O'Sullivan et al, NJP (2020); Kyprianidis et al, Science (2021); Randall et al, Science (2021), Keßler et al, PRL (2021); Kongkhambut et al. PRL (2021); Xiao et al, Nature (2022); etc.]
- First measurement of the **interaction between two discrete time crystals** [Autti et al, Nature Mat. **20**, 171 (2020)]
- **Dissipative continuous time crystal** reported in **atom-cavity experiment** [Kongkhambut et al, Science **377**, 670 (2023)]
- **Discrete and continuous time crystals** also proposed and observed in **classical systems** [Gambetta et al, PRE (2019); Heugel et al, PRL (2019); Yao et al, Nature Physics (2020); Liu et al, Nature Physics (2023)]
- However, a **general approach to engineer custom continuous time-crystal phases** remains elusive so far



DYNAMICAL PHASE TRANSITIONS

- **Spontaneous symmetry breaking** typically found in **critical phenomena …** Ideas extended to **fluctuations**, where **dynamical phase transitions (DPTs)** have been found in the **trajectory statistics** of **classical and quantum systems**

- DPTs appear **when conditioning a system to have a fixed value of some time-integrated observable**, such as, e.g., the **current** or the **activity**

DYNAMICAL PHASE TRANSITIONS

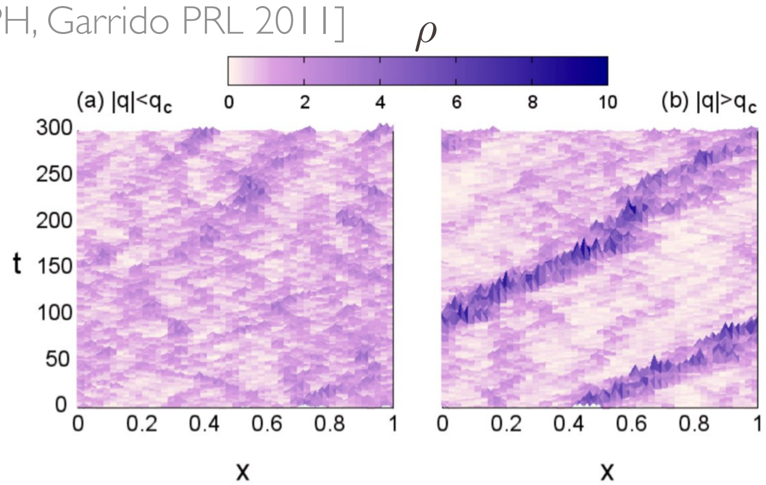
- **Spontaneous symmetry breaking** typically found in **critical phenomena …** Ideas extended to **fluctuations**, where **dynamical phase transitions (DPTs)** have been found in the **trajectory statistics** of **classical and quantum systems**

- DPTs appear **when conditioning a system to have a fixed value of some time-integrated observable**, such as, e.g., the **current** or the **activity**

- **Dynamical phases** correspond to **different types of trajectories**: some may display **emergent order, collective rearrangements, and symmetry-breaking**

Energy diffusion in 1d

[PH, Garrido PRL 2011]



DYNAMICAL PHASE TRANSITIONS

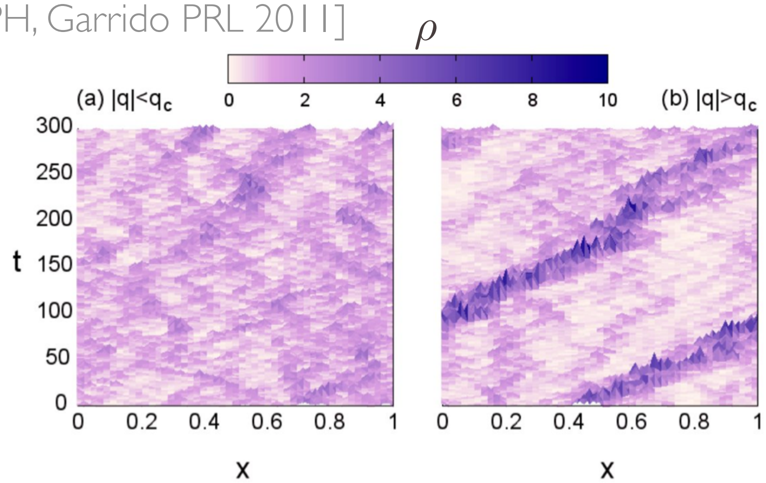
- **Spontaneous symmetry breaking** typically found in **critical phenomena …** Ideas extended to **fluctuations**, where **dynamical phase transitions (DPTs)** have been found in the **trajectory statistics** of **classical and quantum systems**

- DPTs appear **when conditioning a system to have a fixed value of some time-integrated observable**, such as, e.g., the **current** or the **activity**

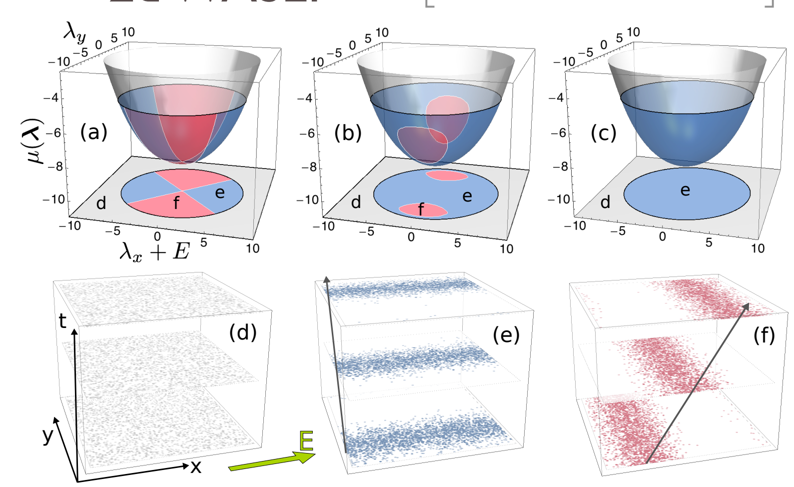
- **Dynamical phases** correspond to **different types of trajectories**: some may display **emergent order, collective rearrangements, and symmetry-breaking**

Energy diffusion in 1d

[PH, Garrido PRL 2011]



2d WASEP [Tizón et al PRL 2017]



DYNAMICAL PHASE TRANSITIONS

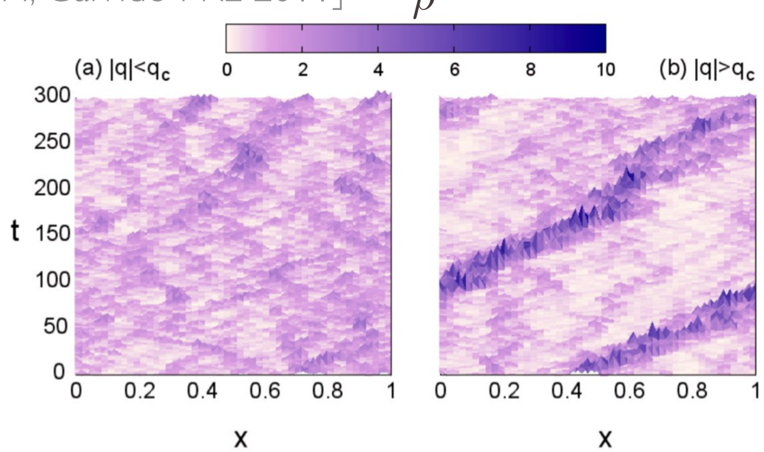
- **Spontaneous symmetry breaking** typically found in **critical phenomena …** Ideas extended to **fluctuations**, where **dynamical phase transitions (DPTs)** have been found in the **trajectory statistics** of **classical and quantum systems**

- DPTs appear **when conditioning a system to have a fixed value of some time-integrated observable**, such as, e.g., the **current** or the **activity**

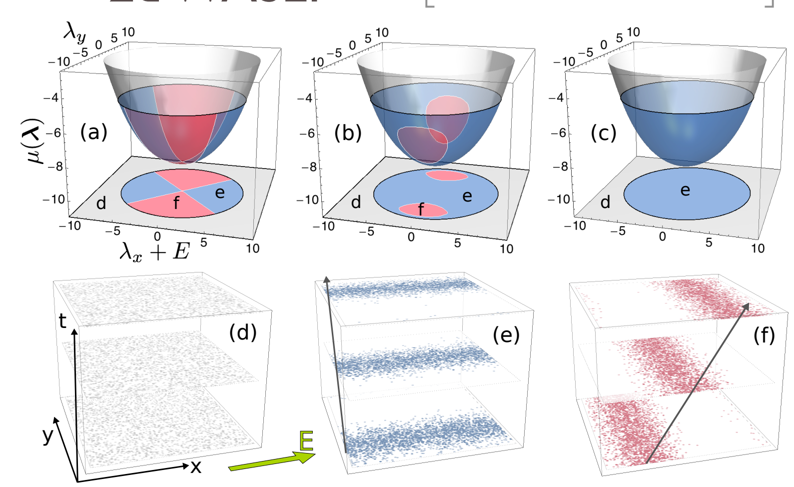
- **Dynamical phases** correspond to **different types of trajectories**: some may display **emergent order, collective rearrangements, and symmetry-breaking**

Energy diffusion in 1d

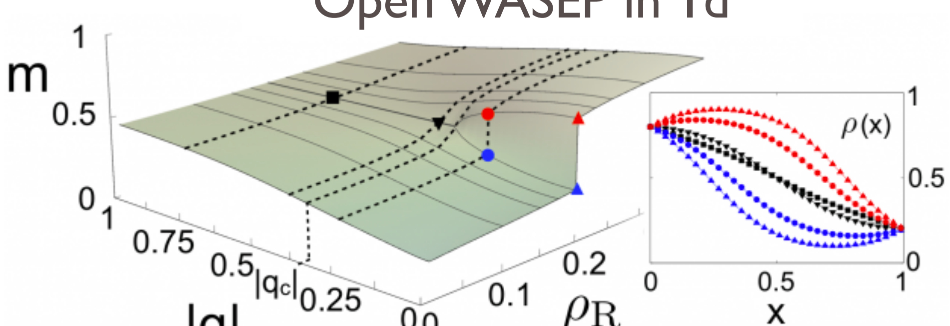
[PH, Garrido PRL 2011]



2d WASEP [Tizón et al PRL 2017]



Open WASEP in 1d



[Espigares et al PRE 2018]

DYNAMICAL PHASE TRANSITIONS

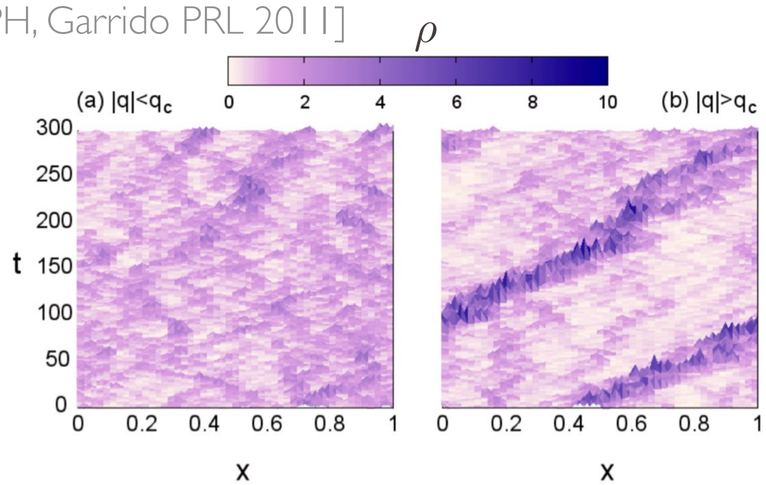
- **Spontaneous symmetry breaking** typically found in **critical phenomena …** Ideas extended to **fluctuations**, where **dynamical phase transitions (DPTs)** have been found in the **trajectory statistics** of **classical and quantum systems**

- DPTs appear **when conditioning a system to have a fixed value of some time-integrated observable**, such as, e.g., the **current** or the **activity**

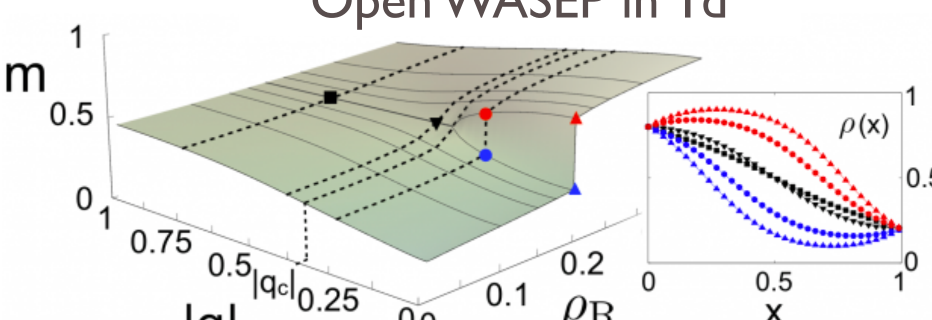
- **Dynamical phases** correspond to **different types of trajectories**: some may display **emergent order, collective rearrangements, and symmetry-breaking**

Energy diffusion in 1d

[PH, Garrido PRL 2011]

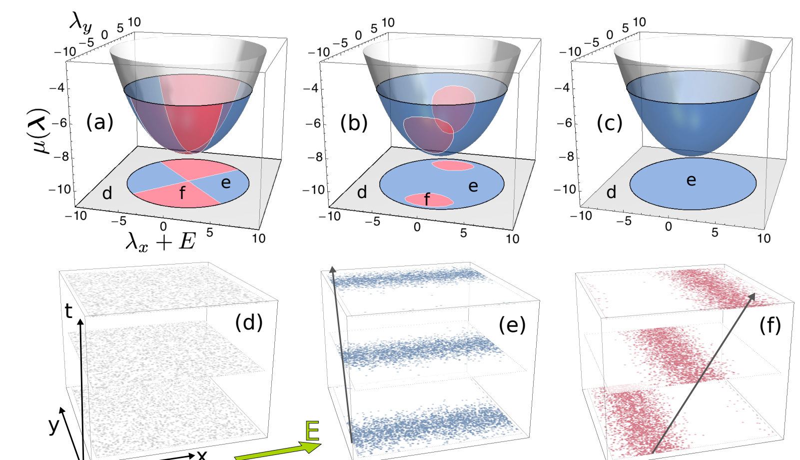


Open WASEP in 1d



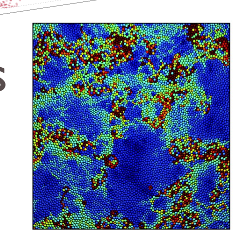
[Espigares et al PRE 2018]

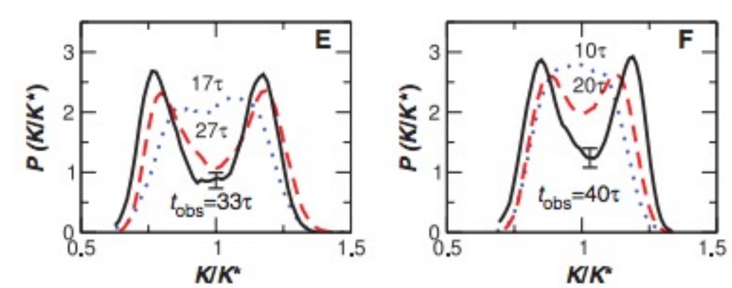
2d WASEP [Tizón et al PRL 2017]



Activity DPT in glasses

[Hedges et al Science 2009]
[Pinchaipat et al PRL 2017]





DYNAMICAL PHASE TRANSITIONS

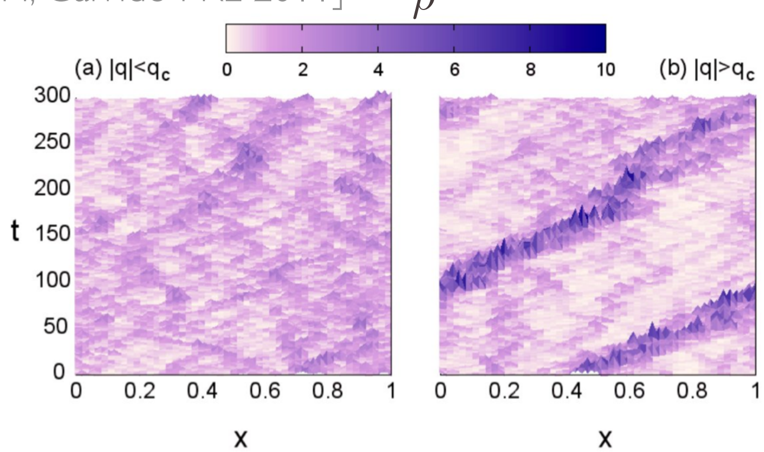
- **Spontaneous symmetry breaking** typically found in **critical phenomena …** Ideas extended to **fluctuations**, where **dynamical phase transitions (DPTs)** have been found in the **trajectory statistics** of **classical and quantum systems**

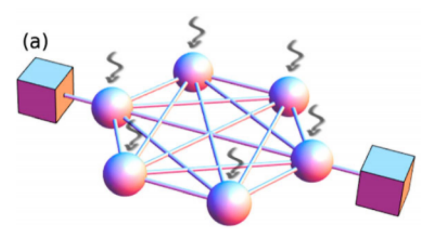
- DPTs appear **when conditioning a system to have a fixed value of some time-integrated observable**, such as, e.g., the **current** or the **activity**

- **Dynamical phases** correspond to **different types of trajectories**: some may display **emergent order, collective rearrangements, and symmetry-breaking**

Energy diffusion in 1d

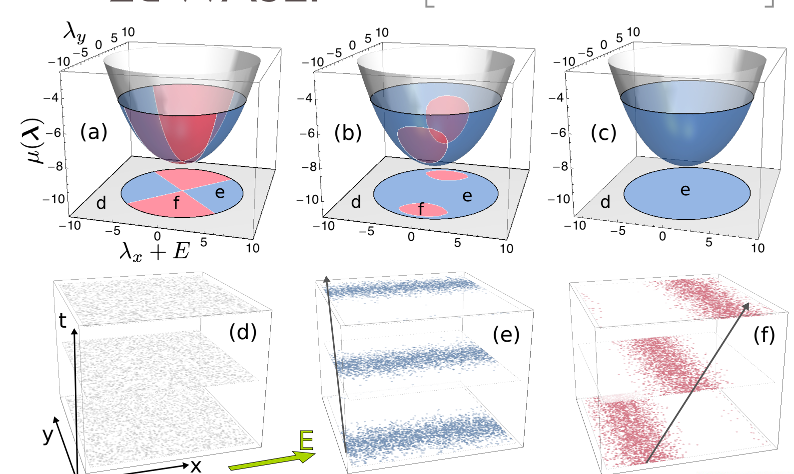
[PH, Garrido PRL 2011]



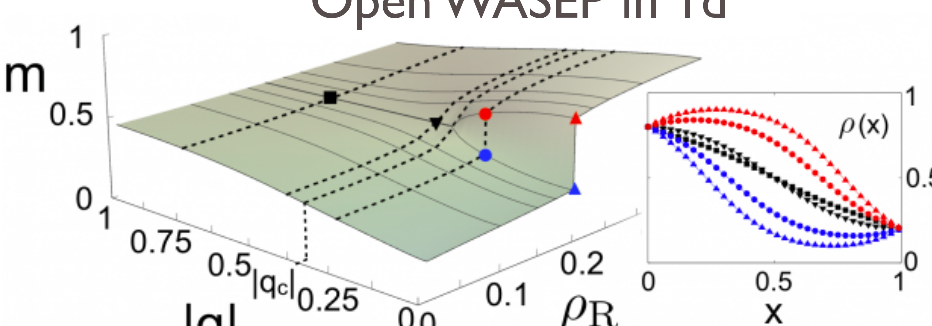


Symmetry-induced DPTs in open quantum systems

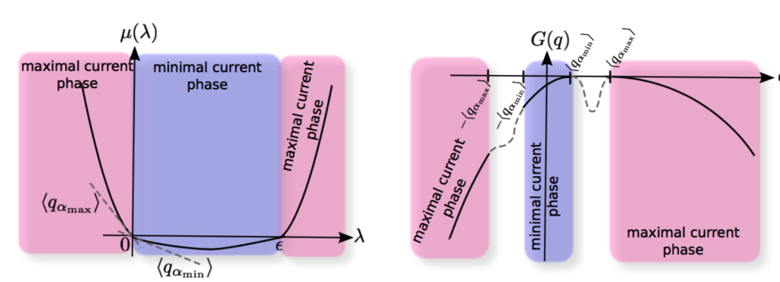
2d WASEP [Tizón et al PRL 2017]



Open WASEP in 1d

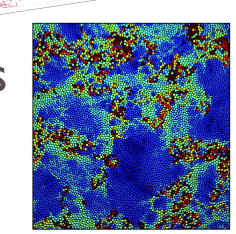


[Espigares et al PRE 2018]

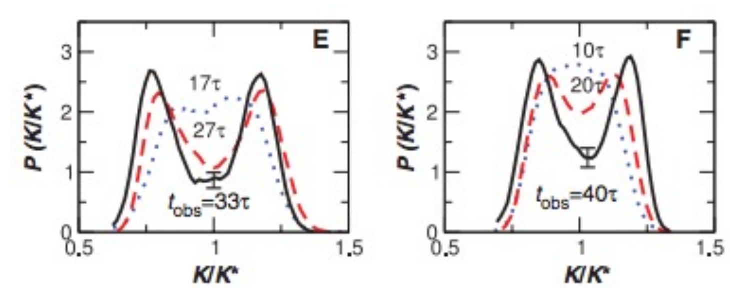


[Manzano, PH PRB 2014]

Activity DPT in glasses



[Hedges et al Science 2009]
[Pinchaipat et al PRL 2017]

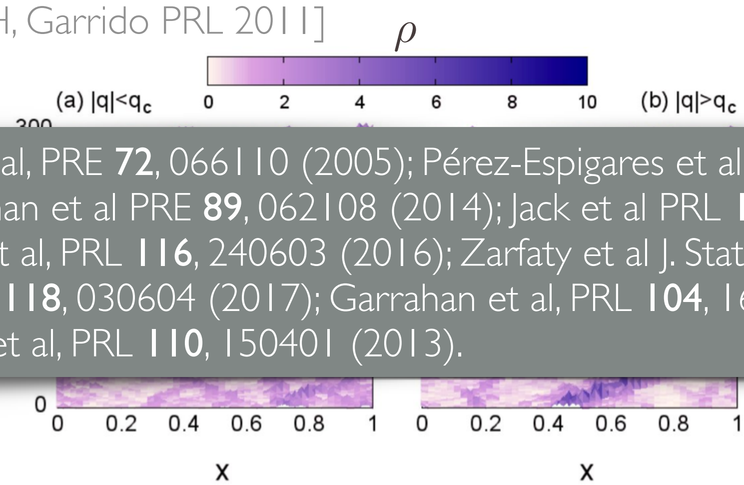


DYNAMICAL PHASE TRANSITIONS

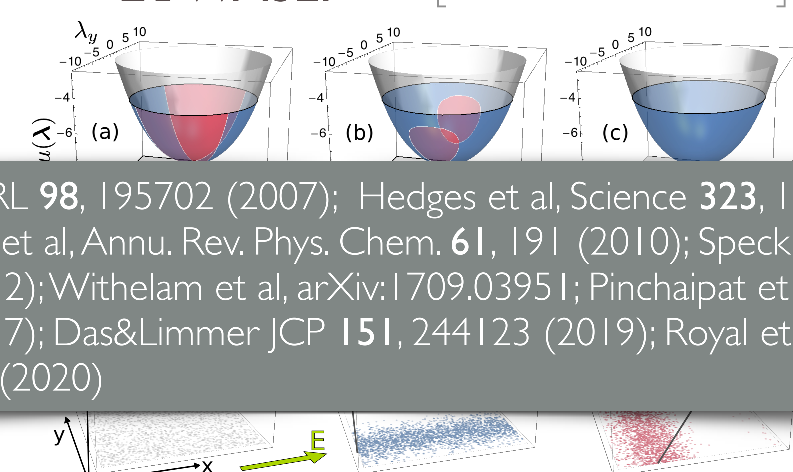
- **Spontaneous symmetry breaking** typically found in **critical phenomena …** Ideas extended to **fluctuations**, where **dynamical phase transitions (DPTs)** have been found in the **trajectory statistics** of **classical and quantum systems**
- DPTs appear **when conditioning a system to have a fixed value of some time-integrated observable**, such as, e.g., the **current** or the **activity**
- **Dynamical phases** correspond to **different types of trajectories**: some may display **emergent order, collective rearrangements, and symmetry-breaking**

Energy diffusion in 1d

[PH, Garrido PRL 2011]



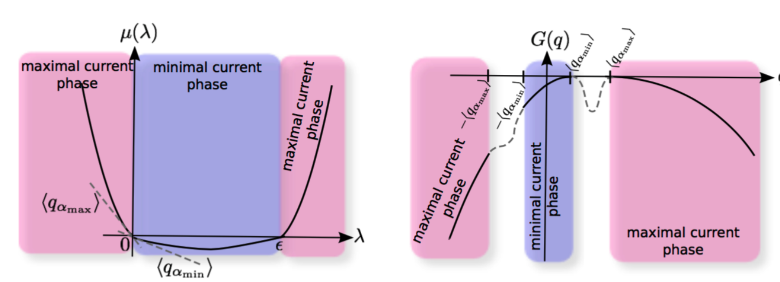
2d WASEP [Tizón et al PRL 2017]



Bodineau at al, PRE **72**, 066110 (2005); Pérez-Espigares et al, PRE **87**, 032115 (2013); Vaikuntanathan et al PRE **89**, 062108 (2014); Jack et al PRL **114**, 060601 (2015); Shpielberg et al, PRL **116**, 240603 (2016); Zarfaty et al J. Stat. Mech. (2016) P033304; Baek al, PRL **118**, 030604 (2017); Garrahan et al, PRL **104**, 160601 (2010); Lesanovsky et al, PRL **110**, 150401 (2013).

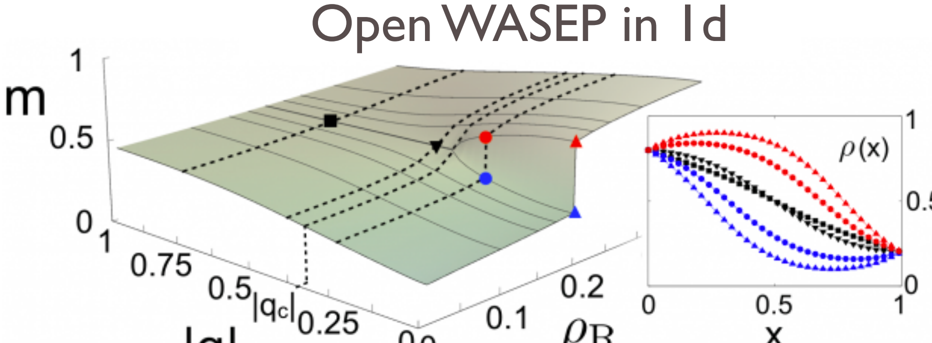
Garrahan et al, PRL **98**, 195702 (2007); Hedges et al, Science **323**, 1309 (2009); Chandler et al, Annu. Rev. Phys. Chem. **61**, 191 (2010); Speck et al, PRL **109**, 195703 (2012); Withelam et al, arXiv:1709.03951; Pinchaipat et al, PRL **119**, 028004 (2017); Das&Limmer JCP **151**, 244123 (2019); Royal et al, JCP **153**, 090901 (2020)

open quantum systems



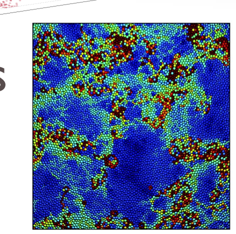
[Manzano, PH PRB 2014]

Open WASEP in 1d

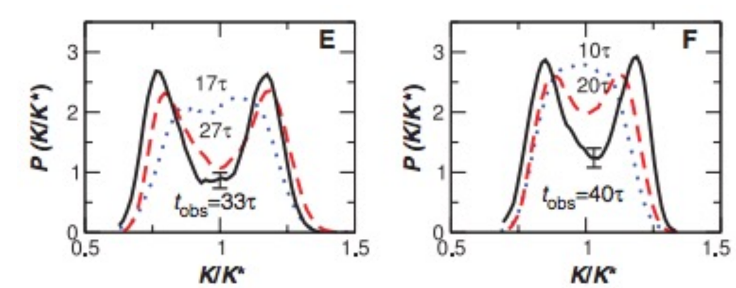


[Espigares et al PRE 2018]

Activity DPT in glasses



[Hedges et al Science 2009]
[Pinchaipat et al PRL 2017]



DYNAMICAL PHASE TRANSITIONS

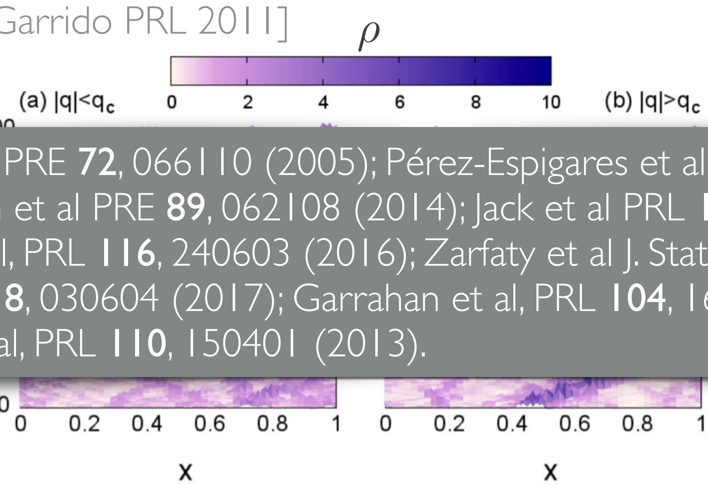
- **Spontaneous symmetry breaking** typically found in **critical phenomena …** Ideas extended to **fluctuations**, where **dynamical phase transitions (DPTs)** have been found in the **trajectory statistics** of **classical and quantum systems**

- DPTs appear **when conditioning a system to have a fixed value of some time-integrated observable**, such as, e.g., the **current** or the **activity**

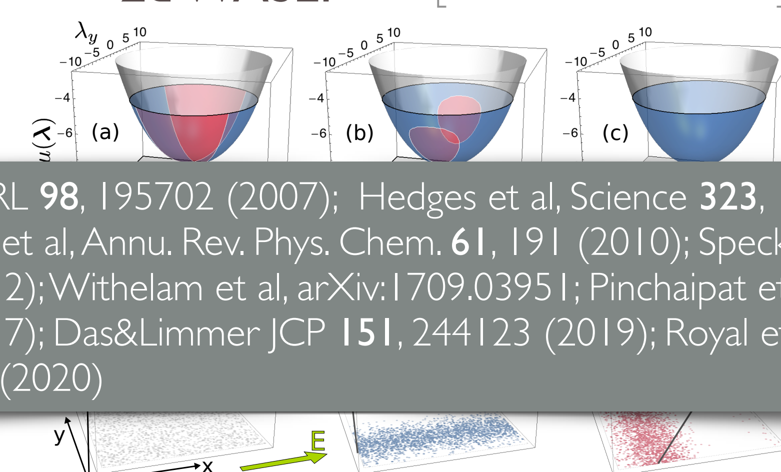
- **Dynamical phases** correspond to **different types of trajectories**: some may display **emergent order, collective rearrangements, and symmetry-breaking**

Energy diffusion in 1d

[PH, Garrido PRL 2011]



2d WASEP [Tizón et al PRL 2017]

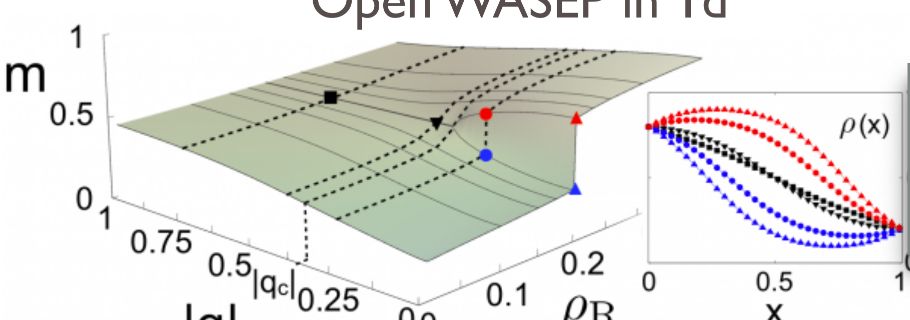


Bodineau at al, PRE **72**, 066110 (2005); Pérez-Espigares et al, PRE **87**, 032115 (2013); Vaikuntanathan et al PRE **89**, 062108 (2014); Jack et al PRL **114**, 060601 (2015); Shpielberg et al, PRL **116**, 240603 (2016); Zarfaty et al J. Stat. Mech. (2016) P033304; Baek al, PRL **118**, 030604 (2017); Garrahan et al, PRL **104**, 160601 (2010); Lesanovsky et al, PRL **110**, 150401 (2013).

Garrahan et al, PRL **98**, 195702 (2007); Hedges et al, Science **323**, 1309 (2009); Chandler et al, Annu. Rev. Phys. Chem. **61**, 191 (2010); Speck et al, PRL **109**, 195703 (2012); Withelam et al, arXiv:1709.03951; Pinchaipat et al, PRL **119**, 028004 (2017); Das&Limmer JCP **151**, 244123 (2019); Royal et al, JCP **153**, 090901 (2020)

open quantum systems

Open WASEP in 1d



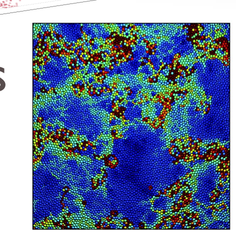
[Espigares et al PRE 2018]

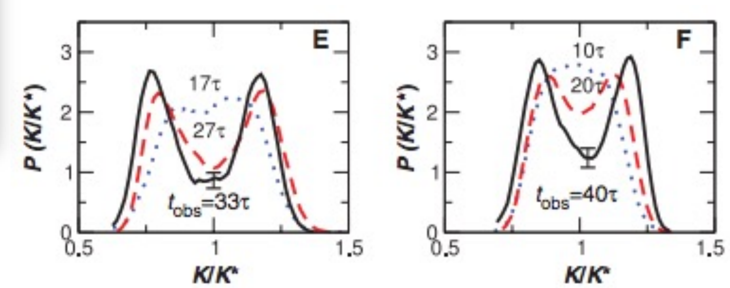
Time crystals in the fluctuations of driven systems?

[Manzano, PH PRB 2014]

Activity DPT in glasses

[Hedges et al Science 2009]
[Pinchaipat et al PRL 2017]

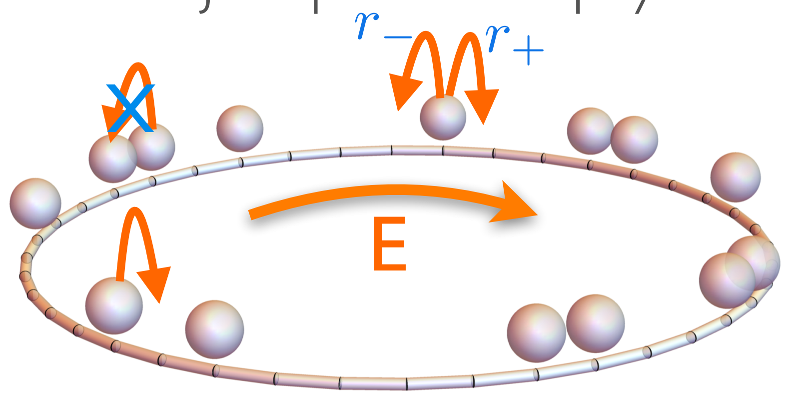




WEAKLY ASYMMETRIC SIMPLE EXCLUSION PROCESS

- **WASEP:** 1d lattice with occupation numbers n_i=0,1 & particle jumps to empty neighbors with rates

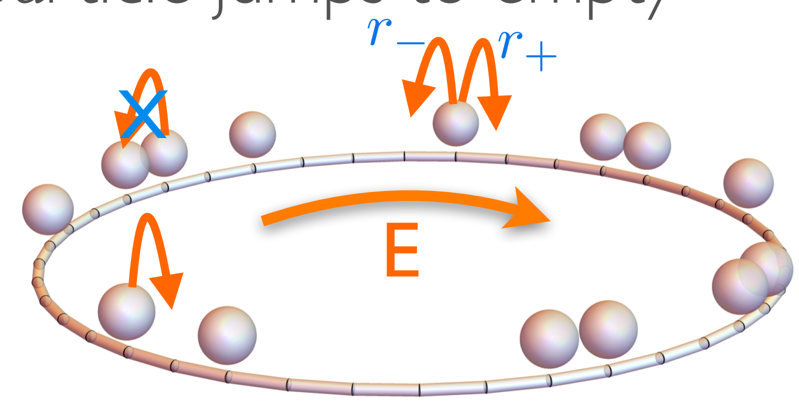
$$r_{\pm} \equiv \frac{1}{2} \exp(\pm E/L)$$



WEAKLY ASYMMETRIC SIMPLE EXCLUSION PROCESS

- **WASEP:** 1d lattice with occupation numbers n_i=0,1 & particle jumps to empty neighbors with rates

$$r_{\pm} \equiv \frac{1}{2}\exp(\pm E/L)$$



- At the **mesoscopic scale**, driven diffusive system

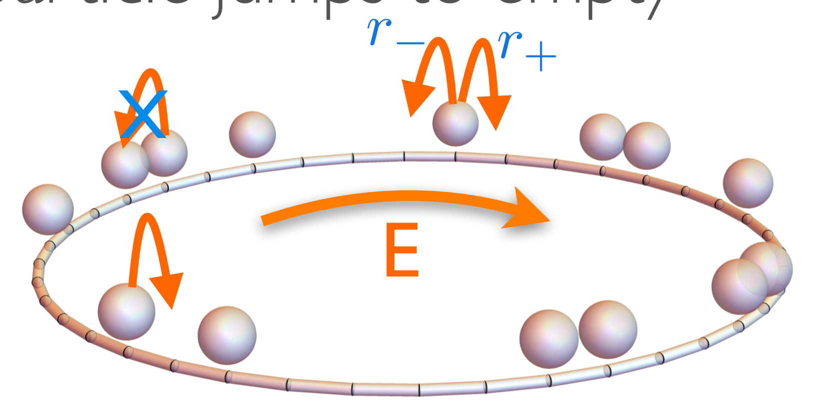
$$\partial_t \rho = -\partial_x \Big(-D(\rho)\partial_x \rho + \sigma(\rho)E + \sqrt{\sigma(\rho)}\xi(x,t)\Big)$$

$$D(\rho) = 1/2 \qquad \sigma(\rho) = \rho(1-\rho)$$

WEAKLY ASYMMETRIC SIMPLE EXCLUSION PROCESS

- **WASEP:** 1d lattice with occupation numbers n_i=0,1 & particle jumps to empty neighbors with rates

$$r_{\pm} \equiv \frac{1}{2} \exp(\pm E/L)$$



- At the **mesoscopic scale**, driven diffusive system

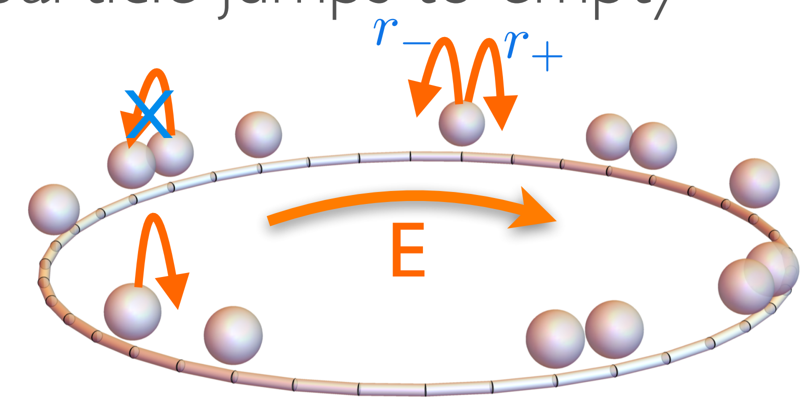
$$\partial_t \rho = -\partial_x \Big(\underbrace{- D(\rho)\partial_x \rho + \sigma(\rho) E + \sqrt{\sigma(\rho)}\xi(x,t)}_{j(x,t)} \Big)$$

$D(\rho) = 1/2 \qquad \sigma(\rho) = \rho(1-\rho)$

WEAKLY ASYMMETRIC SIMPLE EXCLUSION PROCESS

- **WASEP:** 1d lattice with occupation numbers $n_i=0,1$ & particle jumps to empty neighbors with rates

$$r_{\pm} \equiv \frac{1}{2}\exp(\pm E/L)$$



- At the **mesoscopic scale**, driven diffusive system

$$\partial_t \rho = -\partial_x \left(\underbrace{- D(\rho)\partial_x \rho + \sigma(\rho) E + \sqrt{\sigma(\rho)}\xi(x,t)}_{j(x,t)} \right)$$

$$D(\rho) = 1/2 \qquad \sigma(\rho) = \rho(1-\rho)$$

- Question: **probability of a current fluctuation** $q = \tau^{-1} \int_0^\tau dt \int_0^1 dx\, j(x,t)$?

$$P(q) \sim \mathrm{e}^{-\tau L G(q)}$$

$$G(q) = \lim_{\tau\to\infty} \tau^{-1} \min_{\{\rho,j\}_0^\tau} \mathcal{I}_\tau[\rho, j]$$

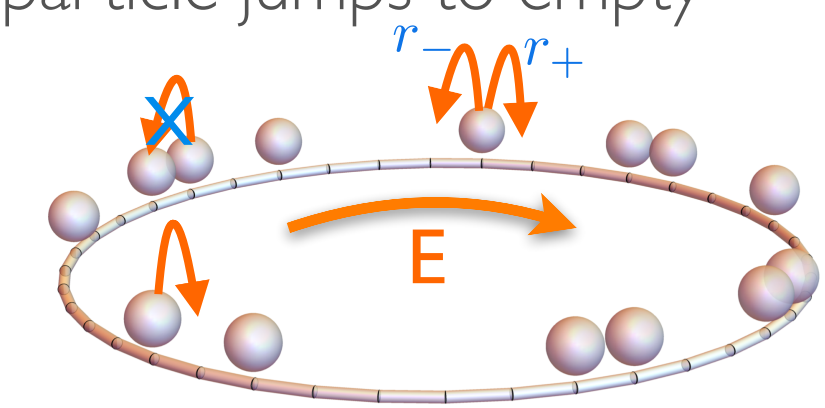
[MFT, Bertini et al Rev. Mod. Phys. 2015]

$$\mathcal{I}_\tau[\rho, j] = \int_0^\tau dt \int_0^1 dx \frac{\left(j + D(\rho)\partial_x\rho - \sigma(\rho)E\right)^2}{2\sigma(\rho)}$$

WEAKLY ASYMMETRIC SIMPLE EXCLUSION PROCESS

- **WASEP:** 1d lattice with occupation numbers n_i=0,1 & particle jumps to empty neighbors with rates

$$r_\pm \equiv \frac{1}{2}\exp(\pm E/L)$$



- At the **mesoscopic scale**, driven diffusive system

$$\partial_t \rho = -\partial_x \left(\boxed{- D(\rho)\partial_x \rho + \sigma(\rho)E + \sqrt{\sigma(\rho)}\xi(x,t)} \right) \quad j(x,t)$$

$$D(\rho) = 1/2 \qquad \sigma(\rho) = \rho(1-\rho)$$

- Question: **probability of a current fluctuation** $q = \tau^{-1}\int_0^\tau dt \int_0^1 dx\, j(x,t)$?

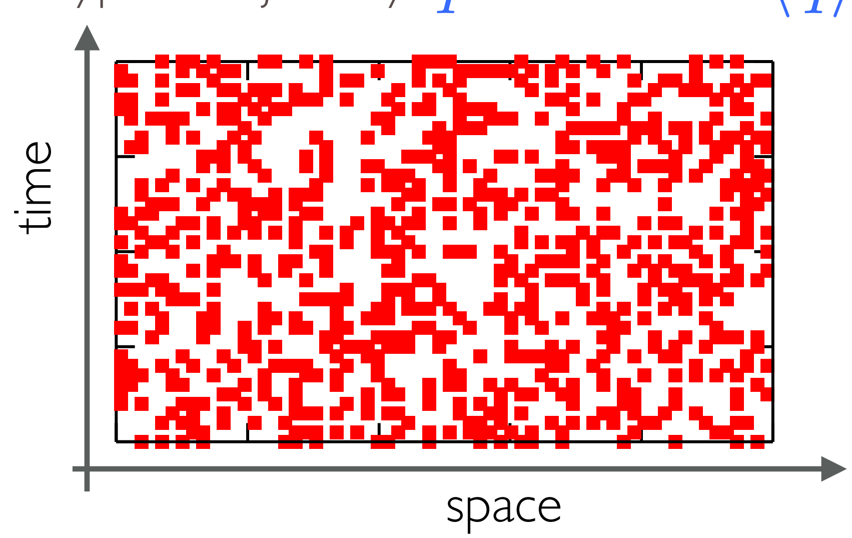
$$P(q) \sim \mathrm{e}^{-\tau L G(q)} \quad \Bigg| \quad G(q) = \lim_{\tau\to\infty} \tau^{-1} \min_{\{\rho,j\}_0^\tau} \mathcal{I}_\tau[\rho,j]$$

[MFT, Bertini et al Rev. Mod. Phys. 2015]

$$\mathcal{I}_\tau[\rho,j] = \int_0^\tau dt \int_0^1 dx \frac{\left(j + D(\rho)\partial_x\rho - \sigma(\rho)E\right)^2}{2\sigma(\rho)}$$

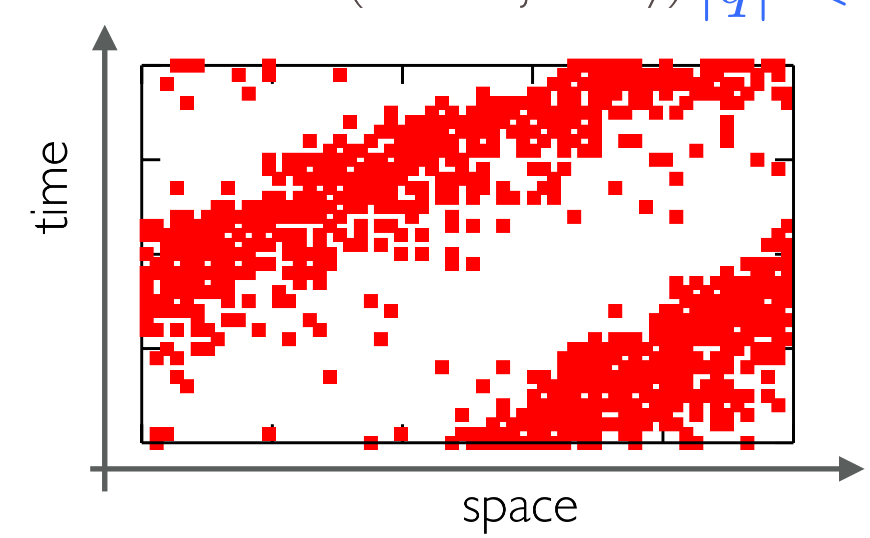
- **Dynamical phase transition** for $|q| < q_c$

Typical trajectory: $q = \sigma E = \langle q \rangle$



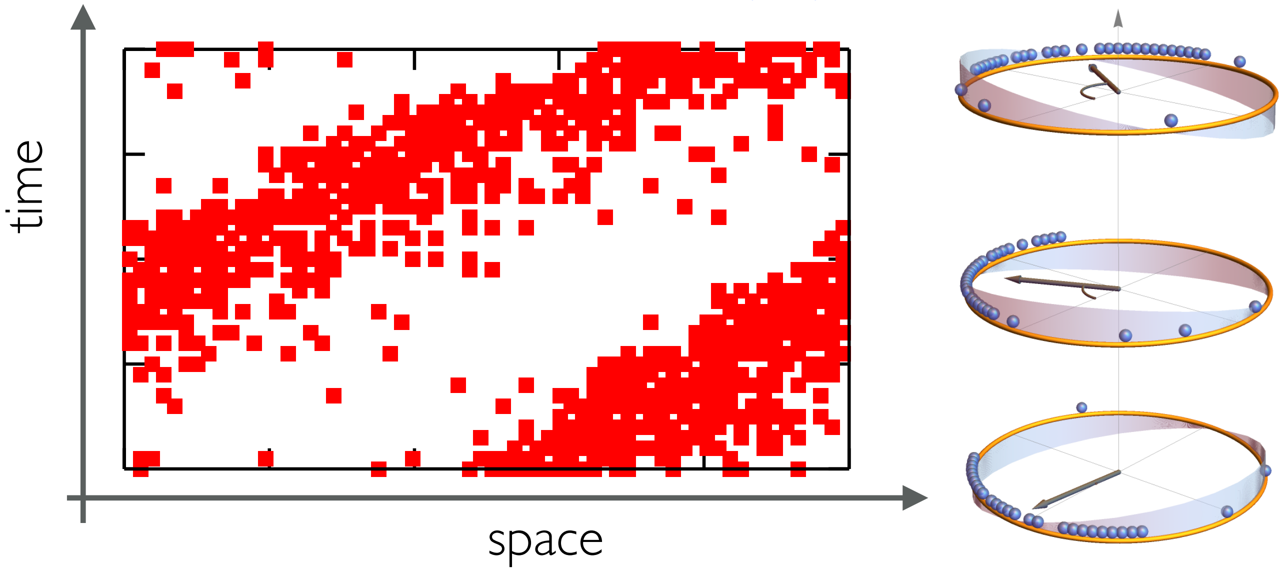
Dynamical phase transition

Small currents (rare trajectory) $|q| < q_c$



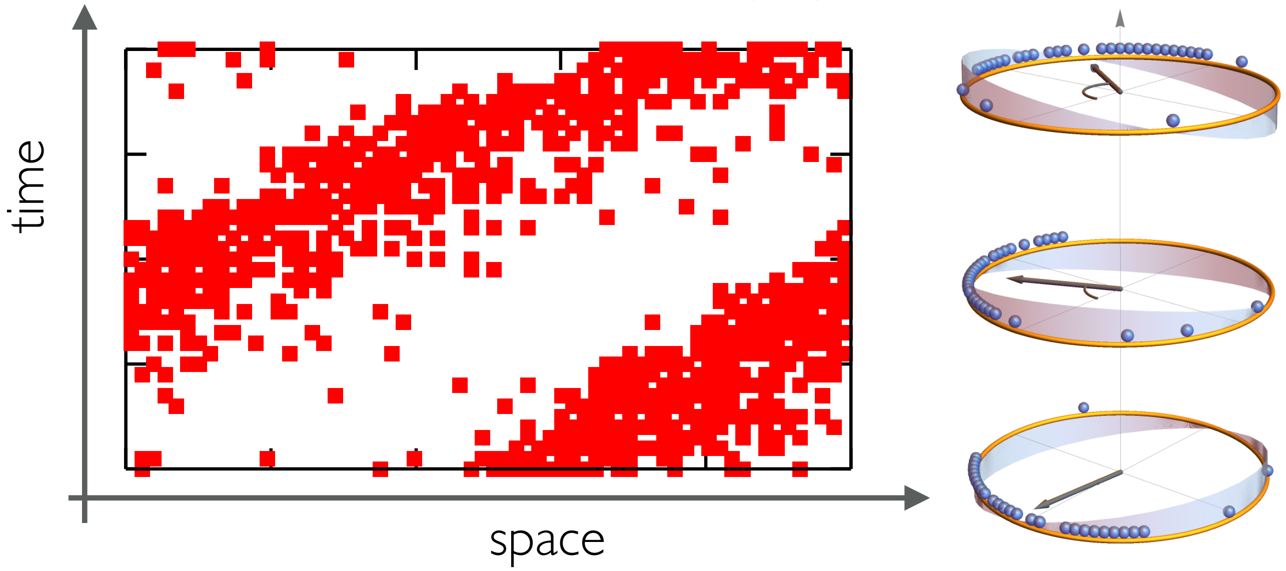
IS THIS A TIME CRYSTAL? [Wilczek, Shapere PRL 2012]

For atypical currents $|q| < q_c$



IS THIS A TIME CRYSTAL? [Wilczek, Shapere PRL 2012]

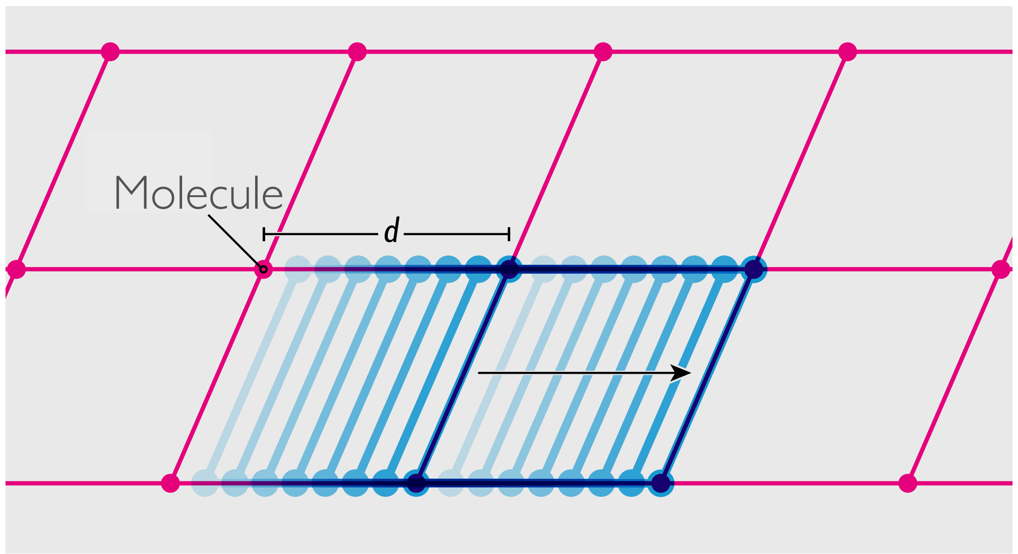
For atypical currents $|q| < q_c$



SPACE CRYSTAL

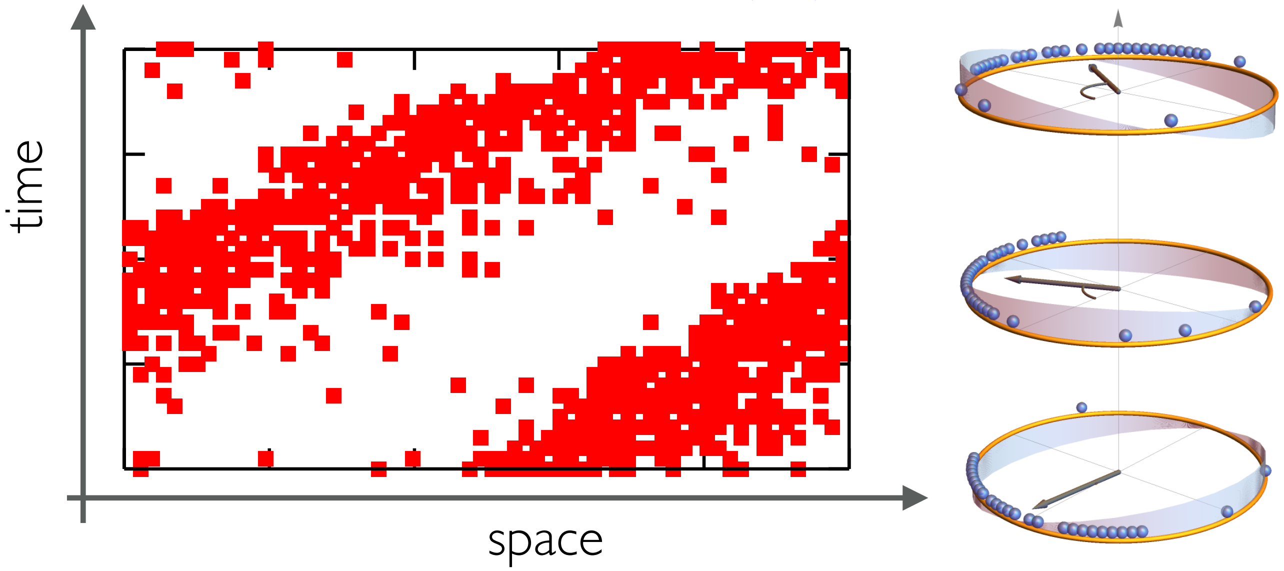
For low temperatures

[Wilczek Sci. Am. 2019]



IS THIS A TIME CRYSTAL? [Wilczek, Shapere PRL 2012]

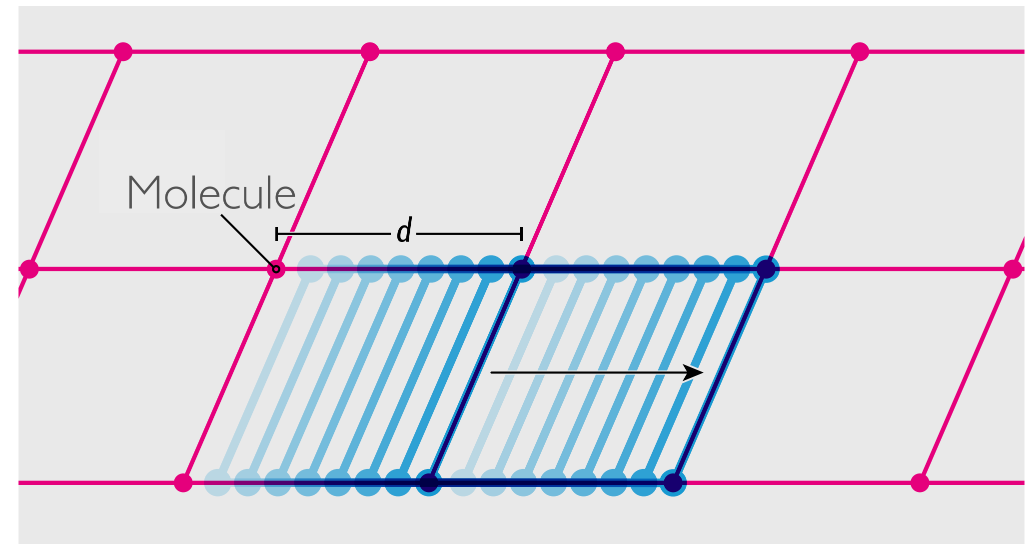
For atypical currents $|q| < q_c$



SPACE CRYSTAL

For low temperatures

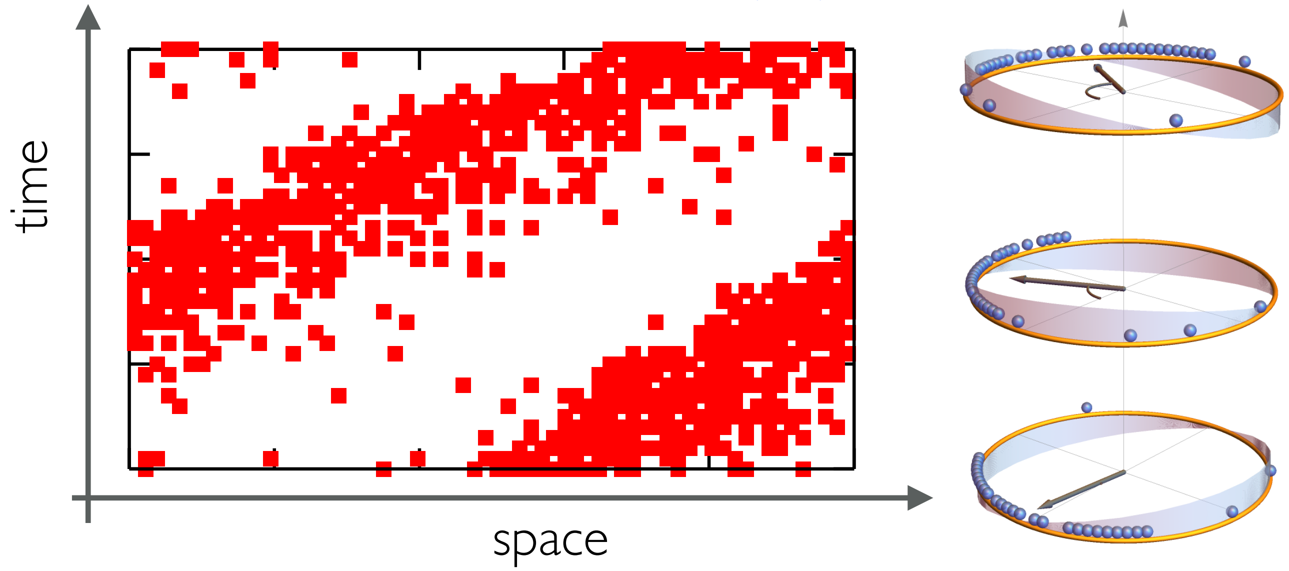
[Wilczek Sci. Am. 2019]



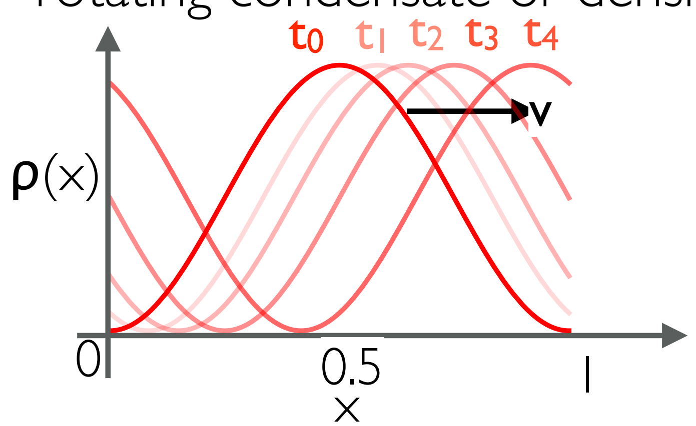
- ☑ **Space-periodic** structure
- ☑ Symmetric for **discrete space translations** d (or nd)
- ☑ **Breaks** spontaneously **continuous space-translation symmetry**

IS THIS A TIME CRYSTAL? [Wilczek, Shapere PRL 2012]

For atypical currents $|q| < q_c$



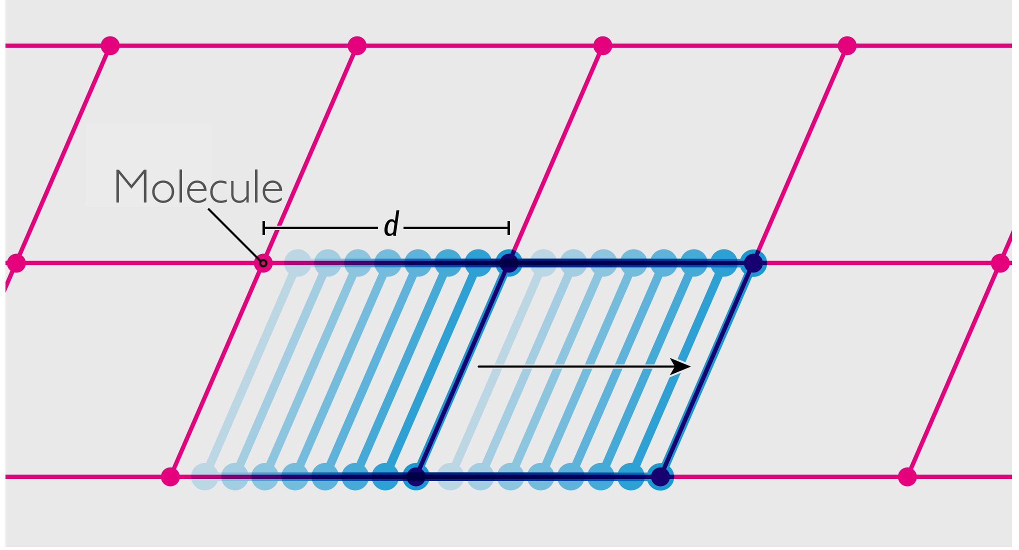
rotating condensate or density wave



SPACE CRYSTAL

For low temperatures

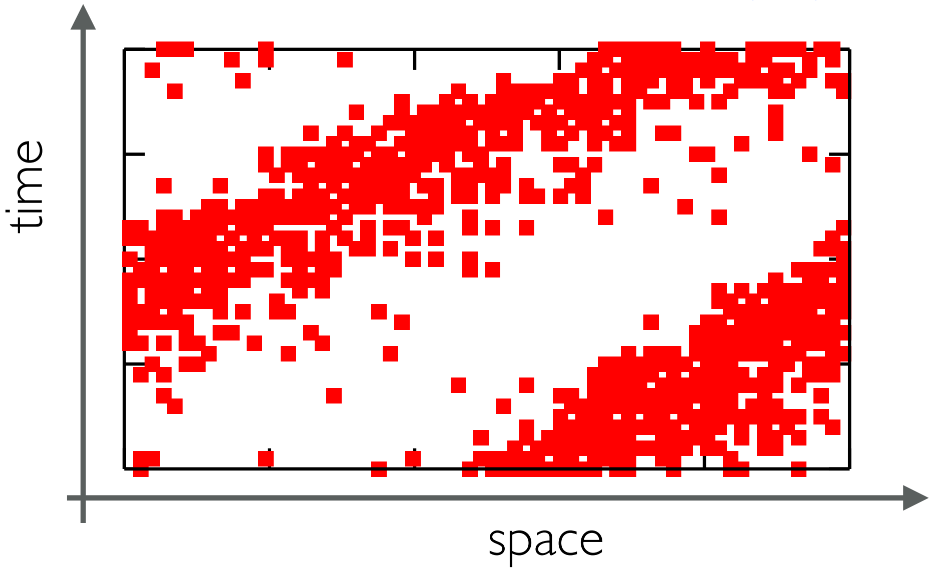
[Wilczek Sci. Am. 2019]

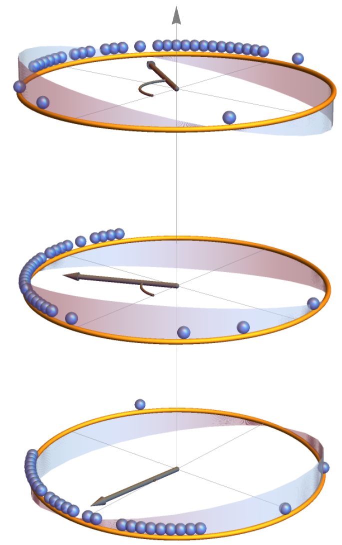


- ☑ **Space-periodic** structure
- ☑ Symmetric for **discrete space translations** d (or nd)
- ☑ **Breaks** spontaneously **continuous space-translation symmetry**

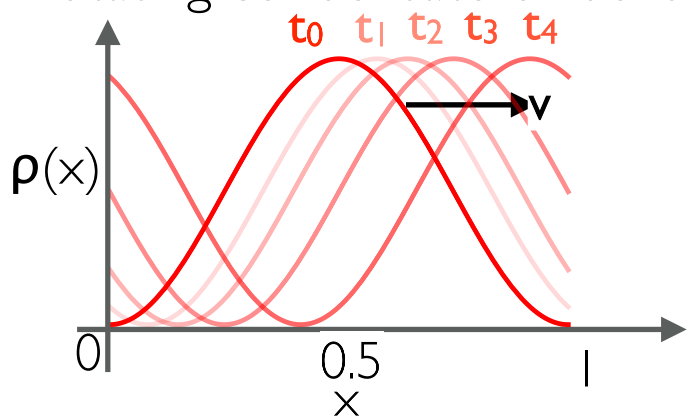
IS THIS A TIME CRYSTAL? [Wilczek, Shapere PRL 2012]

For atypical currents $|q| < q_c$





rotating condensate or density wave



☑ **Time-periodic** density wave

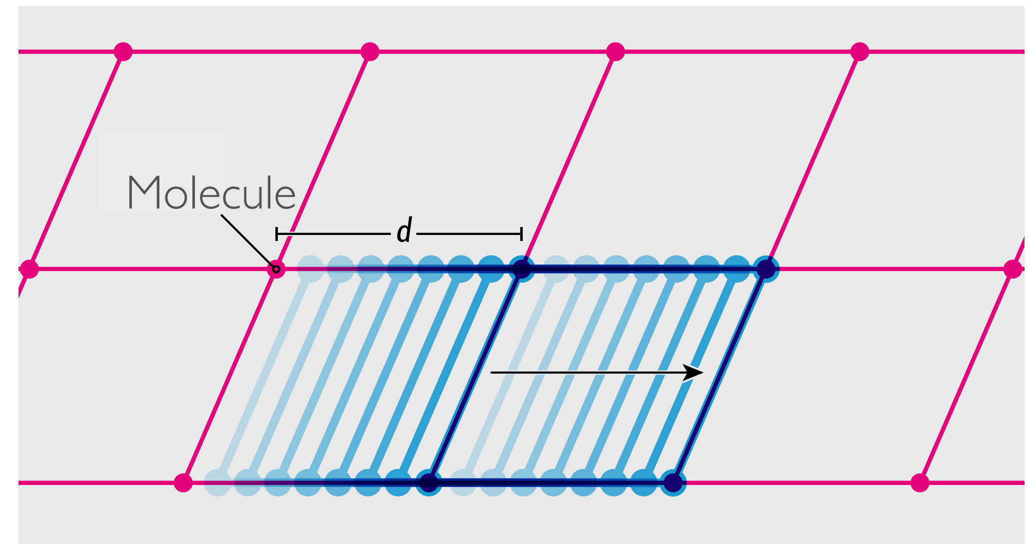
☑ Symmetric for **discrete time translations** T (or nT)

☑ **Breaks** spontaneously **continuous time-translation symmetry**

SPACE CRYSTAL

For low temperatures

[Wilczek Sci. Am. 2019]



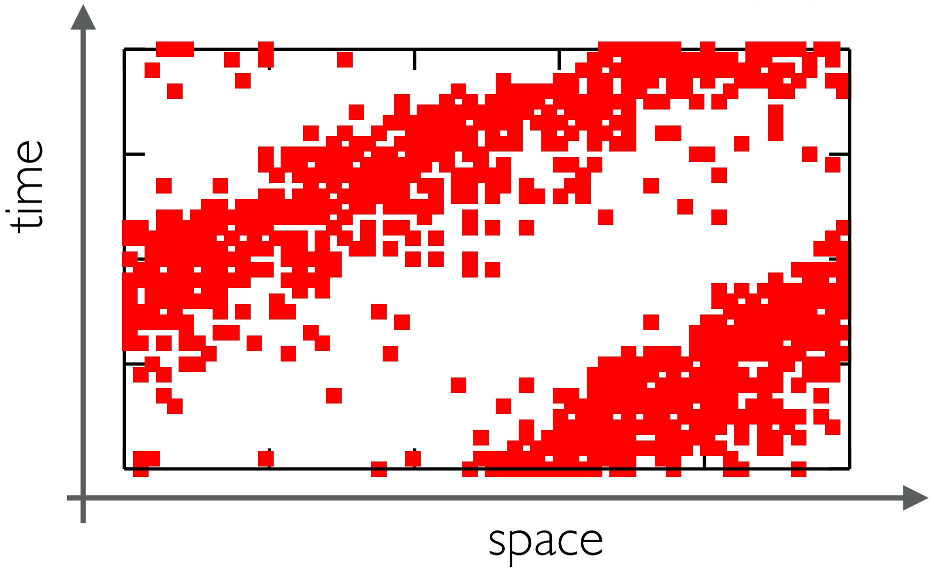
☑ **Space-periodic** structure

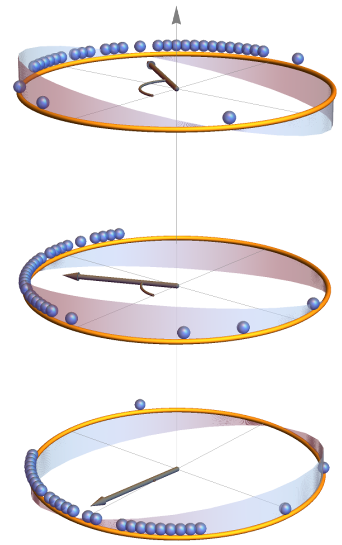
☑ Symmetric for **discrete space translations** d (or nd)

☑ **Breaks** spontaneously **continuous space-translation symmetry**

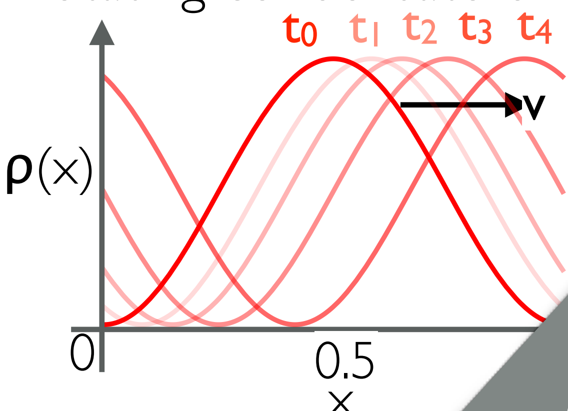
IS THIS A TIME CRYSTAL? [Wilczek, Shapere PRL 2012]

For atypical currents $|q| < q_c$





rotating condensate or density wave



☑ Time-periodic

☑ Symmetri… …e translations T (or nT)

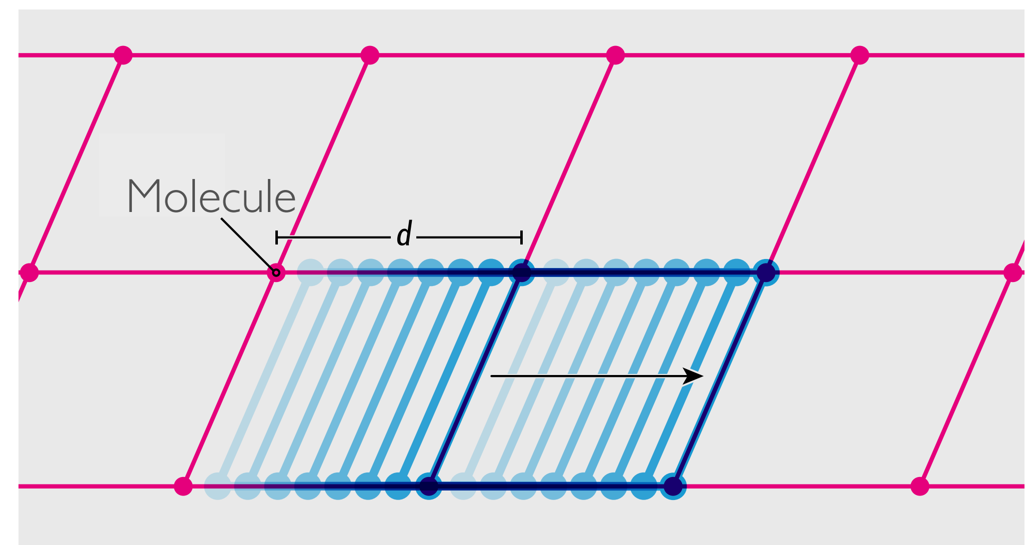
☑ Breaks … …usly continuous time-translation …etry

But it's a rare fluctuation!

SPACE CRYSTAL

For low temperatures

[Wilczek Sci. Am. 2019]



☑ **Space-periodic** structure

☑ Symmetric for **discrete space translations** d (or nd)

☑ **Breaks** spontaneously **continuous space-translation symmetry**

CURRENT FLUCTUATIONS FROM MICROSCOPICS

- **Quantum hamiltonian formalism** for the **master equation** $|P(t)\rangle = \sum_C P(C,t)\,|C\rangle$

$$\partial_t |P(t)\rangle = \mathbb{W}|P(t)\rangle$$

- **Markov generator** $\mathbb{W} = \sum_{C,C'\neq C} W_{C\to C'}\,|C'\rangle\,\langle C| - \sum_C R(C)\,|C\rangle\,\langle C|$

Exit rate

$$R(C) = \sum_{C'} W_{C\to C'}$$

CURRENT FLUCTUATIONS FROM MICROSCOPICS

- **Quantum hamiltonian formalism** for the **master equation** $|P(t)\rangle = \sum_C P(C,t)\,|C\rangle$

$$\partial_t |P(t)\rangle = \mathbb{W}|P(t)\rangle$$

- **Markov generator** $\mathbb{W} = \sum_{C,C'\neq C} W_{C\to C'}\,|C'\rangle\,\langle C| - \sum_C R(C)\,|C\rangle\,\langle C|$

Exit rate

$$R(C) = \sum_{C'} W_{C\to C'}$$

- **Ensemble of trajectories** conditioned on the **current** $Q = \sum_i q_{C_i C_{i-1}}$

$$P_t(Q) \sim \mathrm{e}^{-tG(Q/t)}$$

[Ruelle, Gartner&Ellis, Lebowitz&Spohn, Lecomte et al, and many others]

- **Dynamical partition function:**

$$Z_t(\lambda) = \sum_Q P_t(Q)\mathrm{e}^{\lambda Q} \sim \mathrm{e}^{t\theta(\lambda)}$$

- **Dynamical free energy** $\theta(\lambda) = -\min_q[G(q) - \lambda q]$ **largest eigenvalue of biased generator**

$$\mathbb{W}^\lambda = \sum_{C,C'\neq C} \mathrm{e}^{\lambda q_{C'C}} W_{C\to C'}\,|C'\rangle\,\langle C| - \sum_C R_C|C\rangle\langle C|$$

Biased jumps

No conservation of probability

- **Spectrum of** $\mathbb{W}^\lambda$:

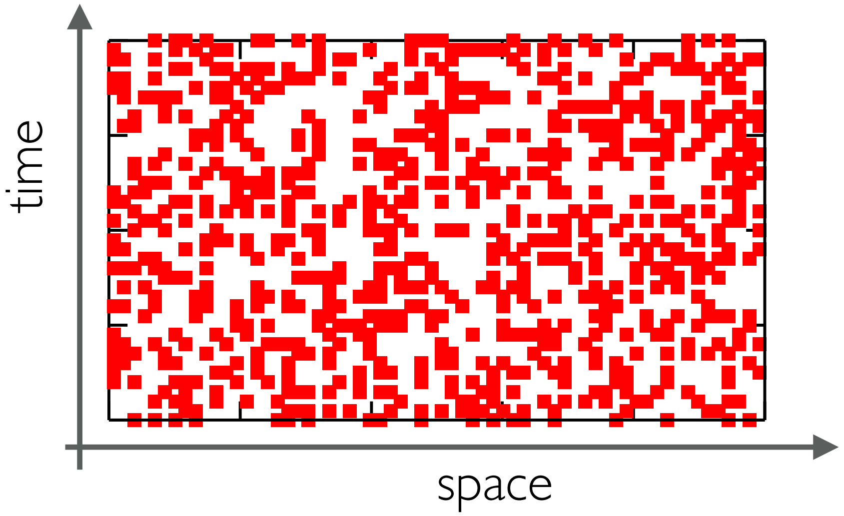
$$\mathbb{W}^\lambda\,|R_i^\lambda\rangle = \theta_i(\lambda)\,|R_i^\lambda\rangle \qquad \langle L_i^\lambda|\,\mathbb{W}^\lambda = \theta_i(\lambda)\,\langle L_i^\lambda| \qquad \theta(\lambda) = \theta_0(\lambda)$$

SPECTRAL SIGNATURES OF THE DPT

[Hurtado-Gutierrez et al, PRE **108**, 014107 (2023)]

L=24, ρ_0=1/3, E=10

Typical trajectory: $q = \sigma E = \langle q \rangle$



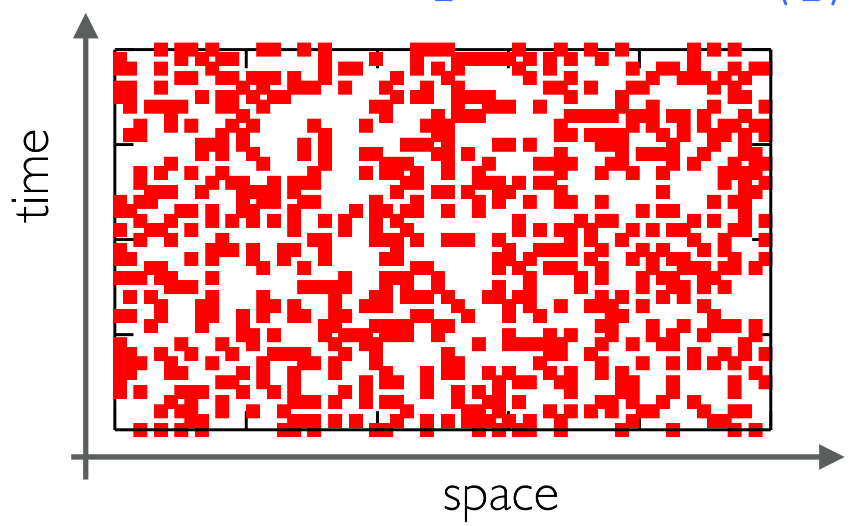
$\mathbb{W}^\lambda$ spectrum changes radically across the DPT

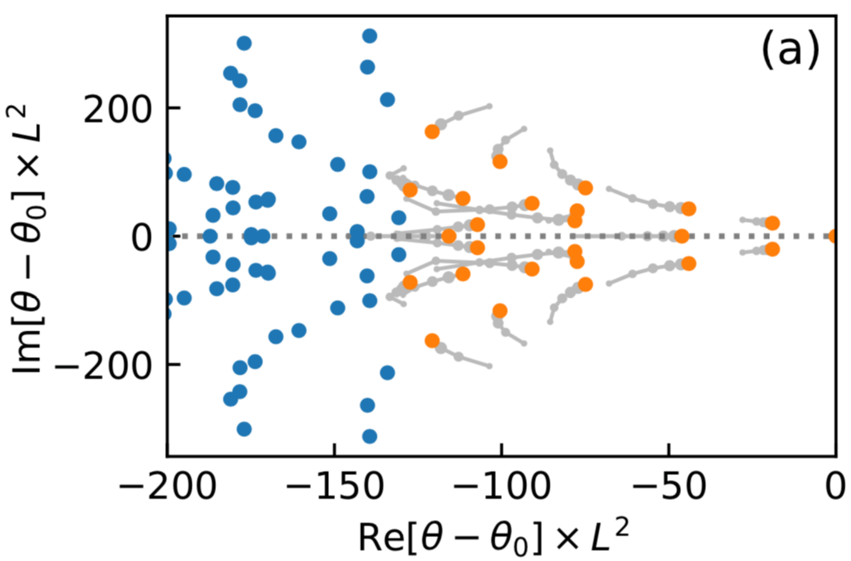
SPECTRAL SIGNATURES OF THE DPT

[Hurtado-Gutierrez et al, PRE **108**, 014107 (2023)]

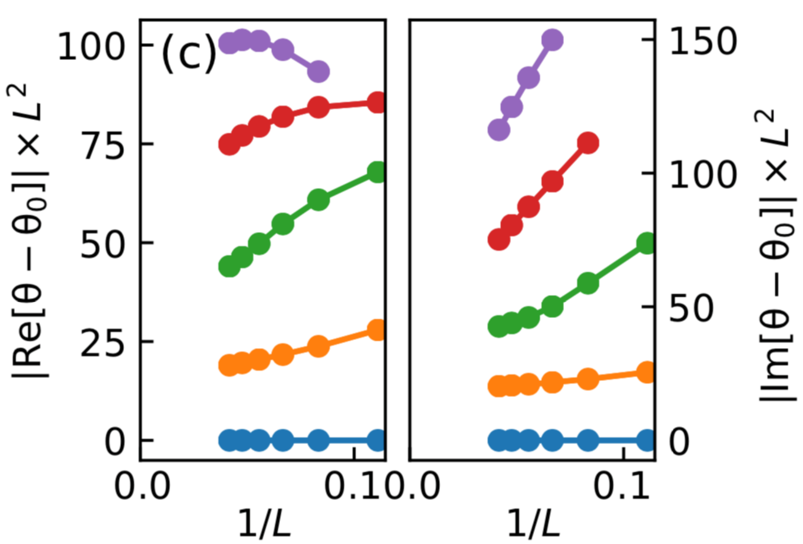
L=24, ρ_0=1/3, E=10

Typical trajectory: $q = \sigma E = \langle q \rangle$





$\mathbb{W}^{\lambda}$ spectrum changes radically across the DPT

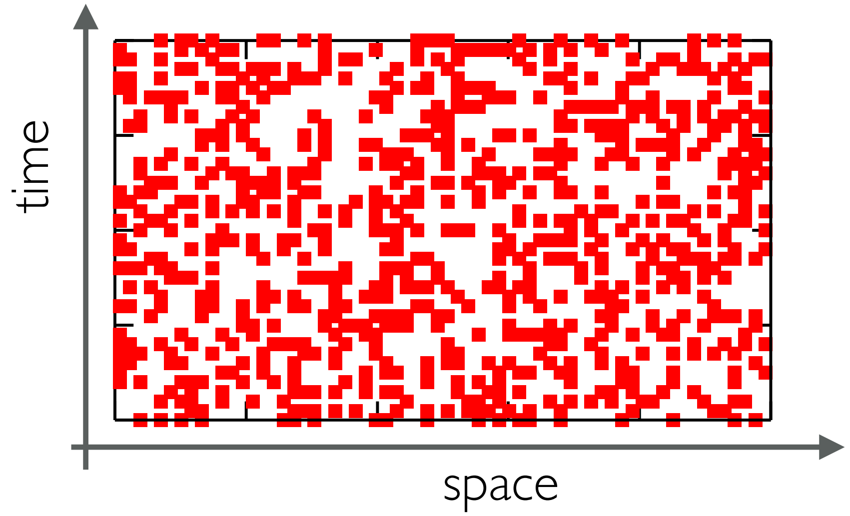


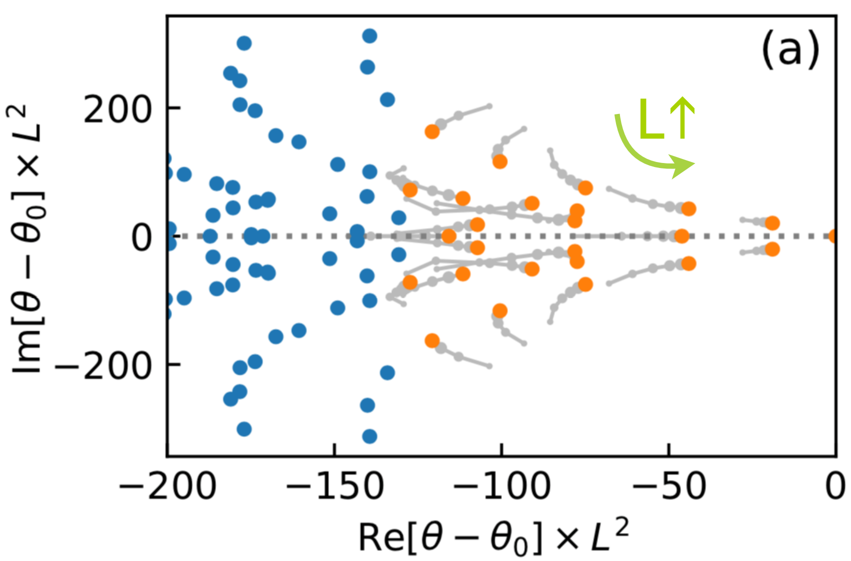
SPECTRAL SIGNATURES OF THE DPT

[Hurtado-Gutierrez et al, PRE **108**, 014107 (2023)]

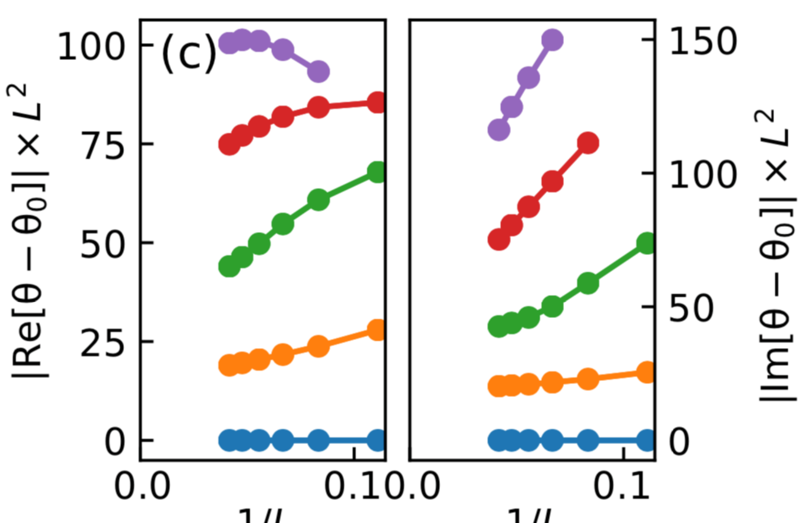
L=24, ρ_0=1/3, E=10

Typical trajectory: $q = \sigma E = \langle q \rangle$





$\mathbb{W}^\lambda$ spectrum changes radically across the DPT

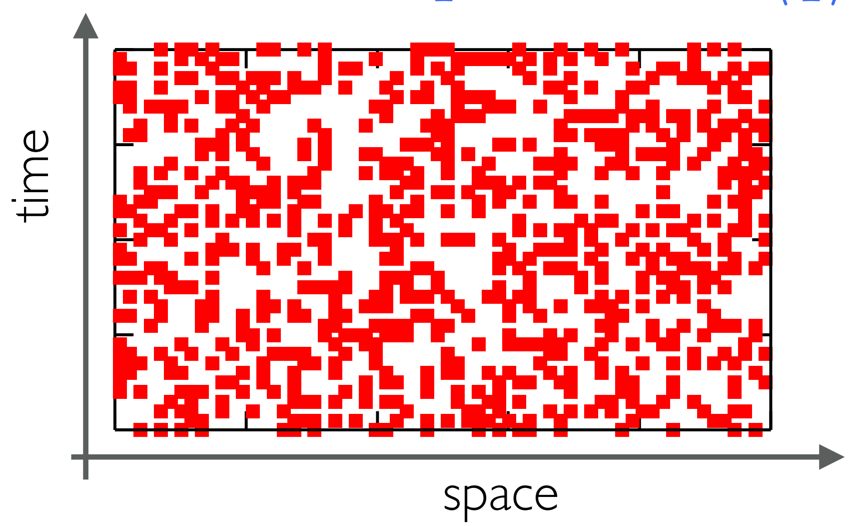


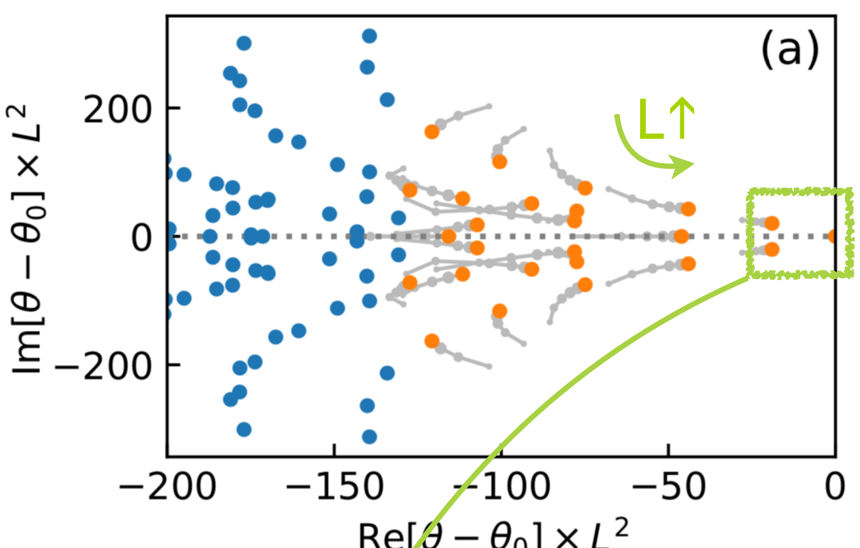
SPECTRAL SIGNATURES OF THE DPT

[Hurtado-Gutierrez et al, PRE **108**, 014107 (2023)]

L=24, ρ_0=1/3, E=10

Typical trajectory: $q = \sigma E = \langle q \rangle$



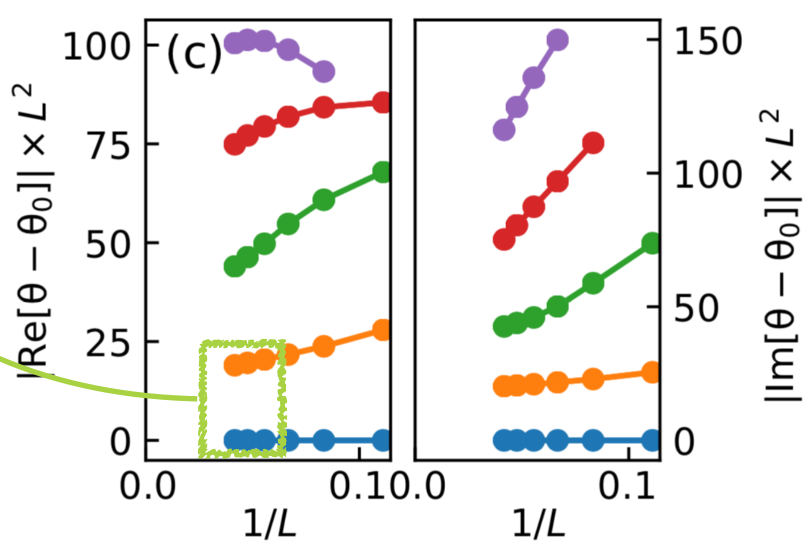


$\mathbb{W}^\lambda$ spectrum changes radically across the DPT

$\mathbb{W}^\lambda$ is gapped

$\Downarrow$

unique steady state $|P_{\rm st}\rangle$

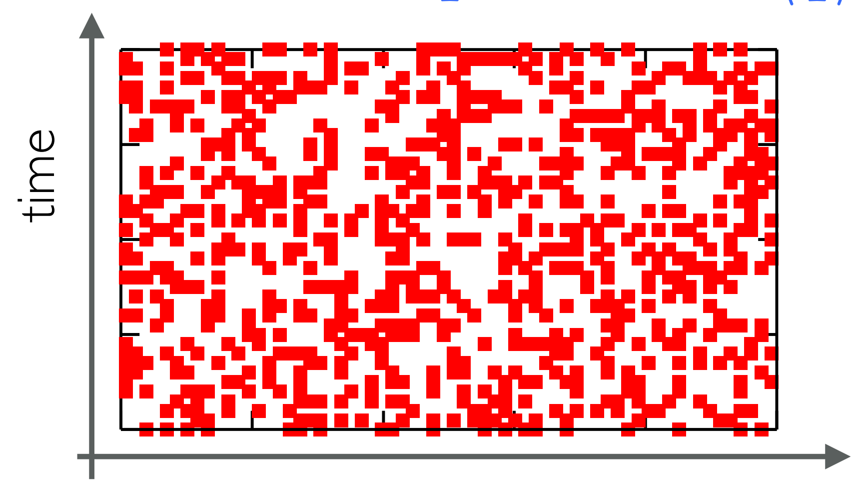


SPECTRAL SIGNATURES OF THE DPT

[Hurtado-Gutierrez et al, PRE **108**, 014107 (2023)]

L=24, ρ_0=1/3, E=10

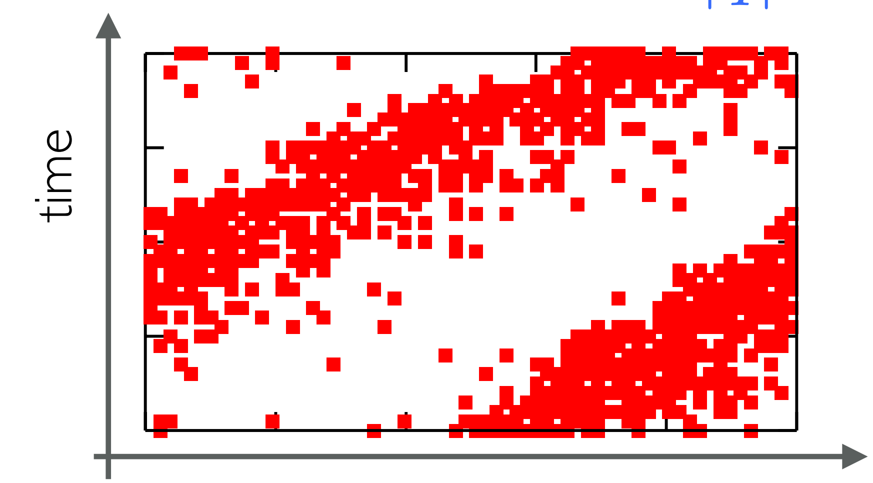
Typical trajectory: $q = \sigma E = \langle q \rangle$

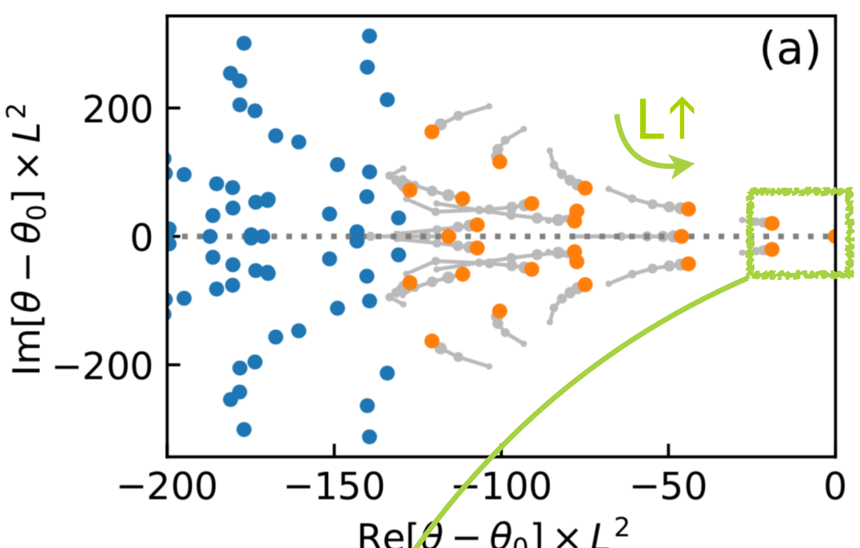


Dynamical phase transition

$\lambda_c^- < \lambda < \lambda_c^+$

Small currents (rare trajectory) $|q| < q_c$



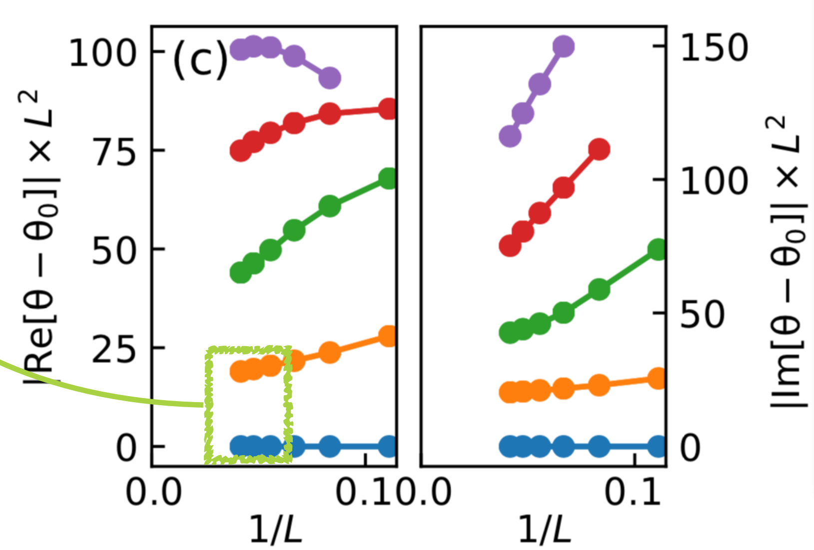


$\mathbb{W}^\lambda$ spectrum changes radically across the DPT

$\mathbb{W}^\lambda$ is gapped

$\Downarrow$

unique steady state $|P_{st}\rangle$

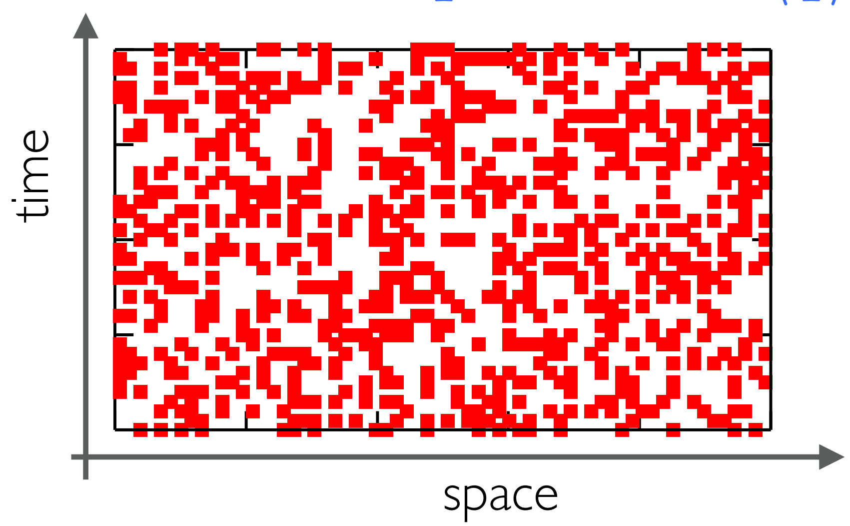


SPECTRAL SIGNATURES OF THE DPT

[Hurtado-Gutierrez et al, PRE **108**, 014107 (2023)]

L=24, ρ_0=1/3, E=10

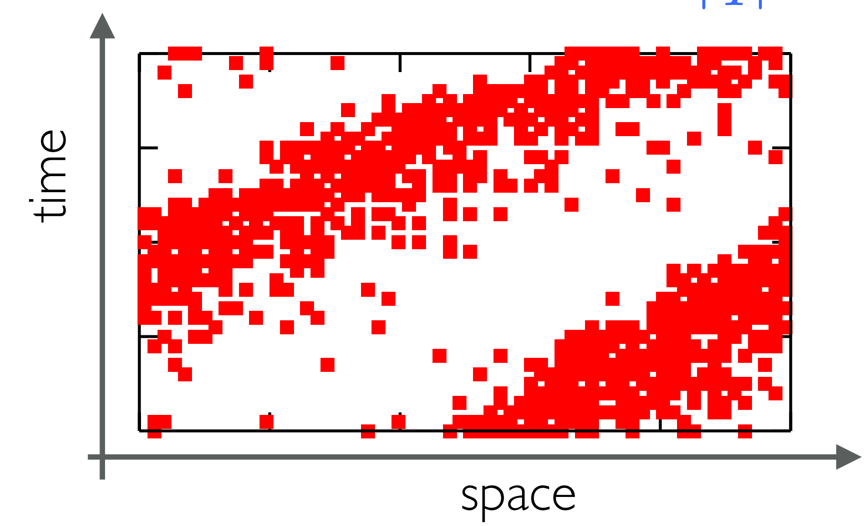
Typical trajectory: $q = \sigma E = \langle q \rangle$

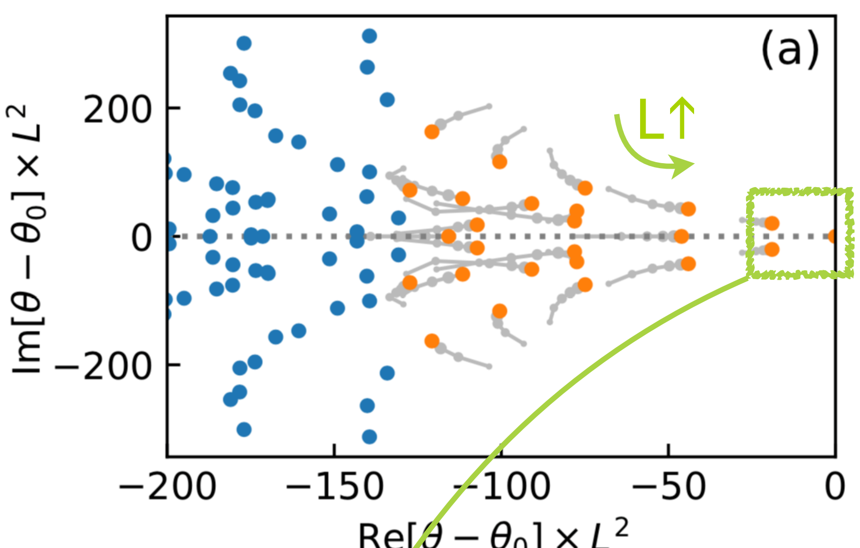


Dynamical phase transition

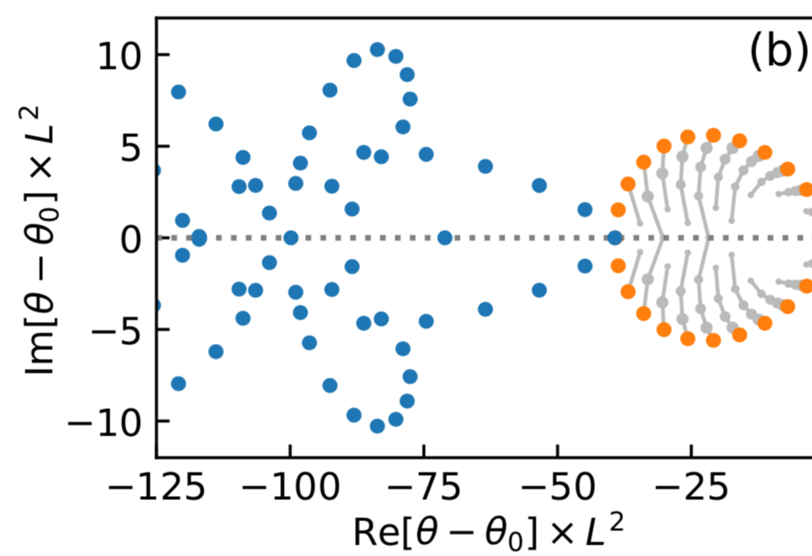
$\lambda_c^- < \lambda < \lambda_c^+$

Small currents (rare trajectory) $|q| < q_c$





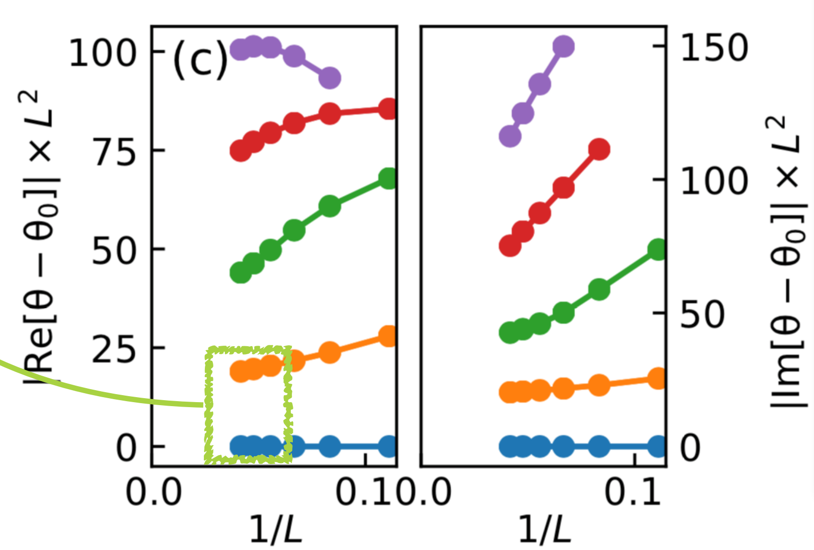
$\mathbb{W}^\lambda$ spectrum changes radically across the DPT

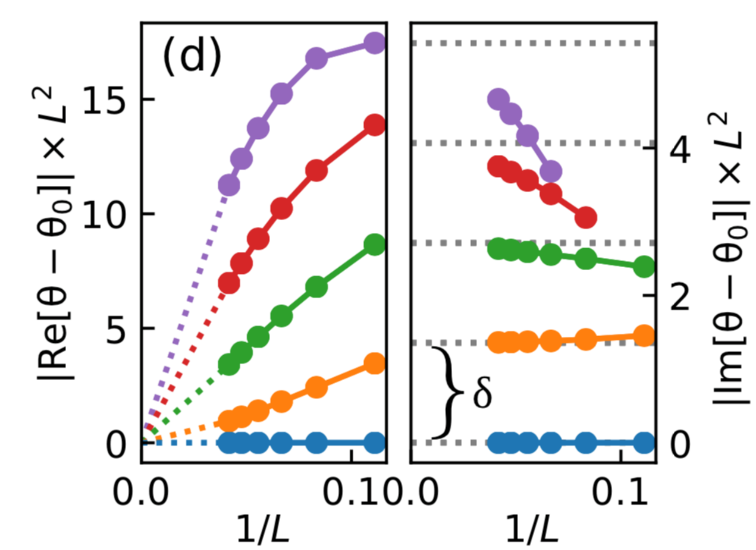


$\mathbb{W}^\lambda$ is gapped

$\Downarrow$

unique steady state $|P_{\rm st}\rangle$



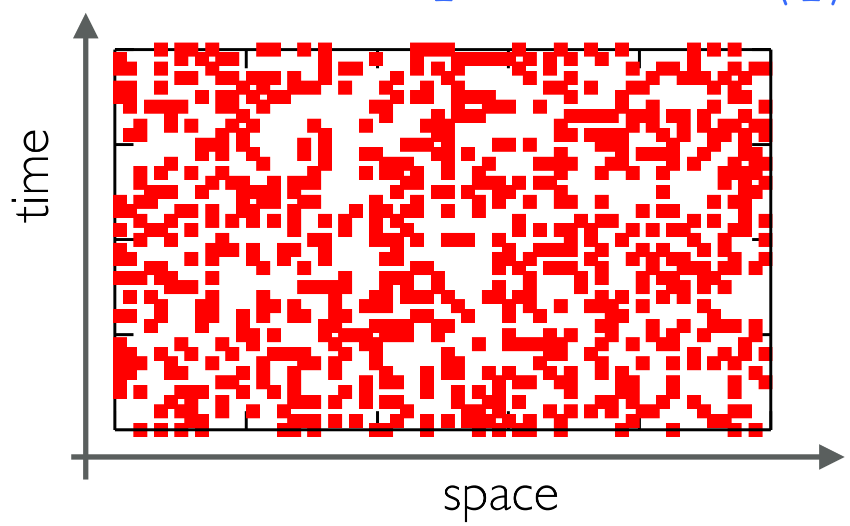


SPECTRAL SIGNATURES OF THE DPT

[Hurtado-Gutierrez et al, PRE **108**, 014107 (2023)]

L=24, ρ_0=1/3, E=10

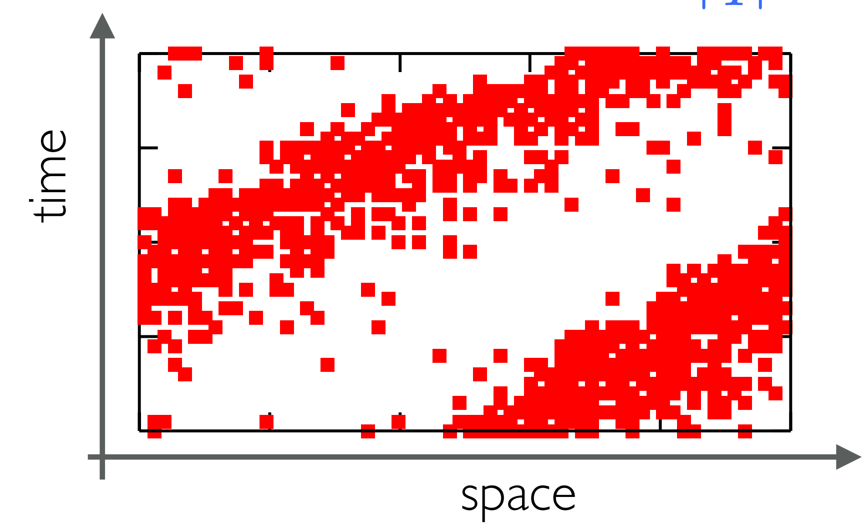
Typical trajectory: $q = \sigma E = \langle q \rangle$

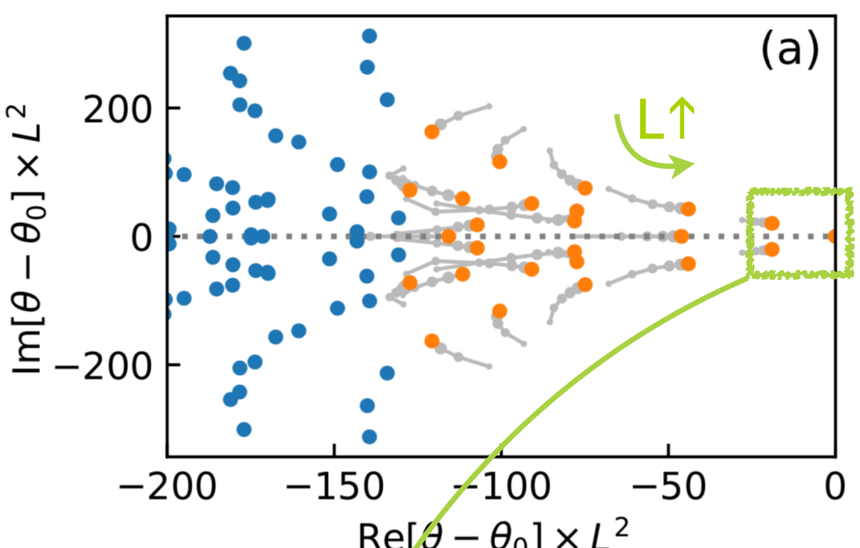


Dynamical phase transition

$\lambda_c^- < \lambda < \lambda_c^+$

Small currents (rare trajectory) $|q| < q_c$



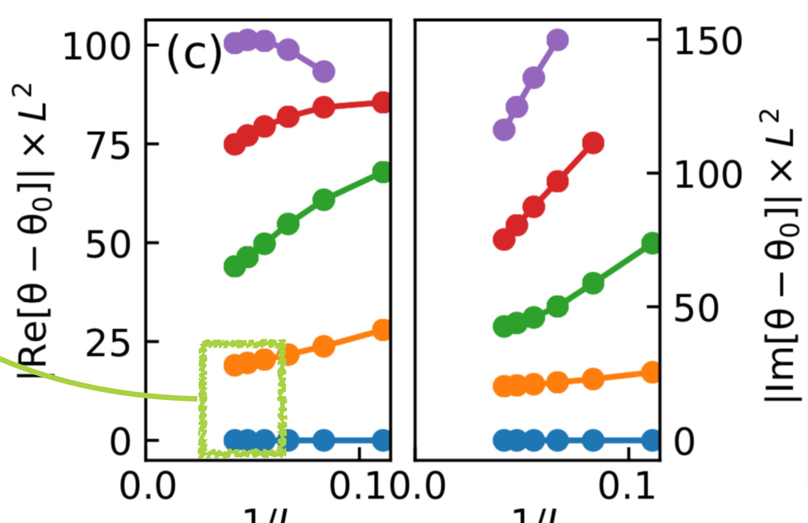


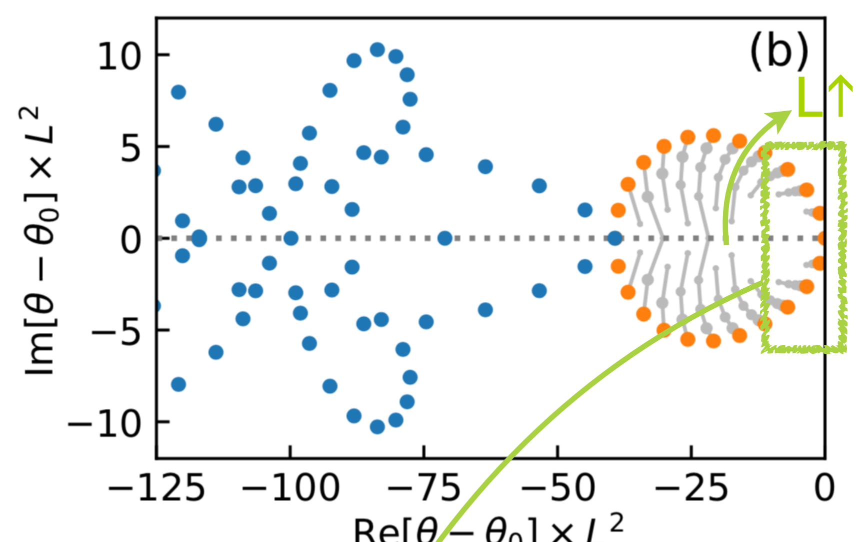
$\mathbb{W}^\lambda$ spectrum changes radically across the DPT

$\mathbb{W}^\lambda$ is gapped

⇓

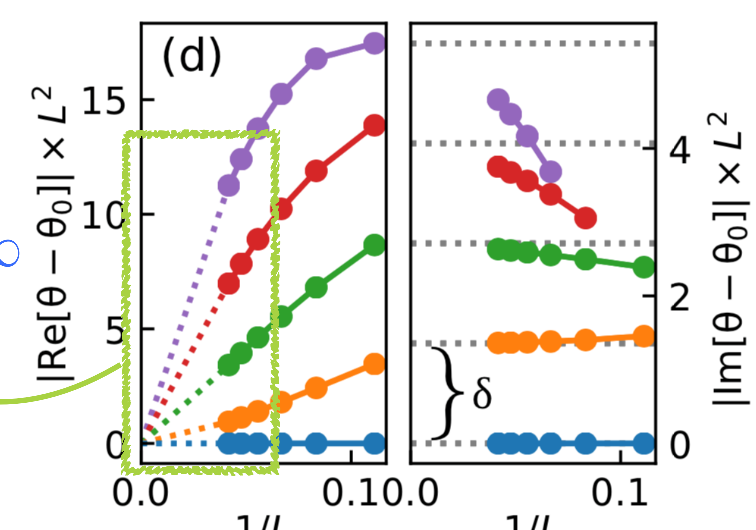
unique steady state $|P_{\rm st}\rangle$





Vanishing spectral gap of $\mathbb{W}^\lambda$ as $L \to \infty$

Power-law decay as L↑

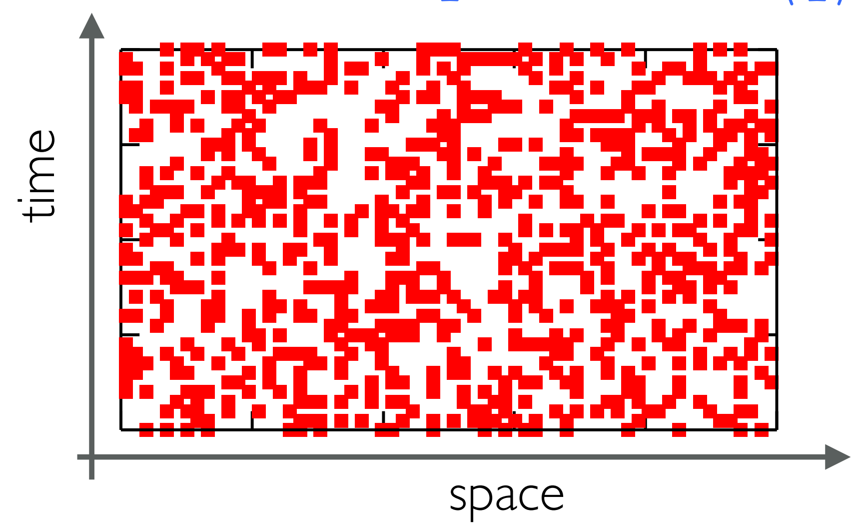


SPECTRAL SIGNATURES OF THE DPT

[Hurtado-Gutierrez et al, PRE **108**, 014107 (2023)]

$L=24$, $\rho_0=1/3$, $E=10$

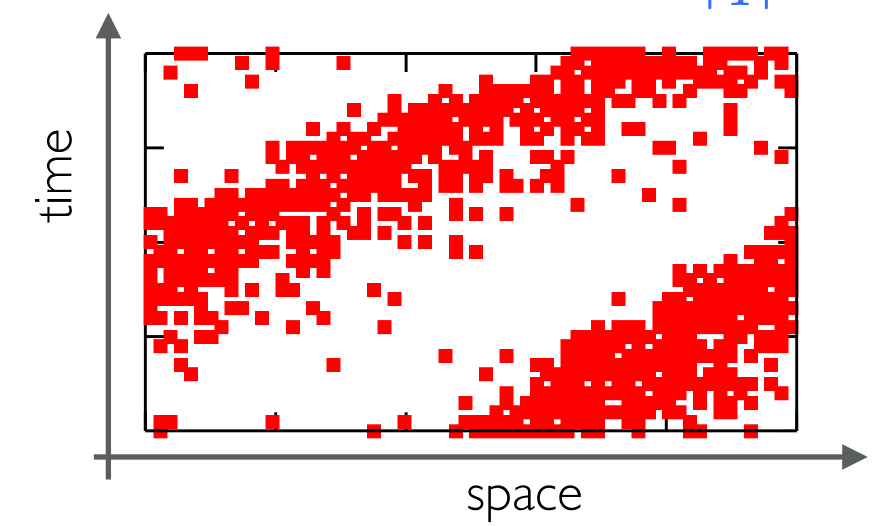
Typical trajectory: $q = \sigma E = \langle q \rangle$

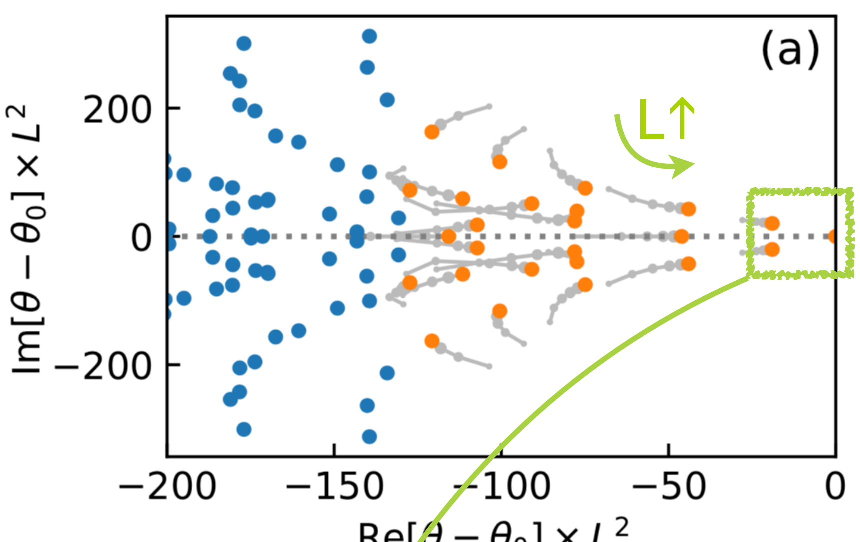


Dynamical phase transition

$\lambda_c^- < \lambda < \lambda_c^+$

Small currents (rare trajectory) $|q| < q_c$



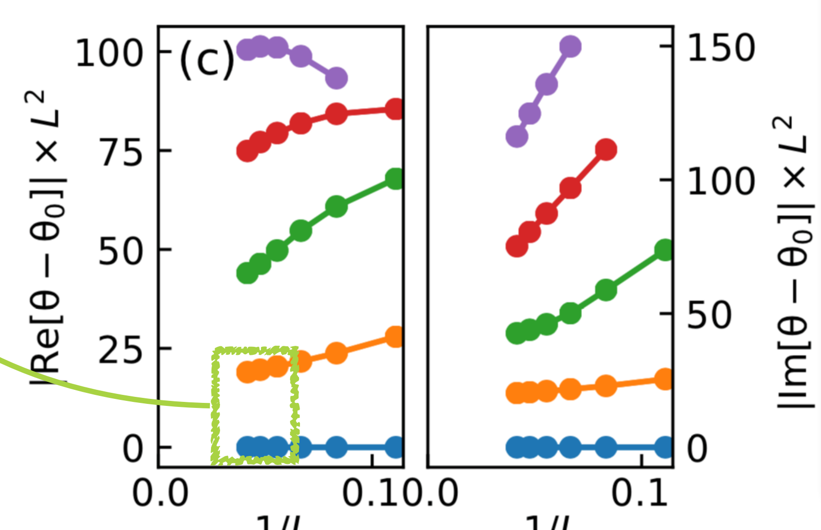


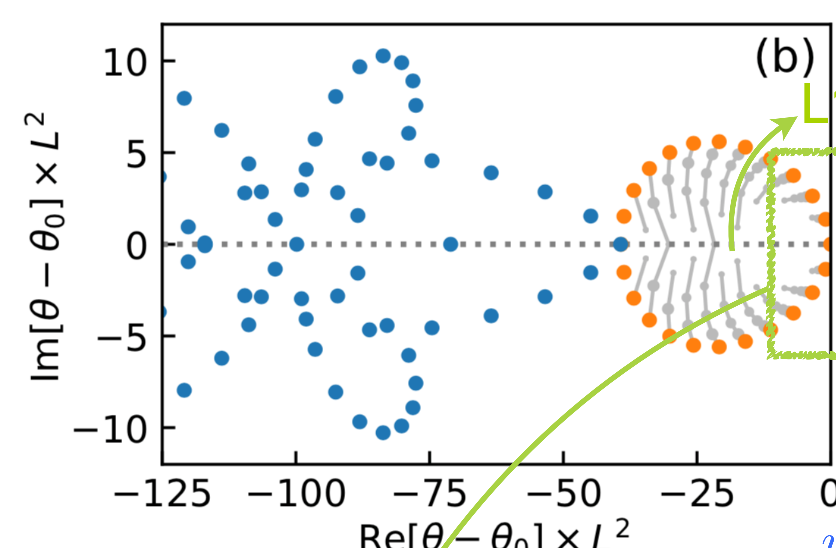
$\mathbb{W}^\lambda$ spectrum changes radically across the DPT

$\mathbb{W}^\lambda$ is gapped

⇓

unique steady state $|P_{\rm st}\rangle$





L↑ $|P(t)\rangle = |P(t+T)\rangle$

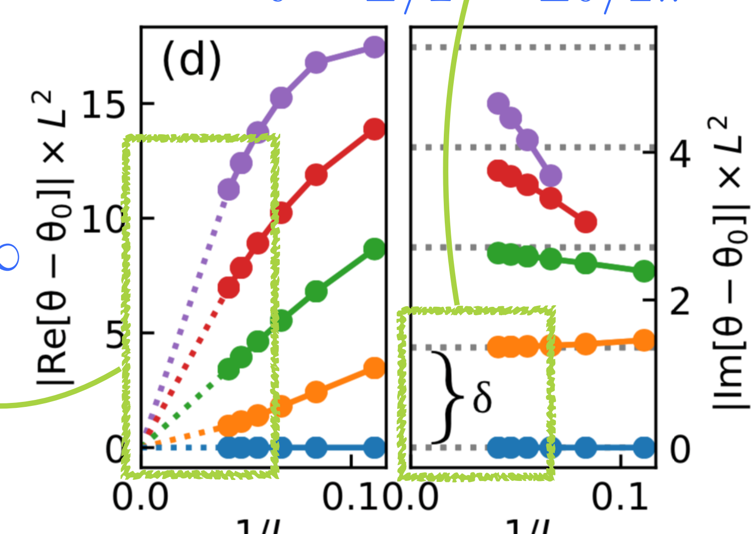
$T = \frac{2\pi}{\delta}$

Band structure in **imaginary part** of gap-closing eigenvals

$v = L/T = L\delta/2\pi$

Vanishing spectral gap of $\mathbb{W}^\lambda$ as $L \to \infty$

Power-law decay as L↑



MAKING RARE EVENTS TYPICAL

- We have a **time crystal for rare current fluctuations** … WHO CARES?
- $\mathbb{W}^\lambda$ generates **atypical trajectories** but $\sum_C \langle C| \, \mathbb{W}^\lambda \neq 0$ (non-physical!)

MAKING RARE EVENTS TYPICAL

- We have a **time crystal for rare current fluctuations** … WHO CARES?
- $\mathbb{W}^\lambda$ generates **atypical trajectories** but $\sum_C \langle C| \mathbb{W}^\lambda \neq 0$ **(non-physical!)**
- We can **make rare events TYPICAL** using **Doob's transform:** [Jack & Sollich 2010, Popkov et al 2010, Chetrite & Touchette 2015]

$$\mathbb{W}_{\mathrm{D}}^\lambda \equiv \mathbb{L}_0 \mathbb{W}^\lambda \mathbb{L}_0^{-1} - \theta_0(\lambda) \qquad \text{with} \quad (\mathbb{L}_0)_{ij} = (\langle L_0^\lambda|)_i \delta_{ij}$$

- $\mathbb{W}_{\mathrm{D}}^\lambda$ now **conserves probability (physical!)**

$$\sum_C \langle C| \mathbb{W}_{\mathrm{D}}^\lambda = 0$$

- $\mathbb{W}_{\mathrm{D}}^\lambda$ **spectrum** is simply related to that of $\mathbb{W}^\lambda$

$$\theta_i^D(\lambda) = \theta_i(\lambda) - \theta_0(\lambda) \qquad |R_{i,\mathrm{D}}^\lambda\rangle = \mathbb{L}_0 |R_i^\lambda\rangle \qquad \langle L_{i,\mathrm{D}}^\lambda| = \langle L_i^\lambda| \mathbb{L}_0^{-1}$$

MAKING RARE EVENTS TYPICAL

- We have a **time crystal for rare current fluctuations** … **WHO CARES?**
- $\mathbb{W}^\lambda$ generates **atypical trajectories** but $\sum_C \langle C| \mathbb{W}^\lambda \neq 0$ **(non-physical!)**
- We can **make rare events TYPICAL** using **Doob's transform:**

[Jack & Sollich 2010, Popkov et al 2010, Chetrite & Touchette 2015]

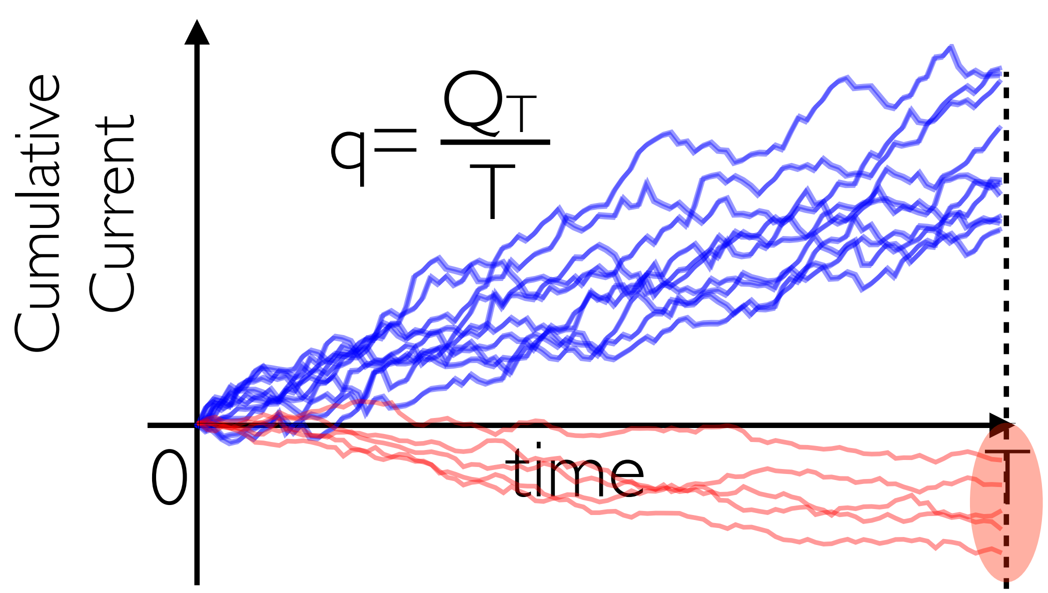
$$\mathbb{W}_{\mathrm{D}}^\lambda \equiv \mathbb{L}_0 \mathbb{W}^\lambda \mathbb{L}_0^{-1} - \theta_0(\lambda) \qquad \text{with} \quad (\mathbb{L}_0)_{ij} = (\langle L_0^\lambda|)_i \delta_{ij}$$

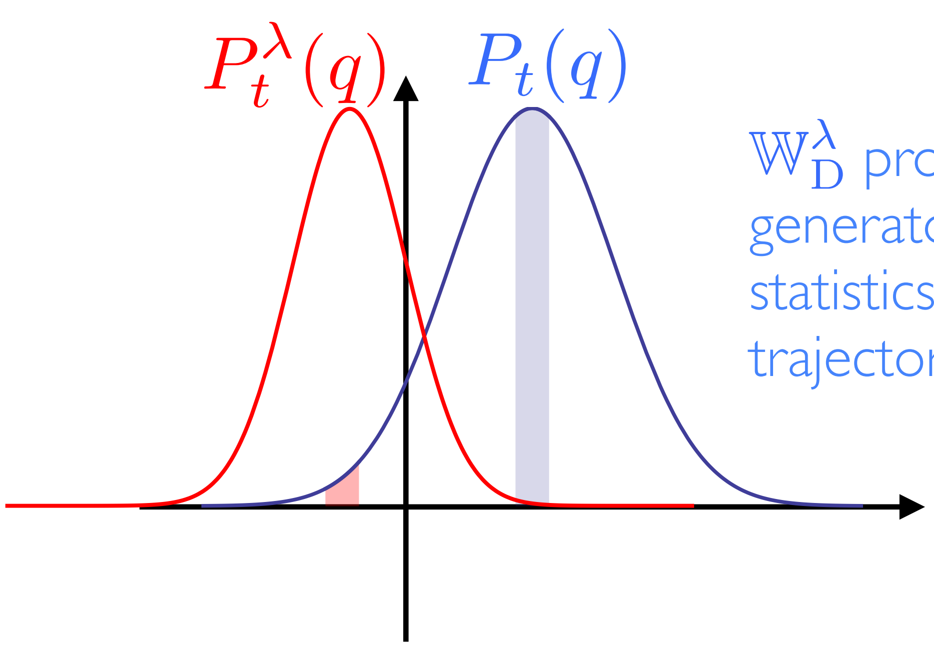
- $\mathbb{W}_{\mathrm{D}}^\lambda$ now **conserves probability** **(physical!)**

$$\sum_C \langle C| \mathbb{W}_{\mathrm{D}}^\lambda = 0$$

- $\mathbb{W}_{\mathrm{D}}^\lambda$ **spectrum** is simply related to that of $\mathbb{W}^\lambda$

$$\theta_i^D(\lambda) = \theta_i(\lambda) - \theta_0(\lambda) \qquad |R_{i,\mathrm{D}}^\lambda\rangle = \mathbb{L}_0 |R_i^\lambda\rangle \qquad \langle L_{i,\mathrm{D}}^\lambda| = \langle L_i^\lambda| \mathbb{L}_0^{-1}$$





$\mathbb{W}_{\mathrm{D}}^\lambda$ proper stochastic generator for the statistics of atypical trajectories

DOOB'S SMART FIELD

- Write **Doob's dynamics** in terms of original WASEP dynamics + smart field E_λ^D

$$(\mathbb{W}_D^\lambda)_{C\to C'} = \mathbb{W}_{C\to C'}\mathrm{e}^{\pm E_\lambda^D/L} \qquad \Rightarrow \qquad (E_\lambda^D)_{C\to C'} = \lambda \pm L\ln\left(\frac{\langle L_0^\lambda | C'\rangle}{\langle L_0^\lambda | C\rangle}\right)$$

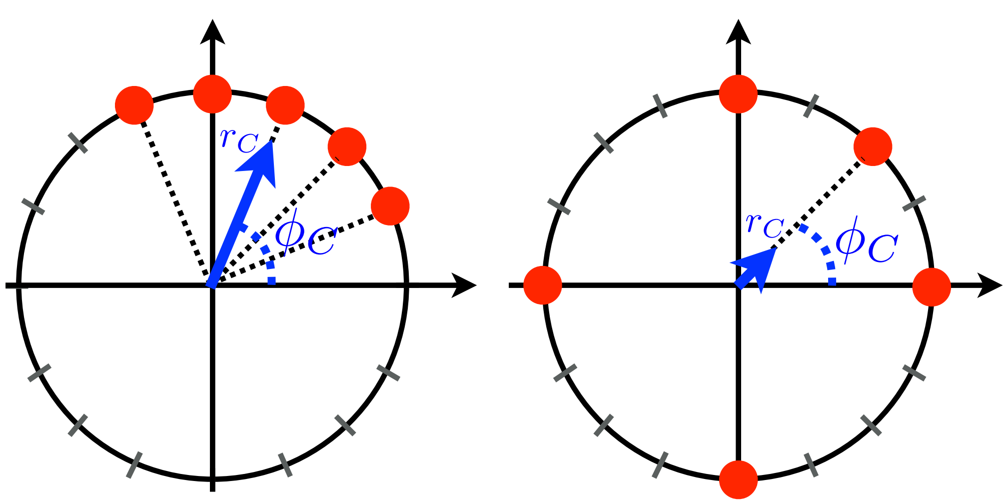
DOOB'S SMART FIELD

- Write **Doob's dynamics** in terms of original **WASEP dynamics + smart field** E_λ^D

$$(\mathbb{W}_D^\lambda)_{C\to C'} = \mathbb{W}_{C\to C'} \mathrm{e}^{\pm E_\lambda^D/L} \qquad \Rightarrow \qquad (E_\lambda^D)_{C\to C'} = \lambda \pm L \ln\left(\frac{\langle L_0^\lambda | C'\rangle}{\langle L_0^\lambda | C\rangle}\right)$$

- Study dependence of E_λ^D on the **packing order parameter** $r_C \equiv |z_C|$

$$z_C = N^{-1}\sum_{k=1}^{L} n_k(C)\mathrm{e}^{i2\pi k/L} = r_c \mathrm{e}^{i\phi_C}$$



$r_C = 0.85$
$\phi_C = 3\pi/8$

$r_C = 0.2$
$\phi_C = \pi/4$

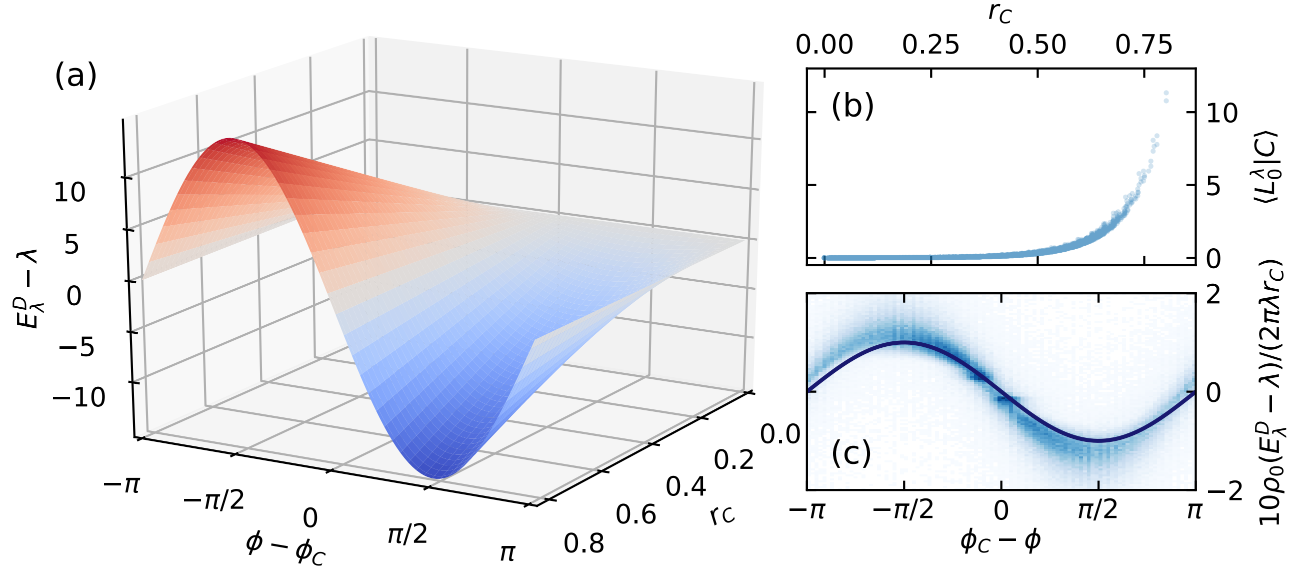
DOOB'S SMART FIELD

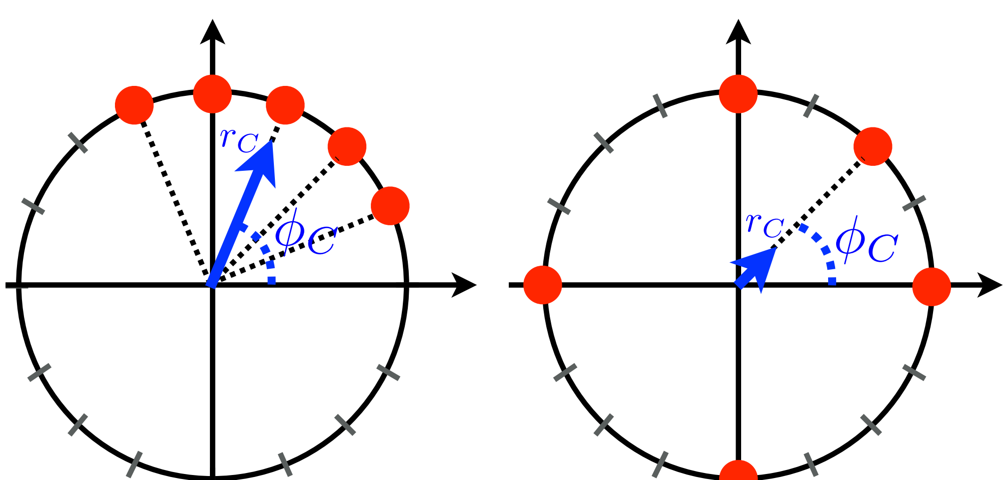
- Write **Doob's dynamics** in terms of original **WASEP dynamics + smart field** E_λ^D

$$(\mathbb{W}_D^\lambda)_{C\to C'} = \mathbb{W}_{C\to C'}\mathrm{e}^{\pm E_\lambda^D/L} \qquad \Rightarrow \qquad (E_\lambda^D)_{C\to C'} = \lambda \pm L \ln\left(\frac{\langle L_0^\lambda | C'\rangle}{\langle L_0^\lambda | C\rangle}\right)$$

- Study dependence of E_λ^D on the **packing order parameter** $r_C \equiv |z_C|$

$$z_C = N^{-1}\sum_{k=1}^{L} n_k(C)\mathrm{e}^{i2\pi k/L} = r_c \mathrm{e}^{i\phi_C}$$





$r_C = 0.85$
$\phi_C = 3\pi/8$

$r_C = 0.2$
$\phi_C = \pi/4$

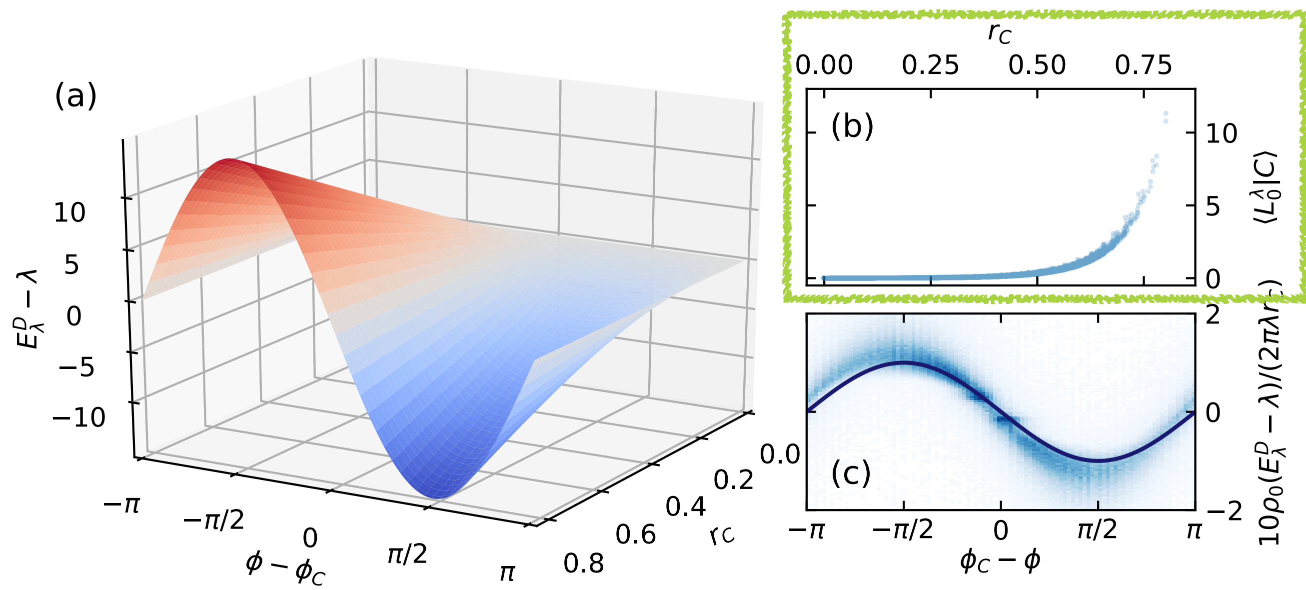
DOOB'S SMART FIELD

- Write **Doob's dynamics** in terms of original **WASEP dynamics + smart field** E_λ^D

$$(\mathbb{W}_D^\lambda)_{C\to C'} = \mathbb{W}_{C\to C'}\mathrm{e}^{\pm E_\lambda^D/L} \quad \Rightarrow \quad (E_\lambda^D)_{C\to C'} = \lambda \pm L \ln\left(\frac{\langle L_0^\lambda | C'\rangle}{\langle L_0^\lambda | C\rangle}\right)$$

- Study dependence of E_λ^D on the **packing order parameter** $r_C \equiv |z_C|$

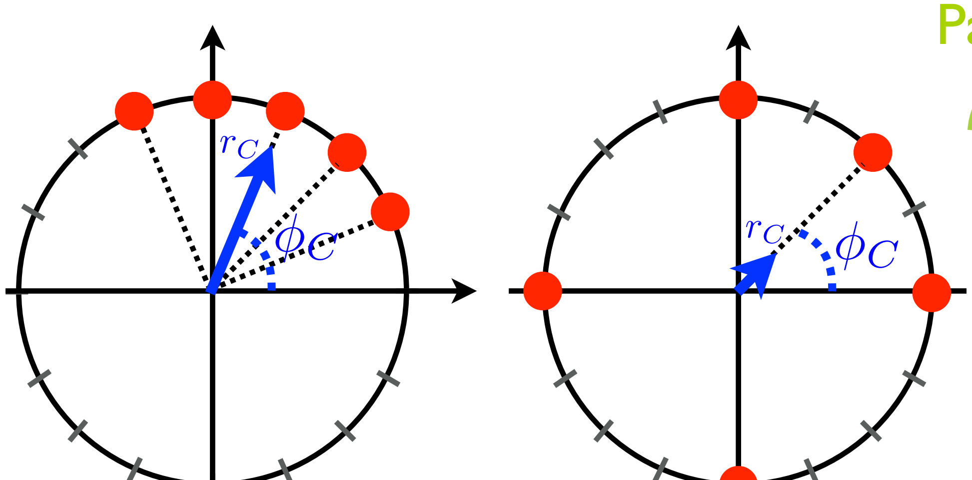
$$z_C = N^{-1}\sum_{k=1}^{L} n_k(C)\mathrm{e}^{i2\pi k/L} = r_c \mathrm{e}^{i\phi_C}$$



$$\langle L_0^\lambda | C\rangle \simeq f_{\lambda,L}(r_C)$$

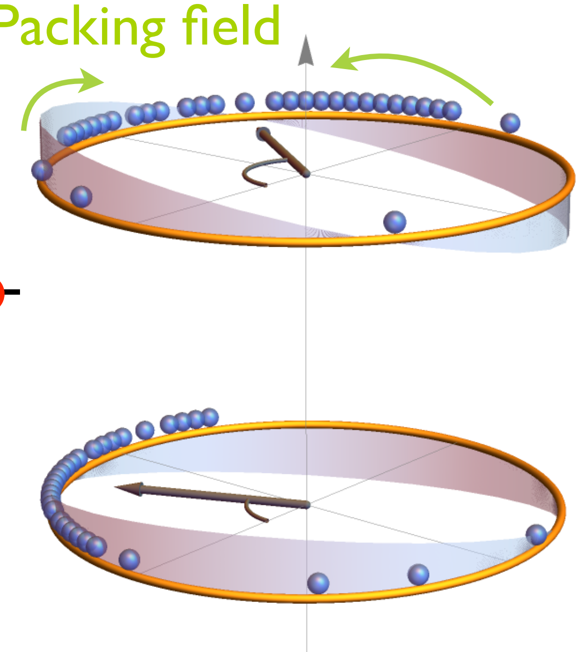
$$\Downarrow$$

$$E_\lambda^D \approx \lambda + \frac{\pi}{5\rho_0}\lambda r_C \sin(\phi_k - \phi_C)$$



$r_C = 0.85$
$\phi_C = 3\pi/8$

$r_C = 0.2$
$\phi_C = \pi/4$

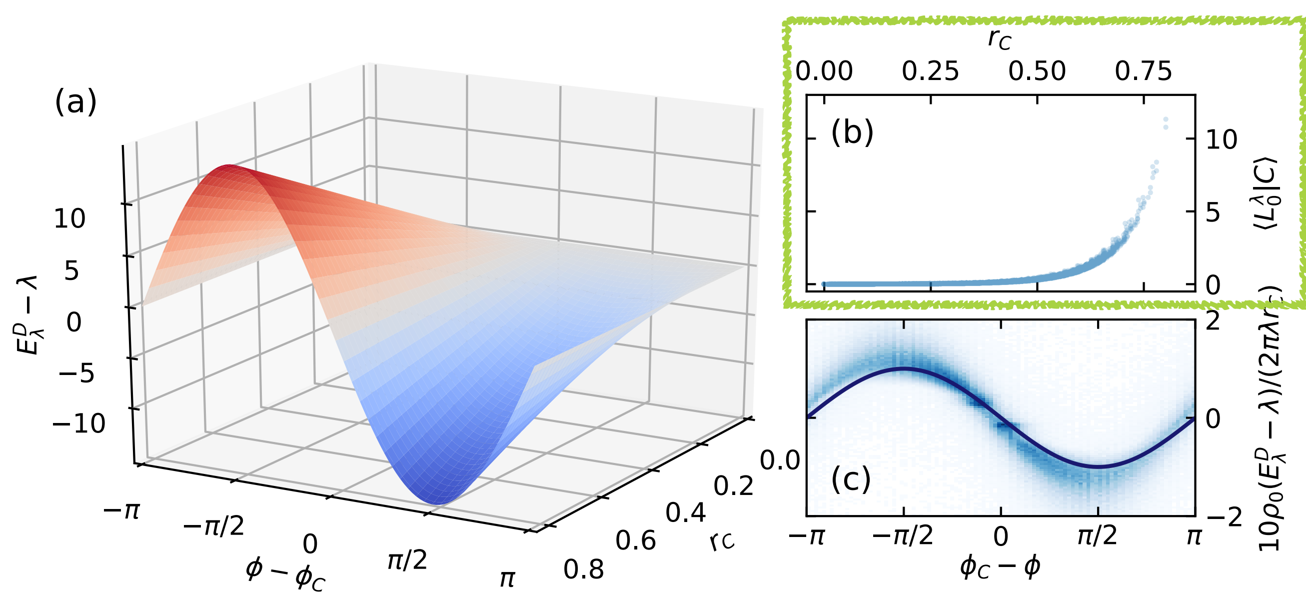


DOOB'S SMART FIELD

- Write **Doob's dynamics** in terms of original **WASEP dynamics + smart field E_λ^D**

$$(\mathbb{W}_D^\lambda)_{C\to C'} = \mathbb{W}_{C\to C'} e^{\pm E_\lambda^D/L} \quad \Rightarrow \quad (E_\lambda^D)_{C\to C'} = \lambda \pm L \ln\left(\frac{\langle L_0^\lambda | C'\rangle}{\langle L_0^\lambda | C\rangle}\right)$$

- Study dependence of E_λ^D on the **packing order parameter** $r_C \equiv |z_C|$

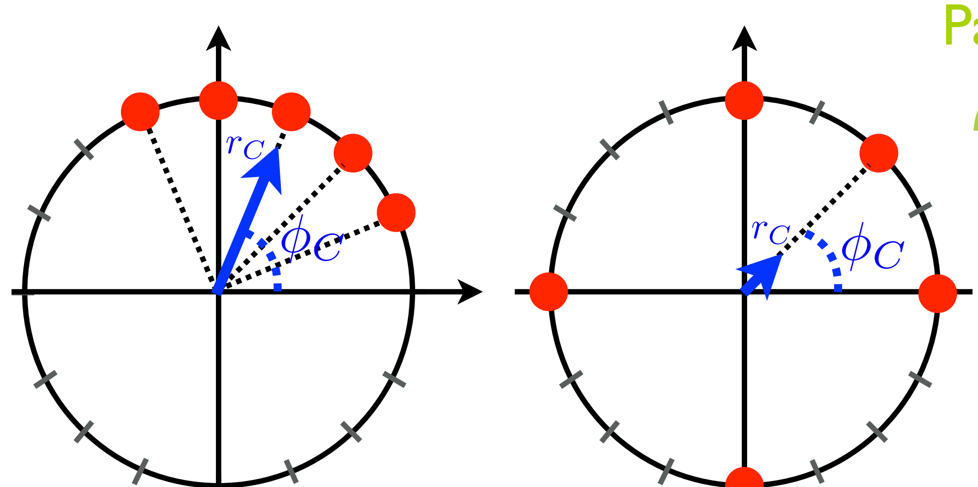


$$z_C = N^{-1} \sum_{k=1}^{L} n_k(C) e^{i2\pi k/L} = r_c e^{i\phi_C}$$

$$\langle L_0^\lambda | C\rangle \simeq f_{\lambda,L}(r_C)$$

$$\Downarrow$$

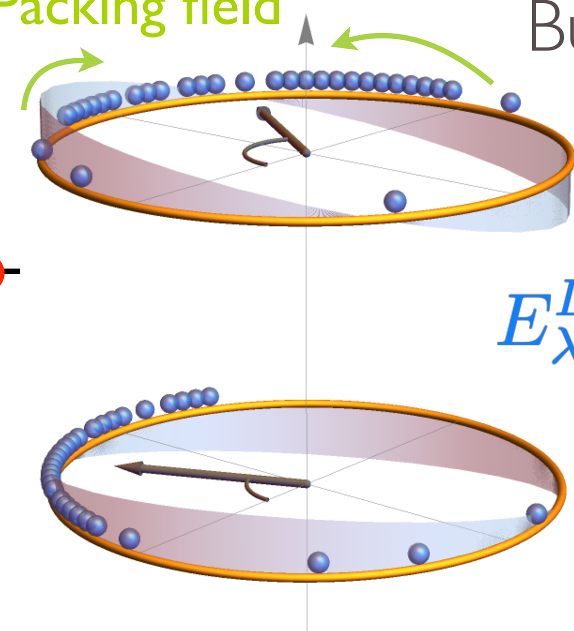
$$E_\lambda^D \approx \lambda + \frac{\pi}{5\rho_0}\lambda r_C \sin(\phi_k - \phi_C)$$



$r_C = 0.85$
$\phi_C = 3\pi/8$

$r_C = 0.2$
$\phi_C = \pi/4$

Packing field



But this is **Kuramoto interaction!!**

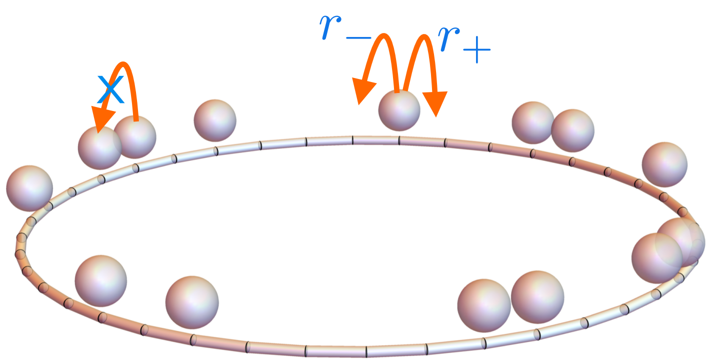
$$\Downarrow$$

$$E_\lambda^D \approx \lambda + \frac{\pi\lambda}{5\rho_0 N} \sum_{j\neq k} \sin(\phi_k - \phi_j)$$

PACKING FIELD MECHANISM

- Inspired by Doob's smart field we now propose a **variant of WASEP** with **configuration-dependent packing field** $E_\lambda(C;k) = \epsilon + \lambda r_C \sin(\phi_k - \phi_C)$

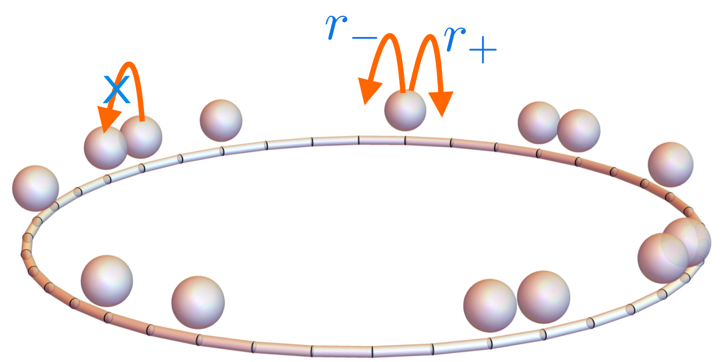
$$r_\pm(C;k) = \frac{1}{2}\mathrm{e}^{\pm E_\lambda(C;k)/L}$$



PACKING FIELD MECHANISM

- Inspired by Doob's smart field we now propose a **variant of WASEP** with **configuration-dependent packing field** $E_\lambda(C;k) = \epsilon + \lambda r_C \sin(\phi_k - \phi_C)$

$$r_\pm(C;k) = \frac{1}{2}\mathrm{e}^{\pm E_\lambda(C;k)/L}$$



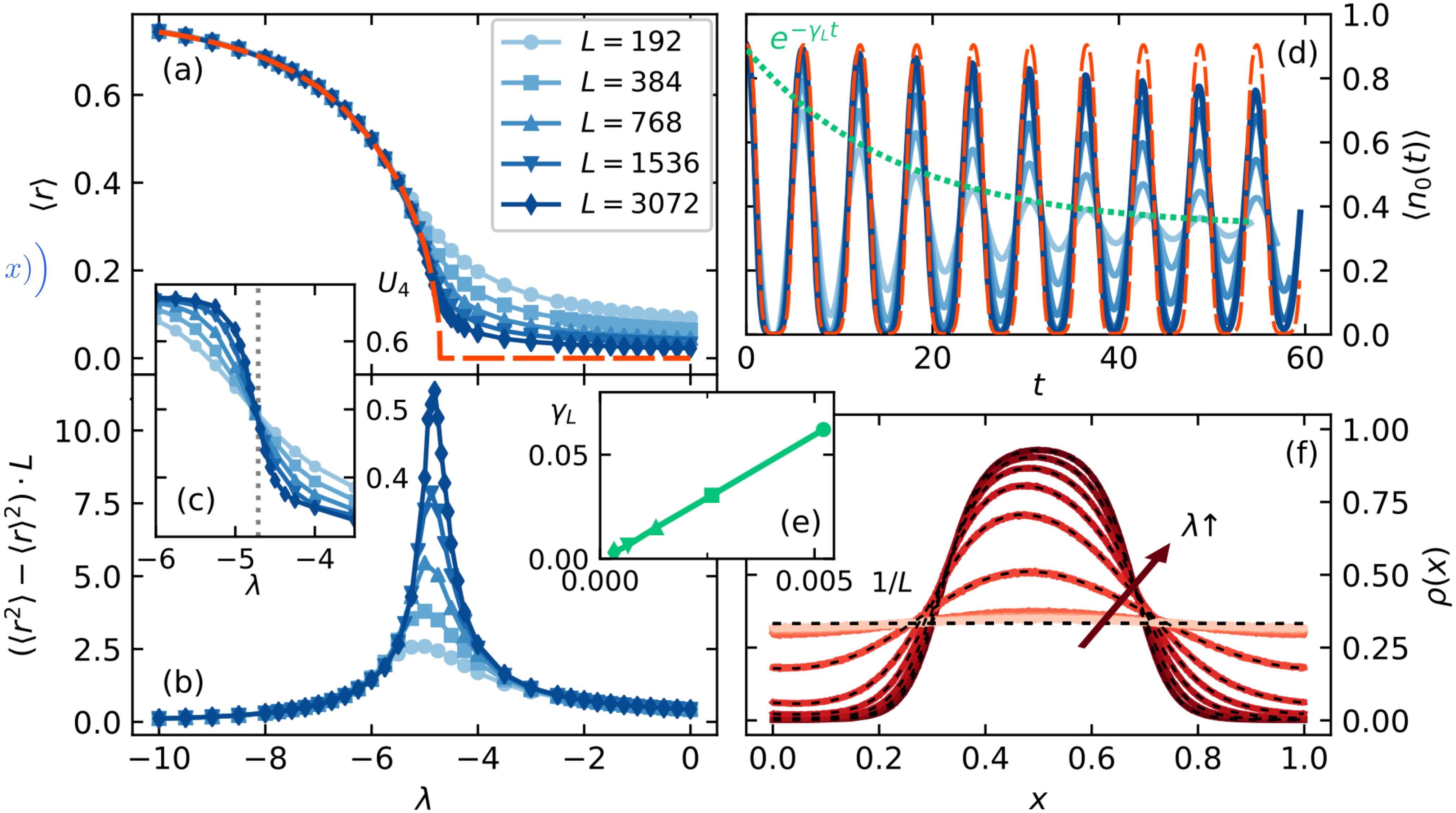
Hydrodynamic limit

$$\partial_t \rho = -\partial_x\Big(-D(\rho)\partial_x\rho + \sigma(\rho)E_\lambda(\rho;x)\Big)$$

$$E_\lambda(\rho;x) = \epsilon + \lambda r_\rho \sin(2\pi x - \phi_\rho)$$

$$z_\rho = \rho_0^{-1}\int_0^1 dx\, \rho(x)\mathrm{e}^{i2\pi x} = r_\rho \mathrm{e}^{i\phi_\rho}$$

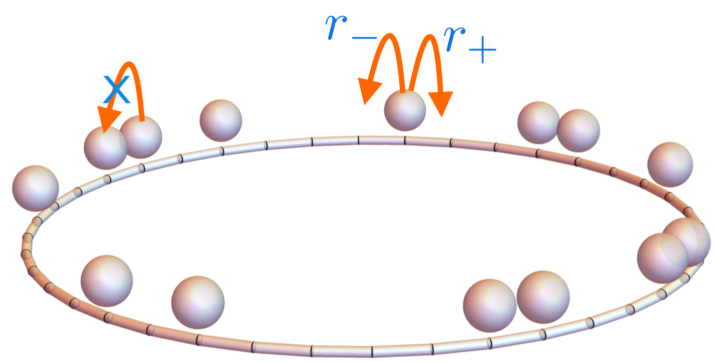
$$\lambda_c = -\pi/(1-\rho_0)$$



PACKING FIELD MECHANISM

- Inspired by Doob's smart field we now propose a **variant of WASEP** with **configuration-dependent packing field** $E_\lambda(C;k) = \epsilon + \lambda r_C \sin(\phi_k - \phi_C)$

$$r_\pm(C;k) = \frac{1}{2}\mathrm{e}^{\pm E_\lambda(C;k)/L}$$



Steady-state phase transition to time-crystal phase

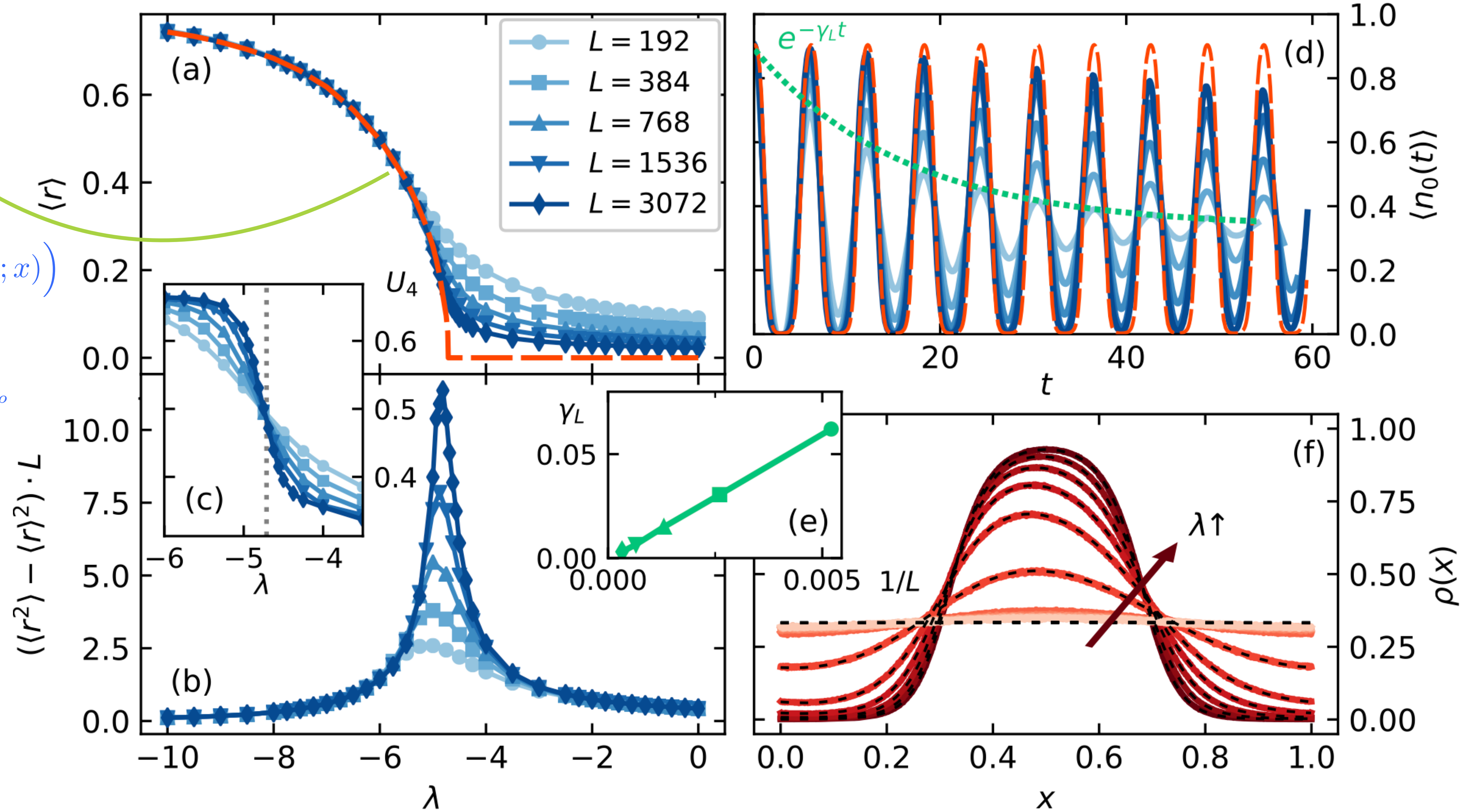
Hydrodynamic limit

$$\partial_t \rho = -\partial_x\Big(-D(\rho)\partial_x\rho + \sigma(\rho)E_\lambda(\rho;x)\Big)$$

$$E_\lambda(\rho;x) = \epsilon + \lambda r_\rho \sin(2\pi x - \phi_\rho)$$

$$z_\rho = \rho_0^{-1}\int_0^1 dx\, \rho(x)\mathrm{e}^{i2\pi x} = r_\rho \mathrm{e}^{i\phi_\rho}$$

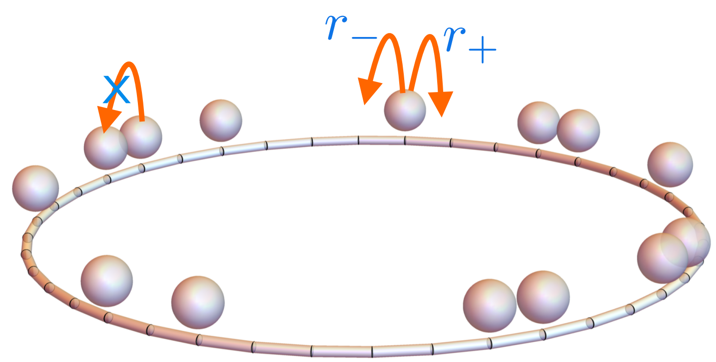
$$\lambda_c = -\pi/(1-\rho_0)$$



PACKING FIELD MECHANISM

- Inspired by Doob's smart field we now propose a **variant of WASEP** with **configuration-dependent packing field** $E_\lambda(C;k) = \epsilon + \lambda r_C \sin(\phi_k - \phi_C)$

$$r_\pm(C;k) = \frac{1}{2}\mathrm{e}^{\pm E_\lambda(C;k)/L}$$



Steady-state phase transition to time-crystal phase

Hydrodynamic limit

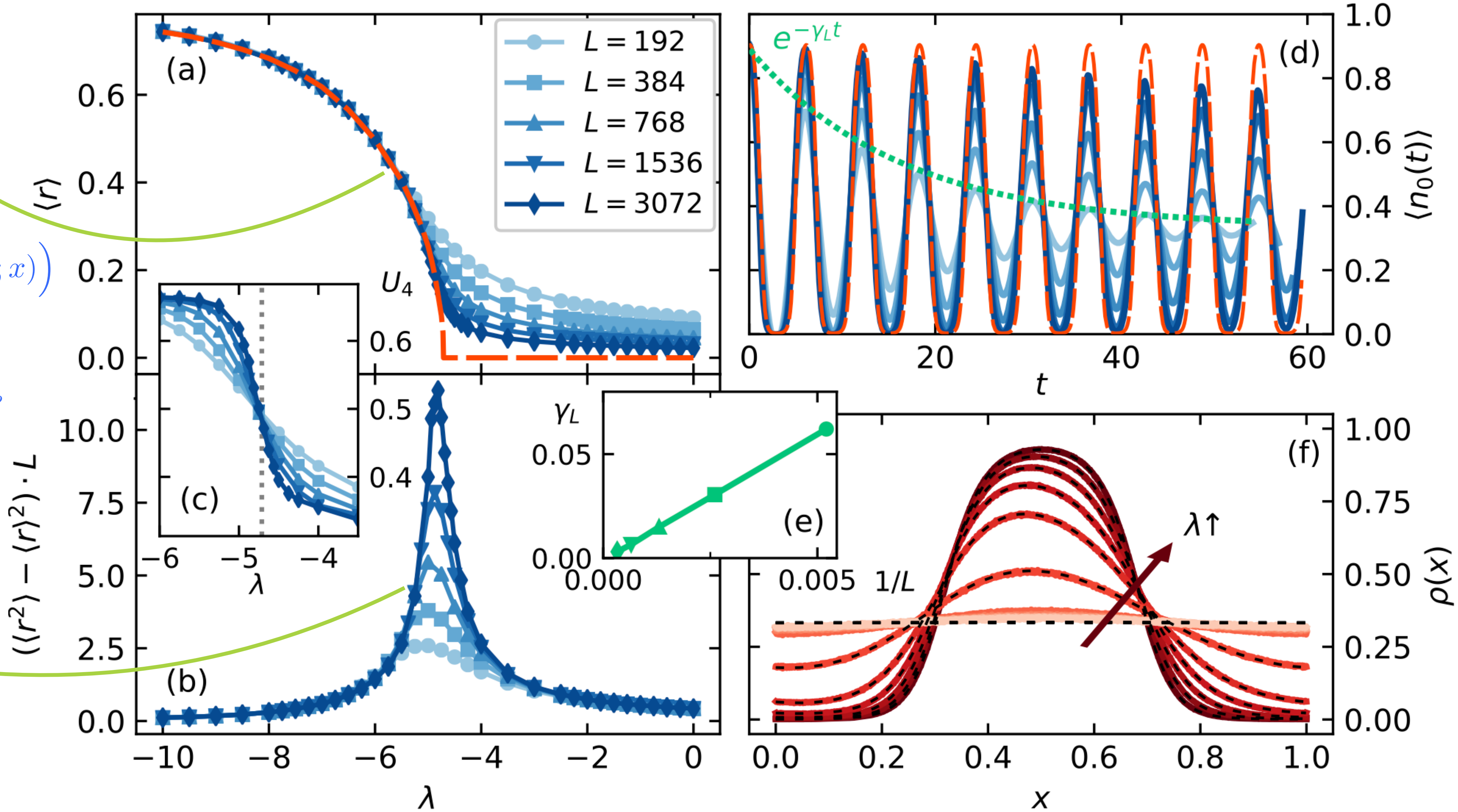
$$\partial_t \rho = -\partial_x \Big(- D(\rho)\partial_x \rho + \sigma(\rho) E_\lambda(\rho;x) \Big)$$

$$E_\lambda(\rho;x) = \epsilon + \lambda r_\rho \sin(2\pi x - \phi_\rho)$$

$$z_\rho = \rho_0^{-1} \int_0^1 dx\, \rho(x) \mathrm{e}^{i2\pi x} = r_\rho \mathrm{e}^{i\phi_\rho}$$

$$\lambda_c = -\pi/(1-\rho_0)$$

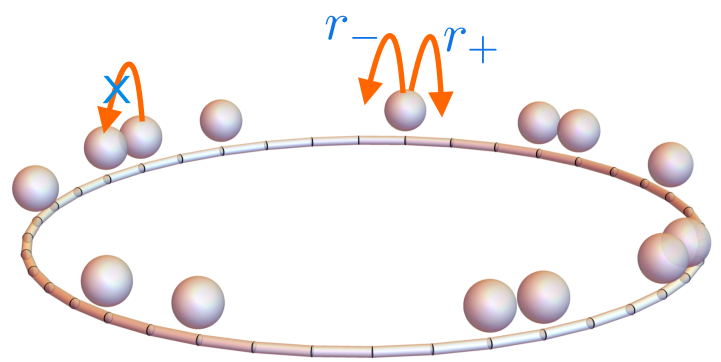
Divergent susceptibility as L grows



PACKING FIELD MECHANISM

- Inspired by Doob's smart field we now propose a **variant of WASEP** with **configuration-dependent packing field** $E_\lambda(C;k) = \epsilon + \lambda r_C \sin(\phi_k - \phi_C)$

$$r_\pm(C;k) = \frac{1}{2}\mathrm{e}^{\pm E_\lambda(C;k)/L}$$



Persistent density oscillations

Steady-state phase transition to time-crystal phase

Hydrodynamic limit

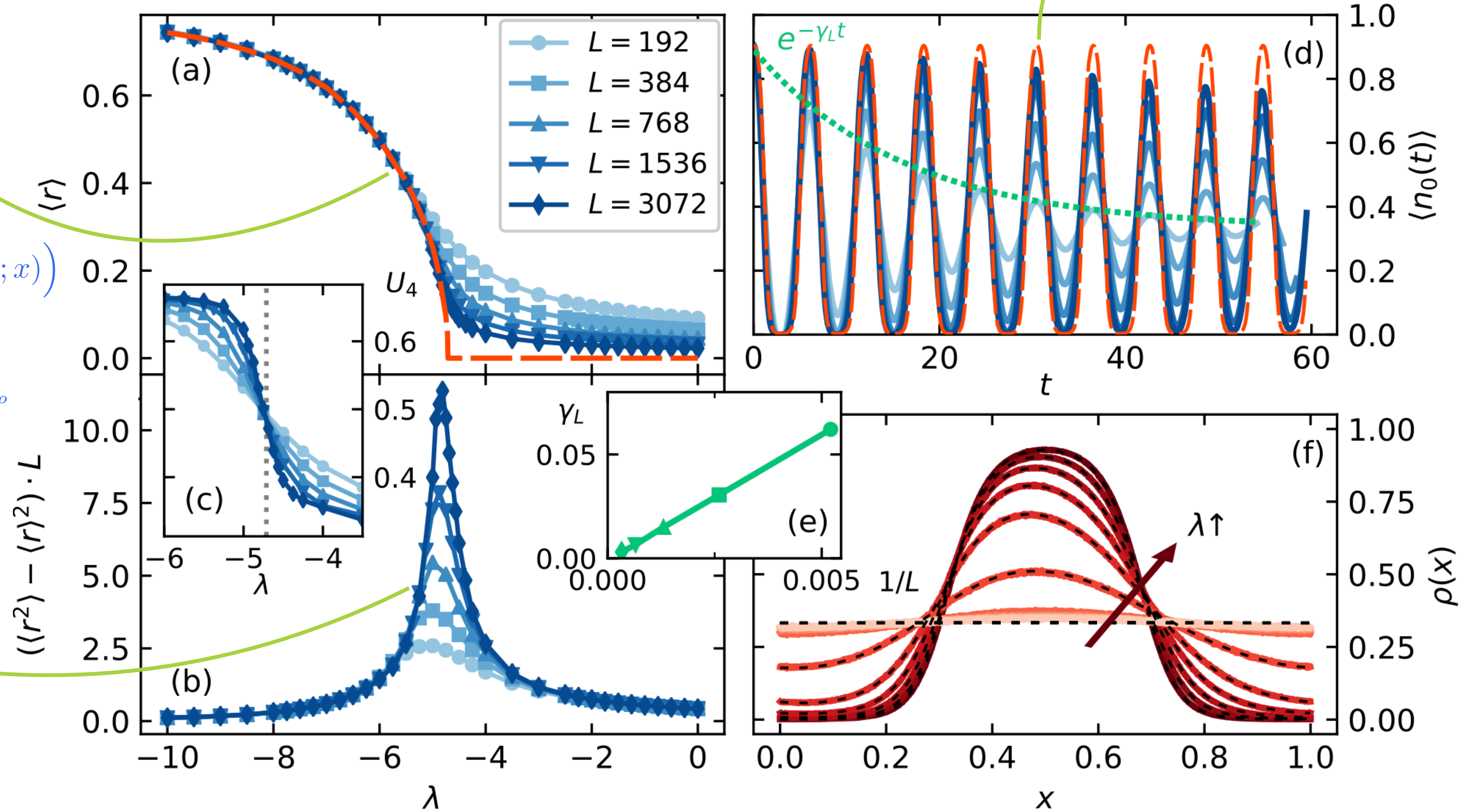
$$\partial_t \rho = -\partial_x\Big(-D(\rho)\partial_x\rho + \sigma(\rho)E_\lambda(\rho;x)\Big)$$

$$E_\lambda(\rho;x) = \epsilon + \lambda r_\rho \sin(2\pi x - \phi_\rho)$$

$$z_\rho = \rho_0^{-1}\int_0^1 dx\, \rho(x)\mathrm{e}^{i2\pi x} = r_\rho \mathrm{e}^{i\phi_\rho}$$

$$\lambda_c = -\pi/(1-\rho_0)$$

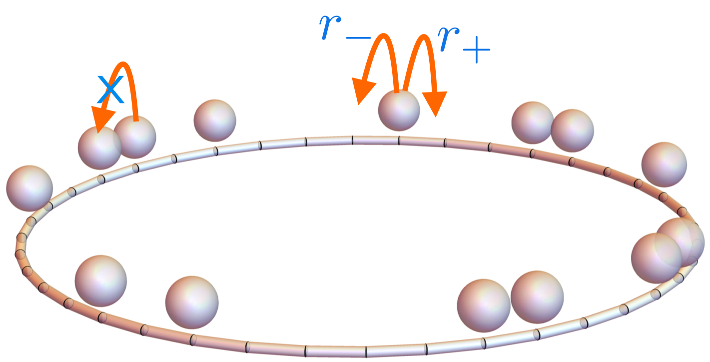
Divergent susceptibility as L grows



PACKING FIELD MECHANISM

- Inspired by Doob's smart field we now propose a **variant of WASEP** with **configuration-dependent packing field** $E_\lambda(C;k) = \epsilon + \lambda r_C \sin(\phi_k - \phi_C)$

$$r_\pm(C;k) = \frac{1}{2} e^{\pm E_\lambda(C;k)/L}$$



Rigidity of long-range spatiotemporal order

Persistent density oscillations

Steady-state phase transition to time-crystal phase

Hydrodynamic limit

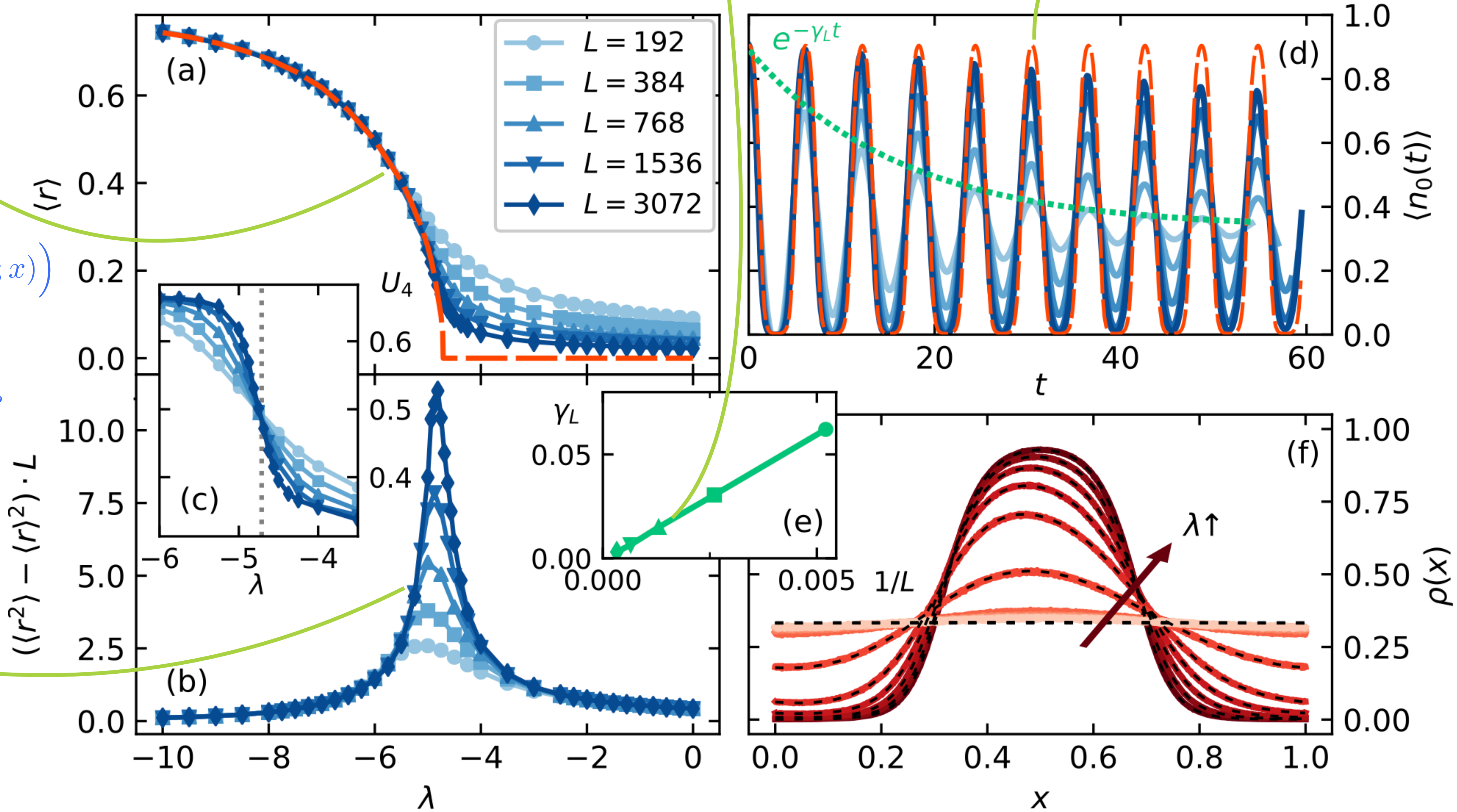
$$\partial_t \rho = -\partial_x \Big(-D(\rho)\partial_x \rho + \sigma(\rho) E_\lambda(\rho;x) \Big)$$

$$E_\lambda(\rho;x) = \epsilon + \lambda r_\rho \sin(2\pi x - \phi_\rho)$$

$$z_\rho = \rho_0^{-1} \int_0^1 dx\, \rho(x) e^{i2\pi x} = r_\rho e^{i\phi_\rho}$$

$$\lambda_c = -\pi/(1-\rho_0)$$

Divergent susceptibility as L grows



WHAT IF …?

- Mathematically, packing field = controlled excitation of the first Fourier mode of the **density field fluctuations around the instantaneous center of mass position**

- Question: What if … we excite any higher-order modes?

WHAT IF ...?

- Mathematically, **packing field = controlled excitation of the first Fourier mode** of the **density field fluctuations around the instantaneous center of mass position**

- Question: **What if ... we excite any higher-order modes?**

$$\partial_t \rho = -\Big(- D(\rho)\partial_x \rho + \sigma(\rho) E_\lambda^{(m)}(\rho; x) \Big)$$

$$E_\lambda^{(m)}(\rho; x) = \epsilon + \lambda \underbrace{\boxed{r_m(\rho)\sin(2\pi m x - \phi_m(\rho))}}_{\mathcal{E}^{(m)}}$$

$$z_m(\rho) = \rho_0^{-1} \int_0^1 dx\, \rho(x,t) e^{i2\pi m x} = r_m(\rho)\, e^{i\phi_m(\rho)}$$

- ☑ Driven diffusive fluid with **mth-order packing field** $E_\lambda^{(m)}(\rho; x)$
- ☑ $\mathcal{E}^{(m)}$ pushes particles locally towards **m** equidistant **emergent localization centers**
- ☑ Located at the complex arguments of the mth-roots of z_m

$$\varphi_m^{(j)} = \arg(\sqrt[m]{z_m}) = \frac{\phi_m + 2\pi j}{m}$$

WHAT IF …?

- Mathematically, **packing field = controlled excitation of the first Fourier mode** of the **density field fluctuations around the instantaneous center of mass position**

- Question: **What if … we excite any higher-order modes?**

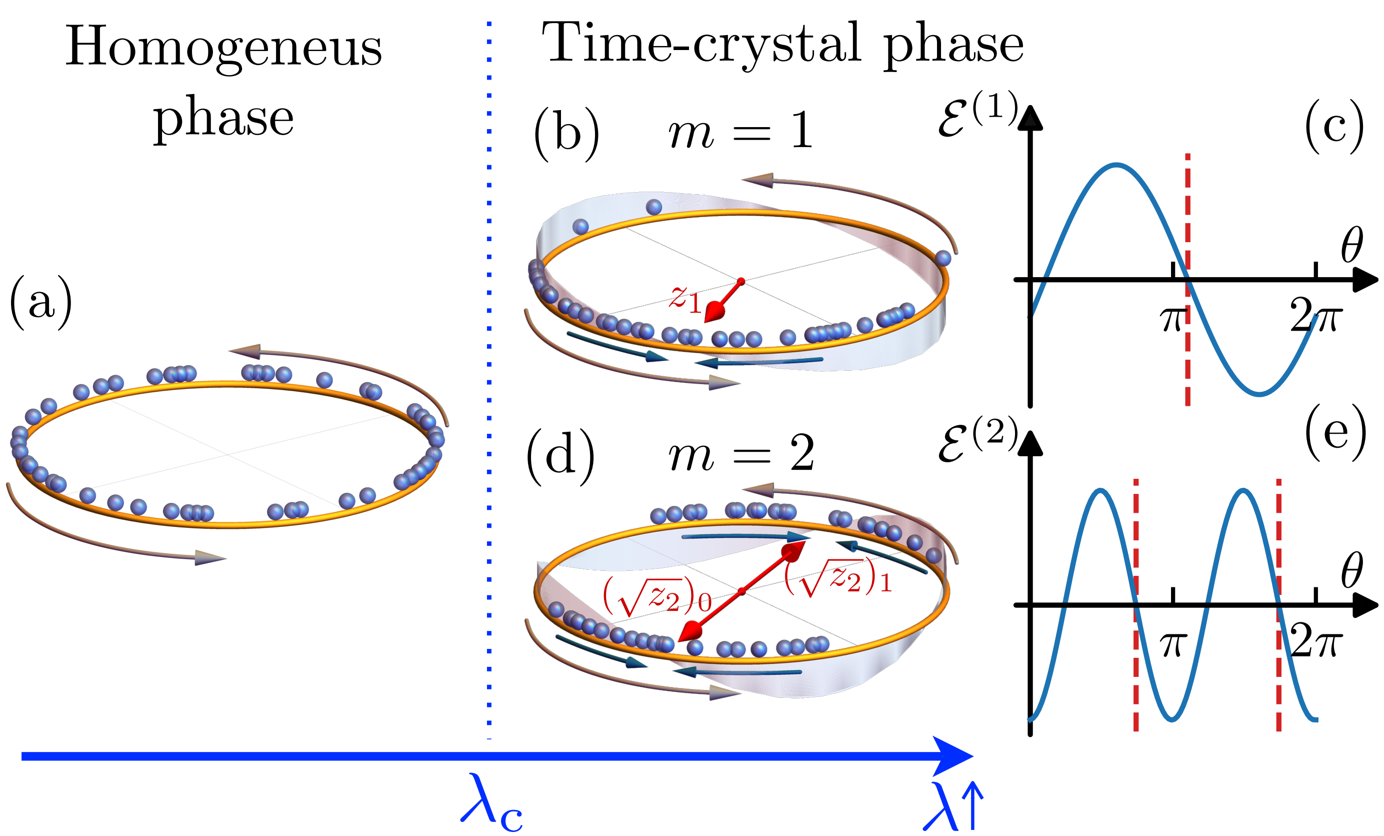
$$\partial_t \rho = -\Big(- D(\rho)\partial_x \rho + \sigma(\rho) E_\lambda^{(m)}(\rho; x) \Big)$$

$$E_\lambda^{(m)}(\rho; x) = \epsilon + \lambda \underbrace{r_m(\rho)\sin(2\pi m x - \phi_m(\rho))}_{\mathcal{E}^{(m)}}$$

$$z_m(\rho) = \rho_0^{-1}\int_0^1 dx\, \rho(x,t) \mathrm{e}^{\mathrm{i}2\pi m x} = r_m(\rho)\, \mathrm{e}^{\mathrm{i}\phi_m(\rho)}$$

- ☑ Driven diffusive fluid with **mth-order packing field** $E_\lambda^{(m)}(\rho; x)$
- ☑ $\mathcal{E}^{(m)}$ pushes particles locally towards **m** equidistant **emergent localization centers**
- ☑ Located at the complex arguments of the mth-roots of z_m

$$\varphi_m^{(j)} = \arg(\sqrt[m]{z_m}) = \frac{\phi_m + 2\pi j}{m}$$

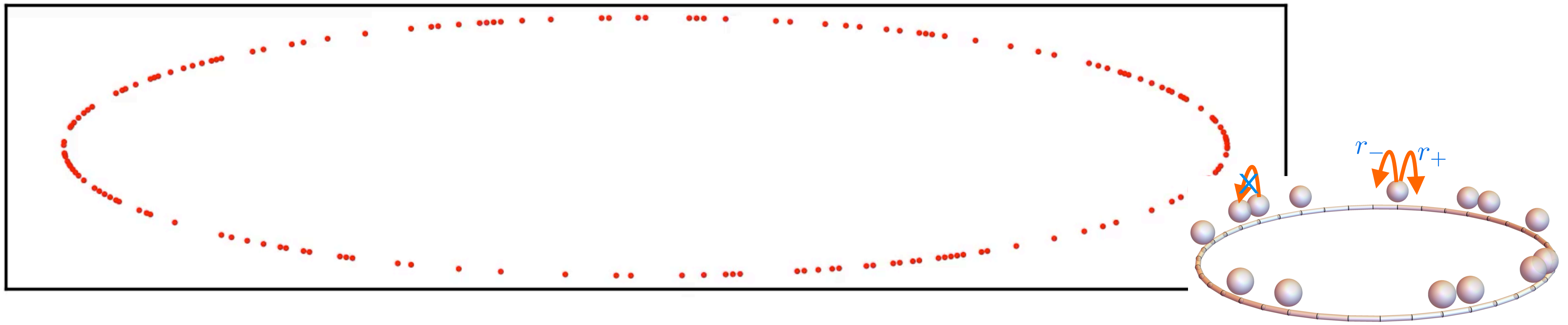


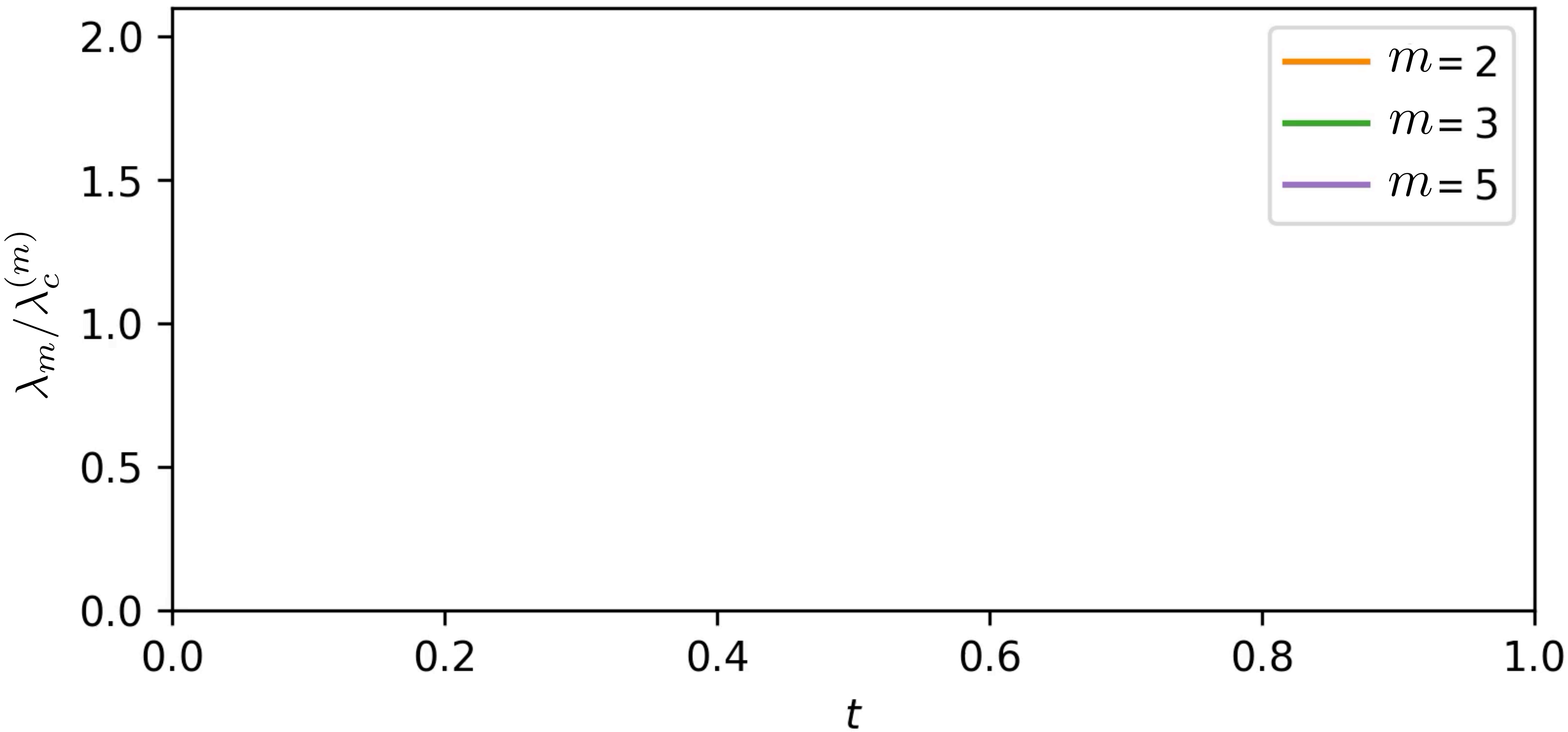
- **Bingo!!** We can **engineer custom multi-mode and fully-controllable continuous time crystals**

MULTIMODE AND CONTROLLABLE TIME CRYSTALS

- Example: switching between different number of condensates in time

$$\partial_t \rho = -\Big(- D(\rho)\partial_x \rho + \sigma(\rho) E_\lambda^{(m)}(\rho;x)\Big) \qquad E_\lambda^{(m)}(\rho;x) = \epsilon + \lambda r_m(\rho)\sin(2\pi m x - \phi_m(\rho))$$

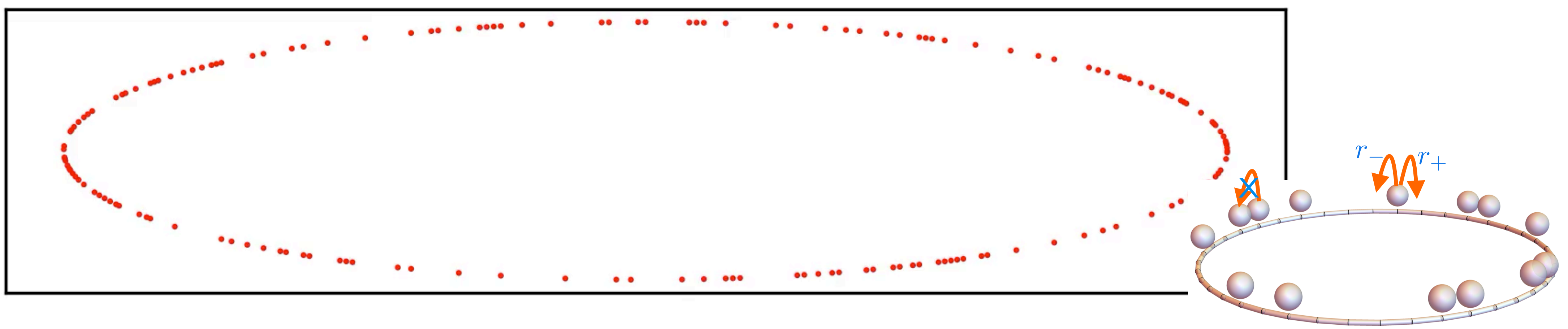


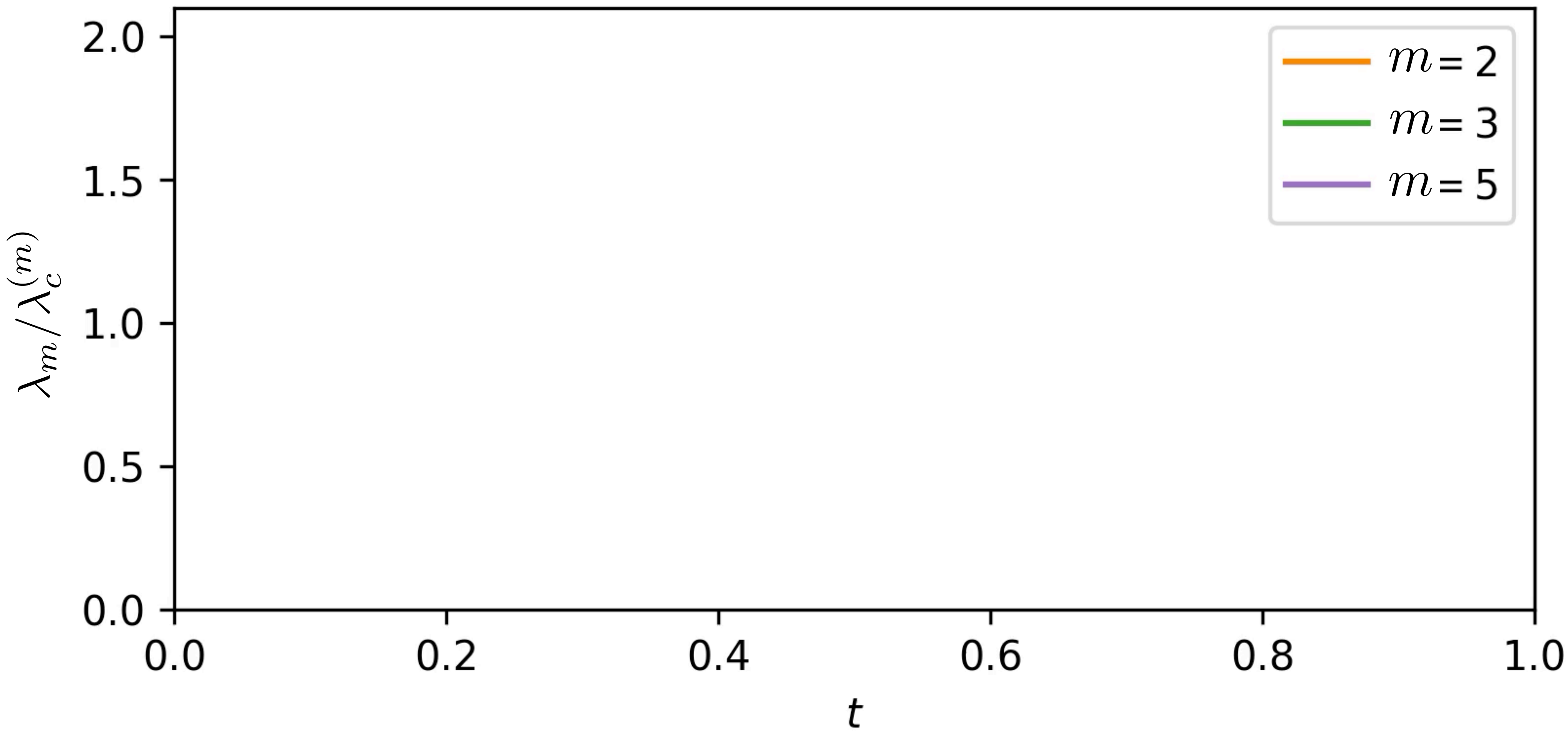


MULTIMODE AND CONTROLLABLE TIME CRYSTALS

- Example: switching between different number of condensates in time

$$\partial_t \rho = -\Big(- D(\rho)\partial_x \rho + \sigma(\rho) E_\lambda^{(m)}(\rho; x)\Big) \qquad E_\lambda^{(m)}(\rho; x) = \epsilon + \lambda r_m(\rho)\sin(2\pi m x - \phi_m(\rho))$$

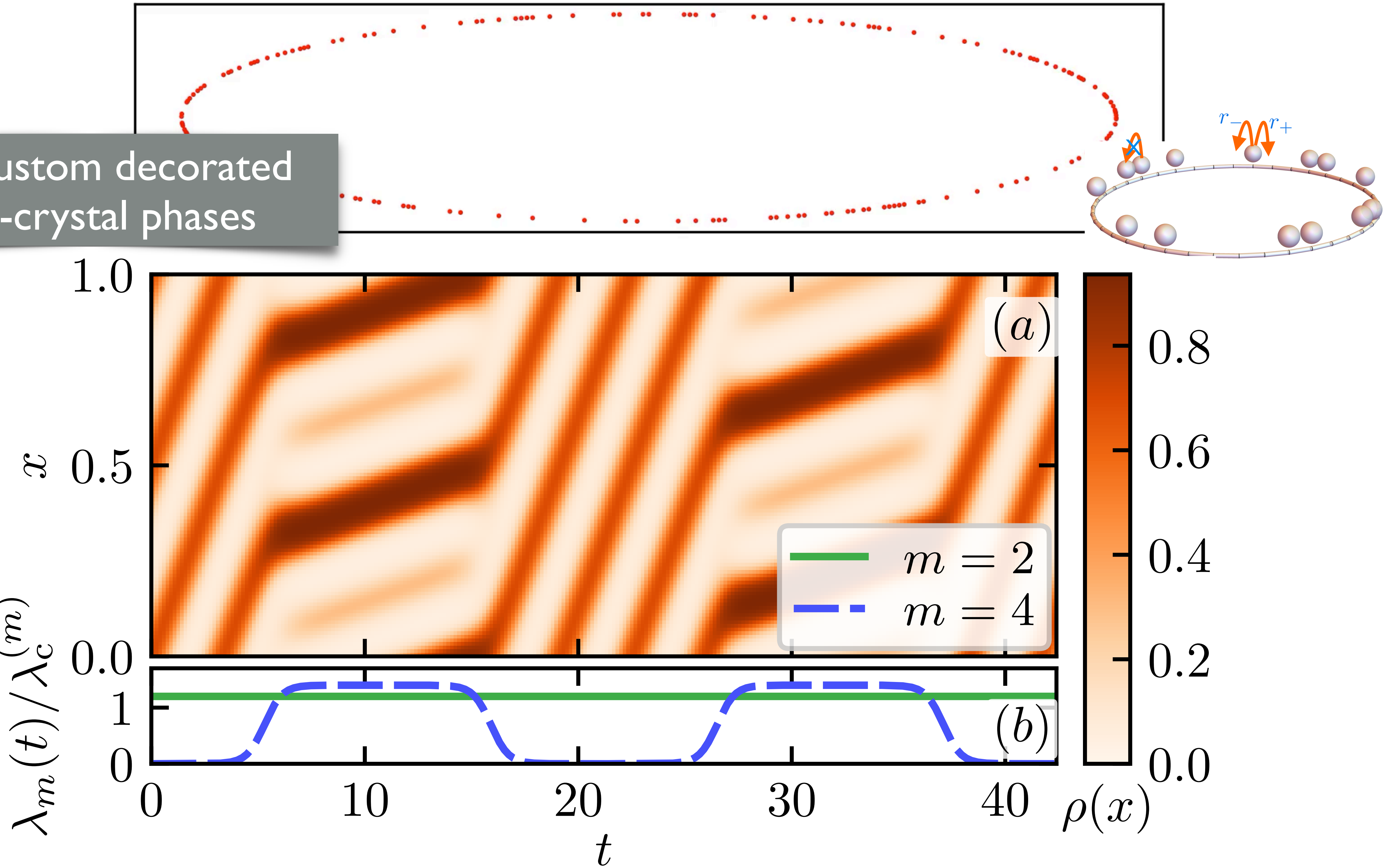




MULTIMODE AND CONTROLLABLE TIME CRYSTALS

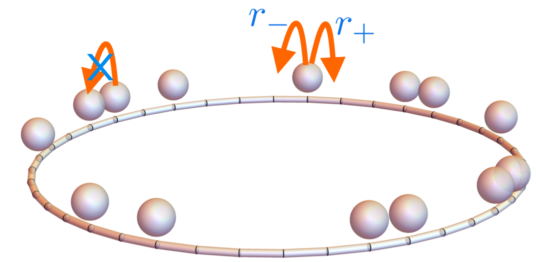
- Example: switching between different number of condensates in time

$$\partial_t \rho = -\Big(- D(\rho)\partial_x \rho + \sigma(\rho) E_\lambda^{(m)}(\rho; x) \Big) \qquad E_\lambda^{(m)}(\rho; x) = \epsilon + \lambda r_m(\rho) \sin(2\pi m x - \phi_m(\rho))$$



Also custom decorated time-crystal phases

PHASE TRANSITION

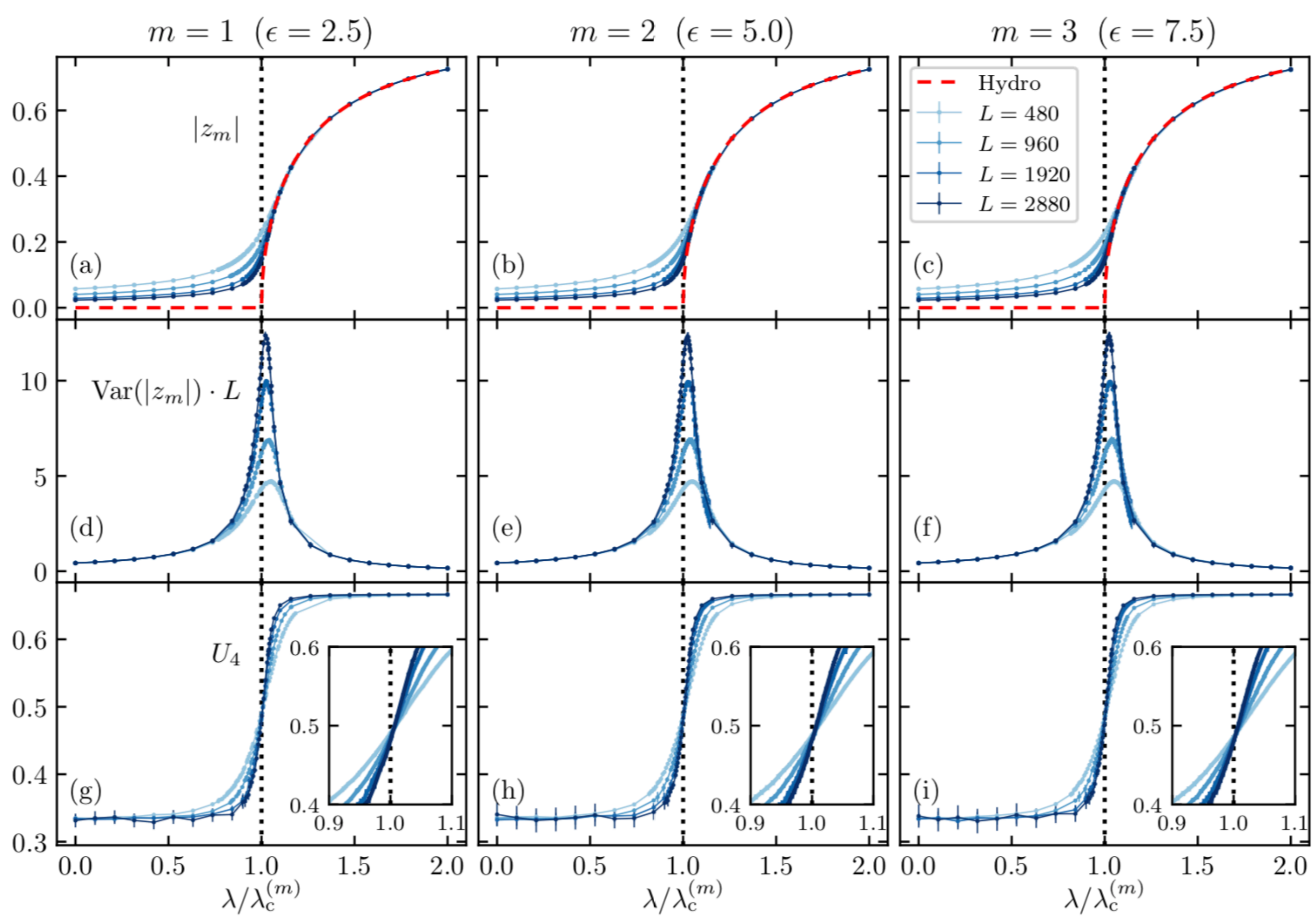


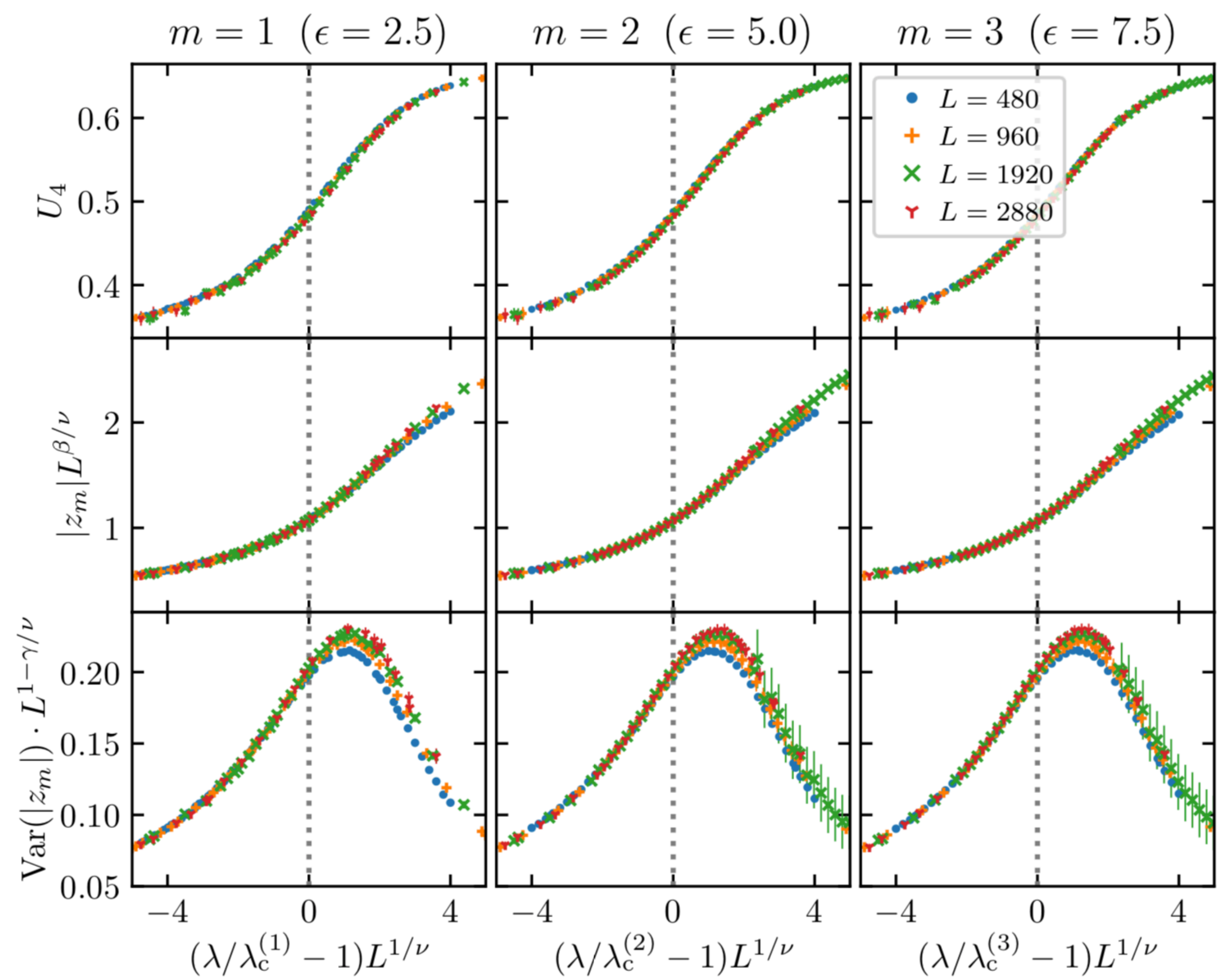
- A **linear stability analysis** of the homogeneous density solution $\rho(x,t) = \rho_0$ yields the **critical threshold for the phase transition**

$$\rho(x,t) = \rho_0 + \delta\rho(x,t) \qquad \rightarrow \qquad \lambda_c^{(m)} = 4\pi m \frac{D(\rho_0)\rho_0}{\sigma(\rho_0)}$$

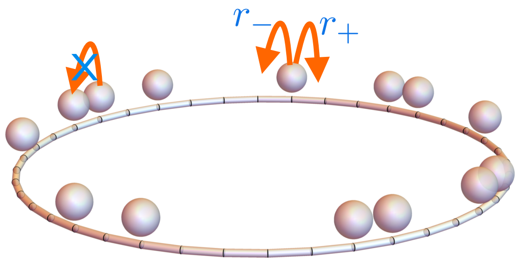
- Finite-size scaling analysis: **Kuramoto universality class**

$$\beta = 1/2, \quad \gamma = 1, \quad \nu = 2$$

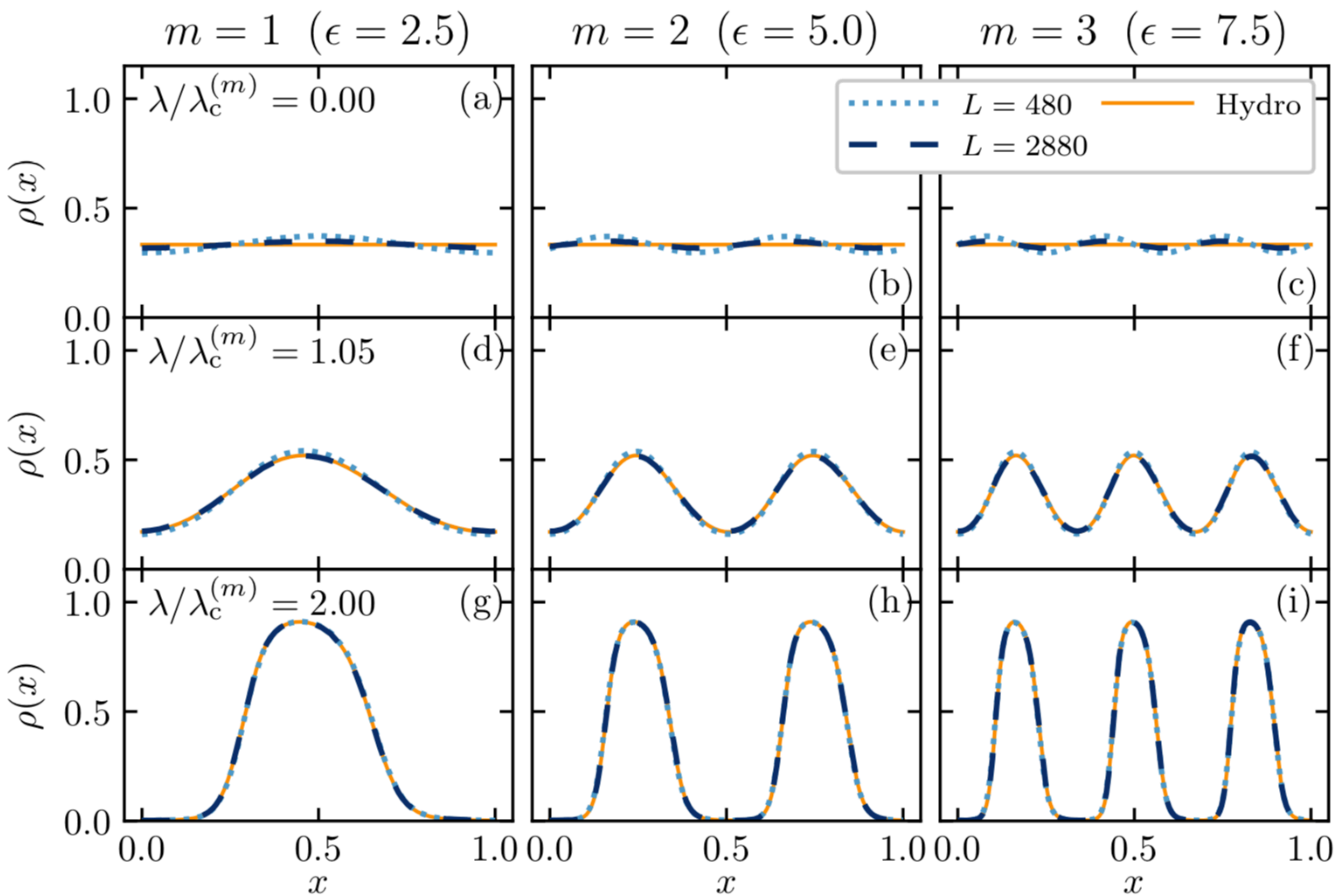




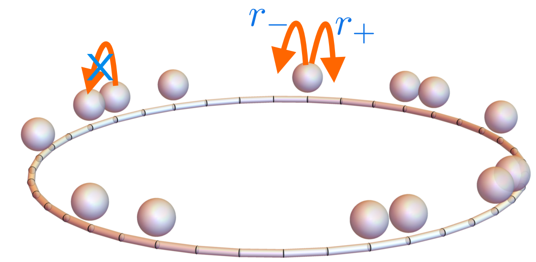
CONDENSATE EQUIVALENCE



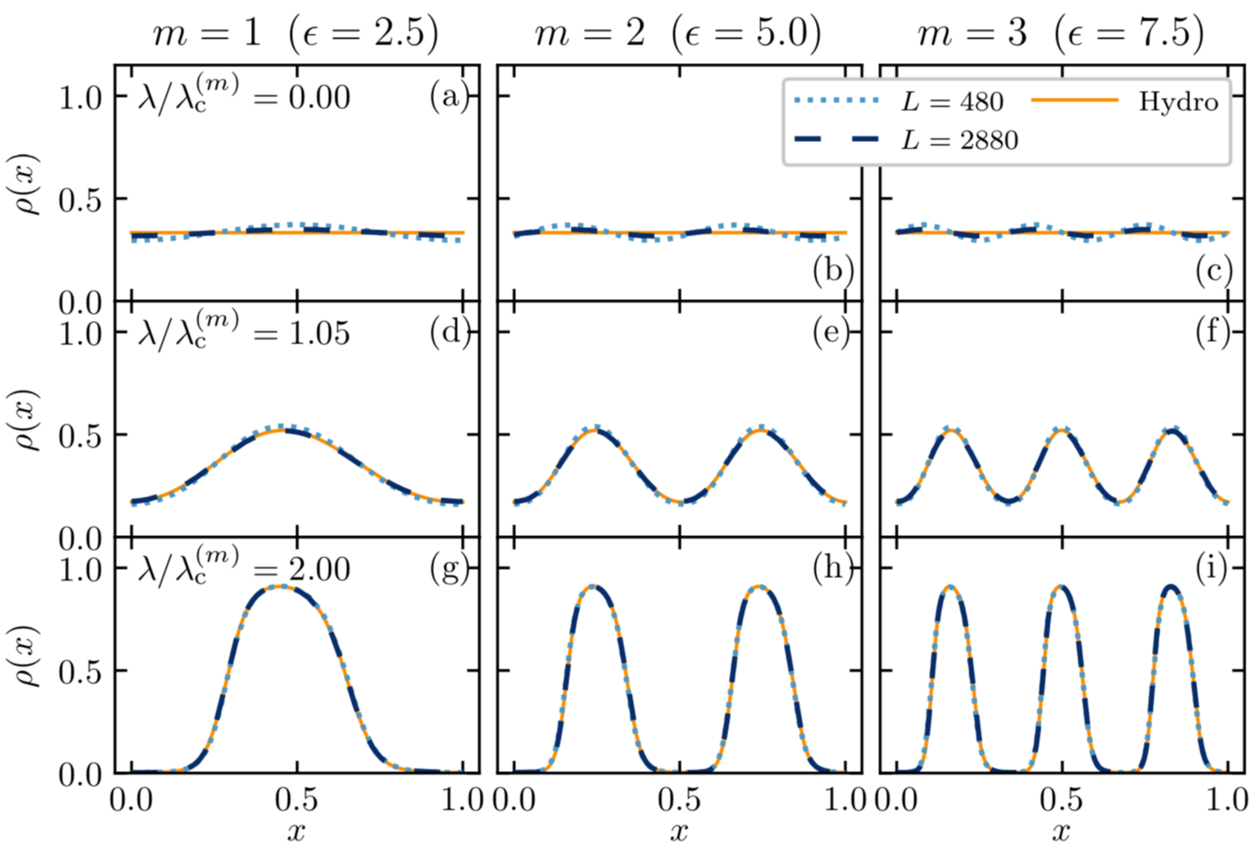
- **Hydrodynamic predictions** fully agree with Monte Carlo simulation measurements

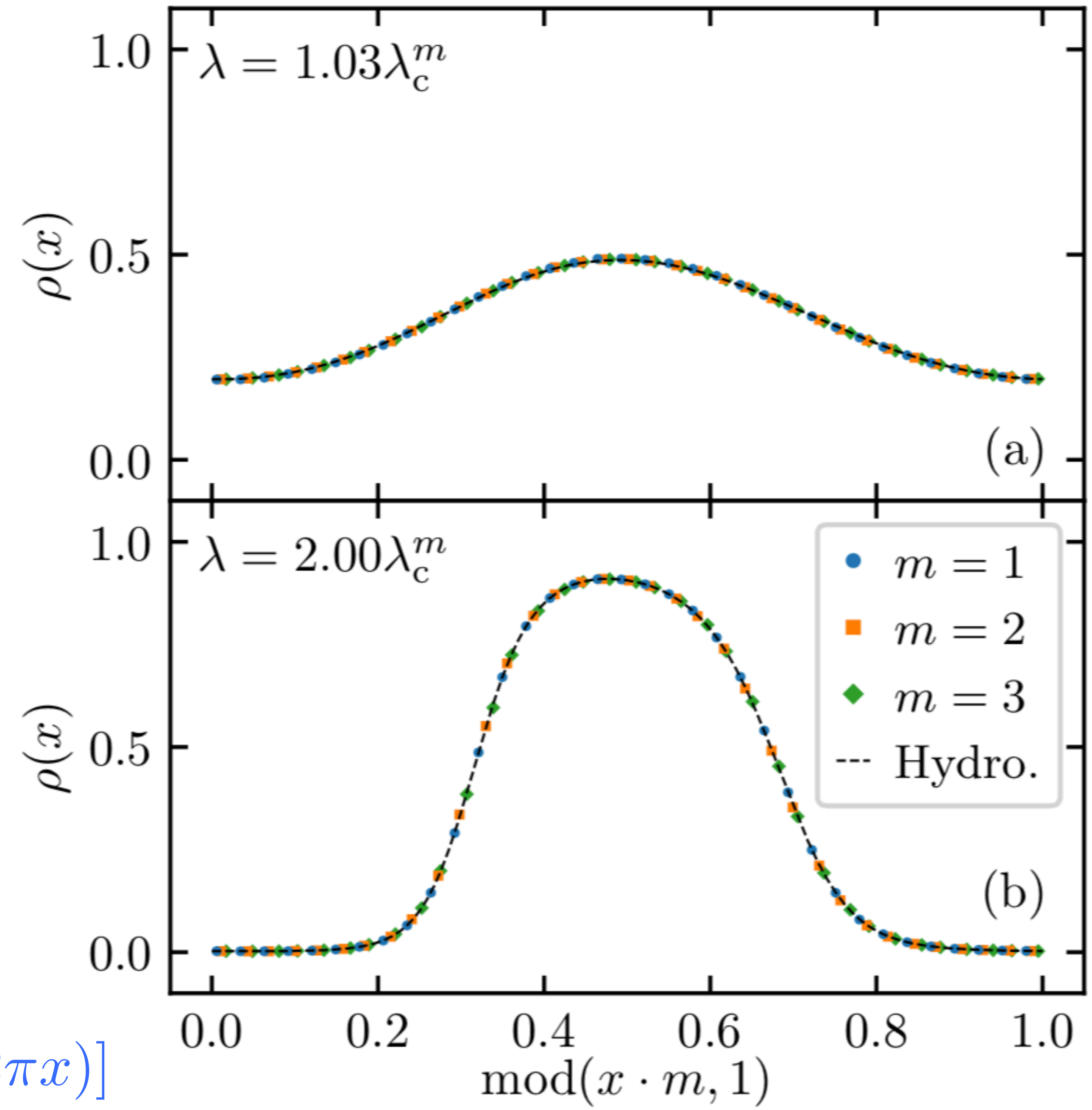


CONDENSATE EQUIVALENCE



- **Hydrodynamic predictions** fully agree with Monte Carlo simulation measurements





- **Condensate equivalence** for different m

$m = 1,\ \epsilon_1,\ \lambda_1$	$m,\ m\epsilon_1,\ m\lambda_1$
$\rho(x,t) = \mathcal{T}(\omega t - 2\pi x)$	$\rho(x,t) = \mathcal{T}[m(\omega t - 2\pi x)]$

EXAMPLES

- To illustrate these ideas, we apply them to **4 paradigmatic lattice gas models**

$$\partial_t \rho = -\Big(- D(\rho)\partial_x \rho + \sigma(\rho) E_\lambda^{(m)}(\rho;x) \Big) \qquad E_\lambda^{(m)}(\rho;x) = \epsilon + \lambda r_m(\rho) \sin(2\pi m x - \phi_m(\rho))$$

EXAMPLES

$$\mathcal{J}(\rho) = \frac{\nu[1+\delta(1-2\rho)] - \eta\sqrt{4\rho(1-\rho)}}{\nu^3} \qquad \nu \equiv \frac{1+\sqrt{(2\rho-1)^2+4\rho(1-\rho)e^{-4\beta}}}{\sqrt{4\rho(1-\rho)}}$$

$$\chi(\rho) = \rho(1-\rho)\sqrt{(2\rho-1)^2+4\rho(1-\rho)e^{-4\beta}} \qquad e^{4\beta} \equiv \frac{1+\eta}{1-\eta}$$

- To illustrate these ideas, we apply them to **4 paradigmatic lattice gas models**

$$\partial_t \rho = -\Big(-D(\rho)\partial_x\rho + \sigma(\rho)E_\lambda^{(m)}(\rho;x)\Big) \qquad E_\lambda^{(m)}(\rho;x) = \epsilon + \lambda r_m(\rho)\sin(2\pi m x - \phi_m(\rho))$$

- ☑ Random walk (RW) fluid
  $D(\rho) = 1/2,\ \sigma(\rho) = \rho$
- ☑ Kipnis-Marchioro-Presutti (KMP) heat transport model
  $D(\rho) = 1/2,\ \sigma(\rho) = \rho^2$
- ☑ Katz-Lebowitz-Spohn (KLS) lattice gas
  $D(\rho) = \mathcal{J}(\rho)/\chi(\rho),\ \sigma(\rho) = 2D(\rho)\chi(\rho)$
- ☑ Weakly asymmetric simple exclusions process (WASEP)
  $D(\rho) = 1/2,\ \sigma(\rho) = \rho(1-\rho)$

EXAMPLES

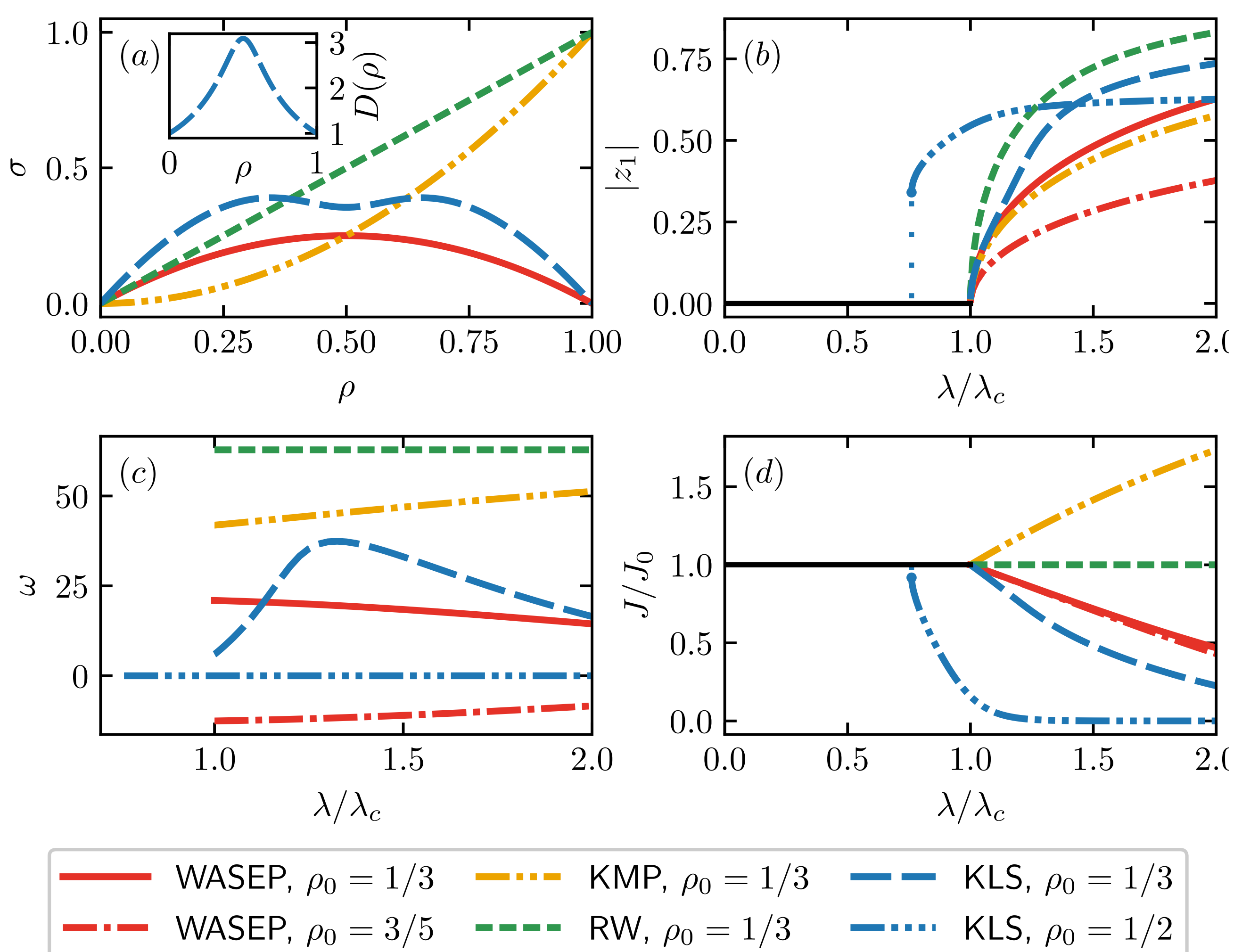
$$\mathcal{J}(\rho) = \frac{\nu[1+\delta(1-2\rho)] - \eta\sqrt{4\rho(1-\rho)}}{\nu^3} \qquad \nu \equiv \frac{1+\sqrt{(2\rho-1)^2+4\rho(1-\rho)e^{-4\beta}}}{\sqrt{4\rho(1-\rho)}}$$

$$\chi(\rho) = \rho(1-\rho)\sqrt{(2\rho-1)^2+4\rho(1-\rho)e^{-4\beta}} \qquad e^{4\beta} \equiv \frac{1+\eta}{1-\eta}$$

- To illustrate these ideas, we apply them to **4 paradigmatic lattice gas models**

$$\partial_t \rho = -\Big(-D(\rho)\partial_x\rho + \sigma(\rho)E_\lambda^{(m)}(\rho;x)\Big) \qquad E_\lambda^{(m)}(\rho;x) = \epsilon + \lambda r_m(\rho)\sin(2\pi m x - \phi_m(\rho))$$

- ☑ Random walk (**RW**) fluid
  $D(\rho) = 1/2,\ \sigma(\rho) = \rho$
- ☑ Katz-Lebowitz-Spohn (**KLS**) lattice gas
  $D(\rho) = \mathcal{J}(\rho)/\chi(\rho),\ \sigma(\rho) = 2D(\rho)\chi(\rho)$
- ☑ Kipnis-Marchioro-Presutti (**KMP**) heat transport model
  $D(\rho) = 1/2,\ \sigma(\rho) = \rho^2$
- ☑ Weakly asymmetric simple exclusions process (**WASEP**)
  $D(\rho) = 1/2,\ \sigma(\rho) = \rho(1-\rho)$



EXAMPLES

$$\mathcal{J}(\rho) = \frac{\nu[1+\delta(1-2\rho)] - \eta\sqrt{4\rho(1-\rho)}}{\nu^3} \qquad \nu \equiv \frac{1+\sqrt{(2\rho-1)^2+4\rho(1-\rho)e^{-4\beta}}}{\sqrt{4\rho(1-\rho)}}$$

$$\chi(\rho) = \rho(1-\rho)\sqrt{(2\rho-1)^2+4\rho(1-\rho)e^{-4\beta}} \qquad e^{4\beta} \equiv \frac{1+\eta}{1-\eta}$$

- To illustrate these ideas, we apply them to **4 paradigmatic lattice gas models**

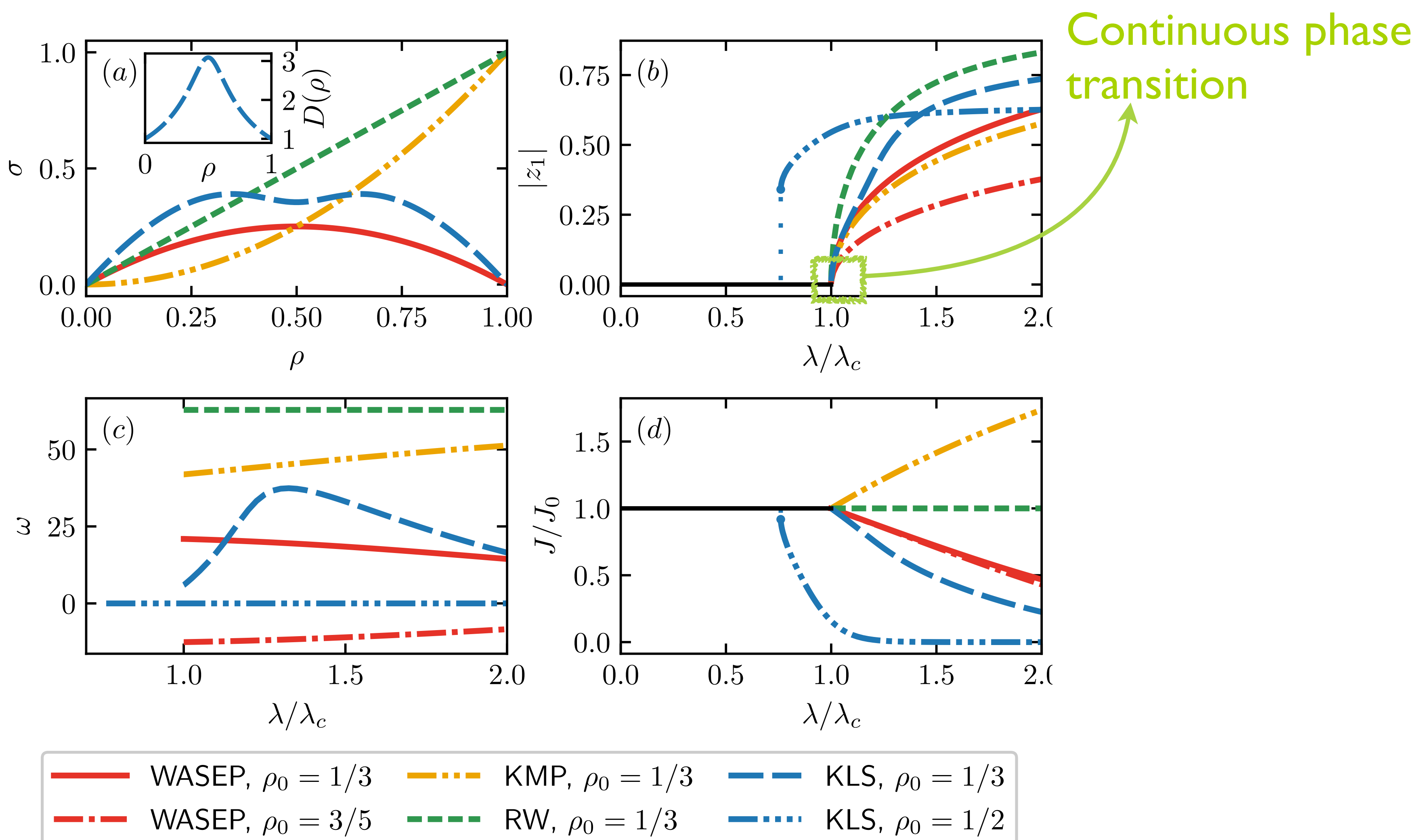
$$\partial_t \rho = -\Big(-D(\rho)\partial_x \rho + \sigma(\rho) E_\lambda^{(m)}(\rho;x)\Big) \qquad E_\lambda^{(m)}(\rho;x) = \epsilon + \lambda r_m(\rho)\sin(2\pi m x - \phi_m(\rho))$$

☑ Random walk (RW) fluid
$D(\rho) = 1/2,\ \sigma(\rho) = \rho$

☑ Katz-Lebowitz-Spohn (KLS) lattice gas
$D(\rho) = \mathcal{J}(\rho)/\chi(\rho),\ \sigma(\rho) = 2D(\rho)\chi(\rho)$

☑ Kipnis-Marchioro-Presutti (KMP) heat transport model
$D(\rho) = 1/2,\ \sigma(\rho) = \rho^2$

☑ Weakly asymmetric simple exclusions process (WASEP)
$D(\rho) = 1/2,\ \sigma(\rho) = \rho(1-\rho)$



EXAMPLES

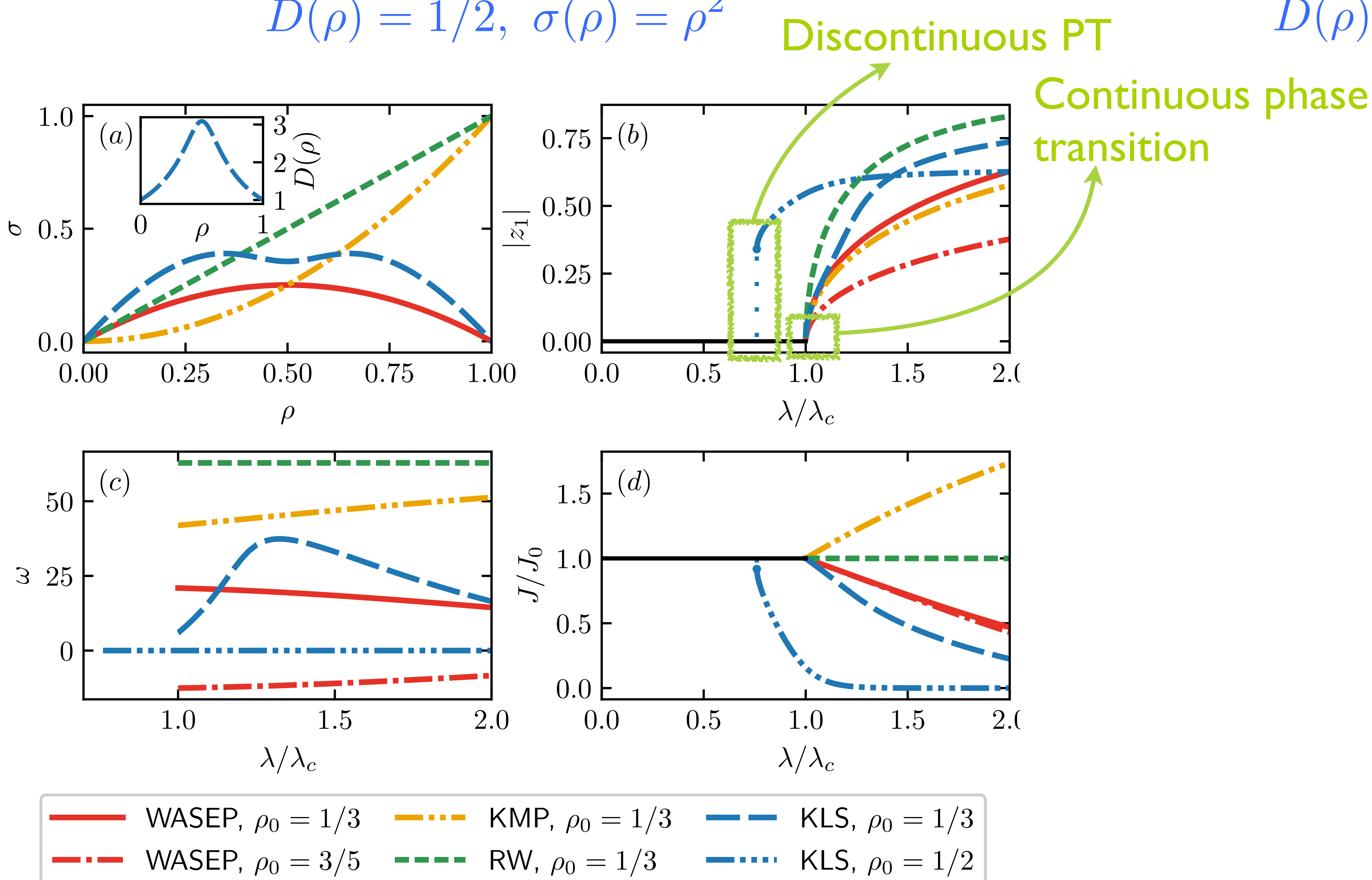
$$\mathcal{J}(\rho) = \frac{\nu[1+\delta(1-2\rho)] - \eta\sqrt{4\rho(1-\rho)}}{\nu^3} \qquad \nu \equiv \frac{1+\sqrt{(2\rho-1)^2+4\rho(1-\rho)e^{-4\beta}}}{\sqrt{4\rho(1-\rho)}}$$

$$\chi(\rho) = \rho(1-\rho)\sqrt{(2\rho-1)^2+4\rho(1-\rho)e^{-4\beta}} \qquad e^{4\beta} \equiv \frac{1+\eta}{1-\eta}$$

- To illustrate these ideas, we apply them to **4 paradigmatic lattice gas models**

$$\partial_t \rho = -\Big(-D(\rho)\partial_x\rho + \sigma(\rho)E_\lambda^{(m)}(\rho;x)\Big) \qquad E_\lambda^{(m)}(\rho;x) = \epsilon + \lambda r_m(\rho)\sin(2\pi m x - \phi_m(\rho))$$

- ☑ Random walk (**RW**) fluid
  $D(\rho)=1/2,\ \sigma(\rho)=\rho$
- ☑ Katz-Lebowitz-Spohn (**KLS**) lattice gas
  $D(\rho)=\mathcal{J}(\rho)/\chi(\rho),\ \sigma(\rho)=2D(\rho)\chi(\rho)$
- ☑ Kipnis-Marchioro-Presutti (**KMP**) heat transport model
  $D(\rho)=1/2,\ \sigma(\rho)=\rho^2$
- ☑ Weakly asymmetric simple exclusions process (**WASEP**)
  $D(\rho)=1/2,\ \sigma(\rho)=\rho(1-\rho)$



EXAMPLES

$$\mathcal{J}(\rho) = \frac{\nu[1+\delta(1-2\rho)] - \eta\sqrt{4\rho(1-\rho)}}{\nu^3} \qquad \nu \equiv \frac{1+\sqrt{(2\rho-1)^2+4\rho(1-\rho)e^{-4\beta}}}{\sqrt{4\rho(1-\rho)}}$$

$$\chi(\rho) = \rho(1-\rho)\sqrt{(2\rho-1)^2+4\rho(1-\rho)e^{-4\beta}} \qquad e^{4\beta} \equiv \frac{1+\eta}{1-\eta}$$

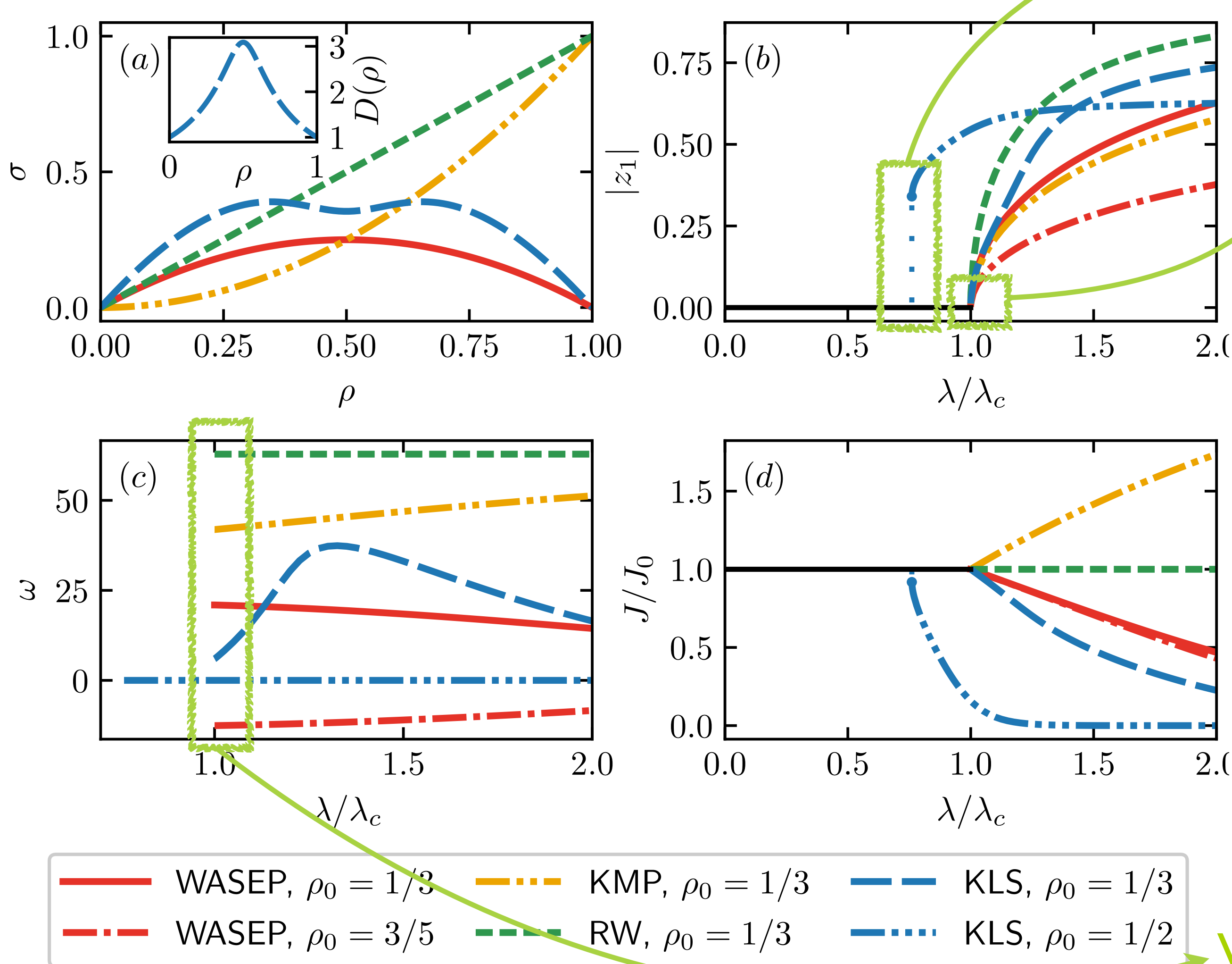
- To illustrate these ideas, we apply them to **4 paradigmatic lattice gas models**

$$\partial_t \rho = -\Big(-D(\rho)\partial_x\rho + \sigma(\rho)E_\lambda^{(m)}(\rho;x)\Big) \qquad E_\lambda^{(m)}(\rho;x) = \epsilon + \lambda r_m(\rho)\sin(2\pi m x - \phi_m(\rho))$$

- ☑ Random walk (**RW**) fluid
  $D(\rho) = 1/2,\ \sigma(\rho) = \rho$
- ☑ Katz-Lebowitz-Spohn (**KLS**) lattice gas
  $D(\rho) = \mathcal{J}(\rho)/\chi(\rho),\ \sigma(\rho) = 2D(\rho)\chi(\rho)$
- ☑ Kipnis-Marchioro-Presutti (**KMP**) heat transport model
  $D(\rho) = 1/2,\ \sigma(\rho) = \rho^2$
- ☑ Weakly asymmetric simple exclusions process (**WASEP**)
  $D(\rho) = 1/2,\ \sigma(\rho) = \rho(1-\rho)$

Discontinuous PT

Continuous phase transition



Velocity ω initially proportional to $\sigma'(\rho_0)$

EXAMPLES

$$\mathcal{J}(\rho) = \frac{\nu[1+\delta(1-2\rho)] - \eta\sqrt{4\rho(1-\rho)}}{\nu^3} \qquad \nu \equiv \frac{1+\sqrt{(2\rho-1)^2+4\rho(1-\rho)e^{-4\beta}}}{\sqrt{4\rho(1-\rho)}}$$

$$\chi(\rho) = \rho(1-\rho)\sqrt{(2\rho-1)^2+4\rho(1-\rho)e^{-4\beta}} \qquad e^{4\beta} \equiv \frac{1+\eta}{1-\eta}$$

- To illustrate these ideas, we apply them to **4 paradigmatic lattice gas models**

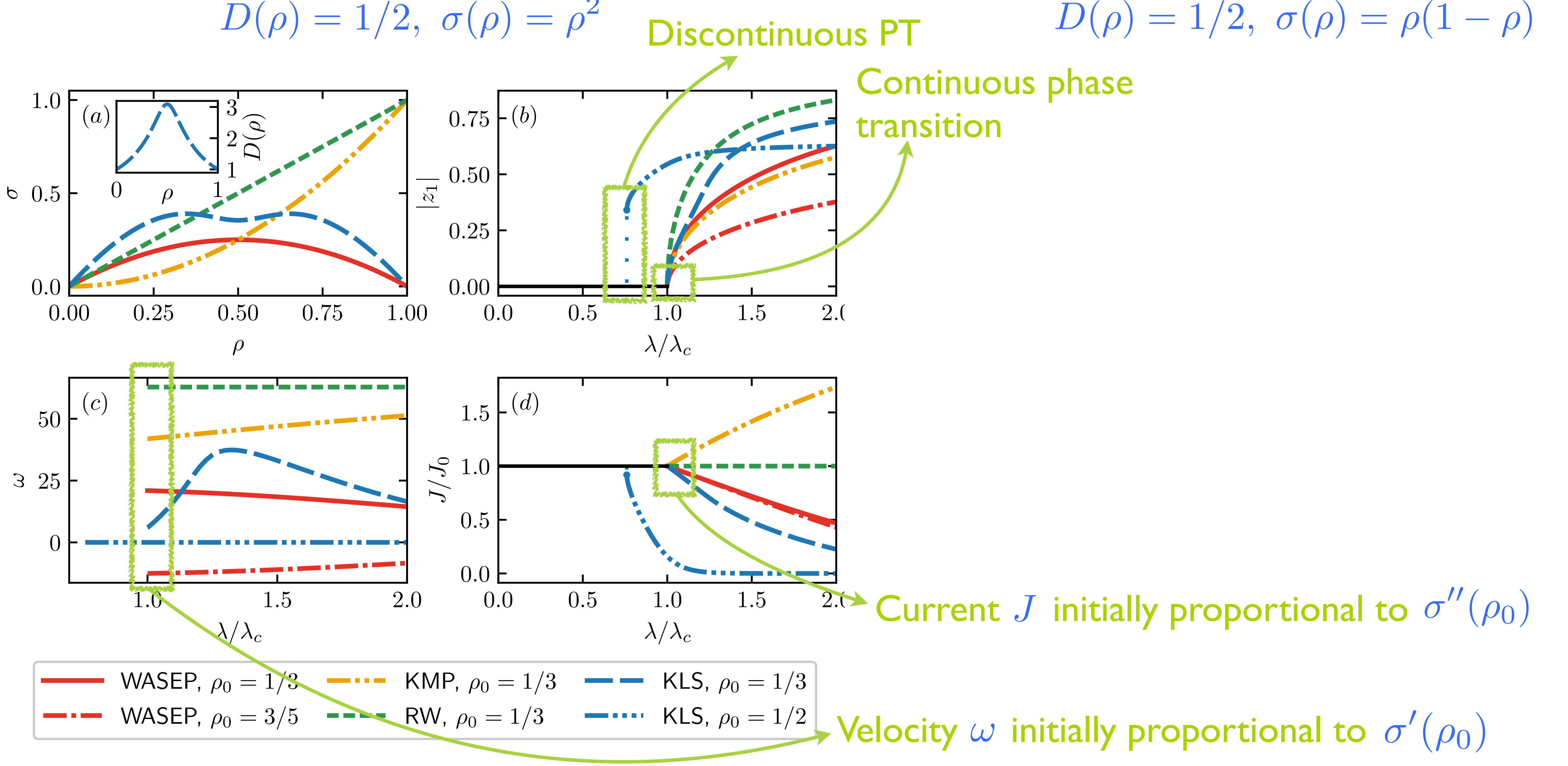
$$\partial_t\rho = -\Big(-D(\rho)\partial_x\rho + \sigma(\rho)E_\lambda^{(m)}(\rho;x)\Big) \qquad E_\lambda^{(m)}(\rho;x) = \epsilon + \lambda r_m(\rho)\sin(2\pi m x - \phi_m(\rho))$$

☑ Random walk (RW) fluid
$D(\rho) = 1/2,\ \sigma(\rho) = \rho$

☑ Katz-Lebowitz-Spohn (KLS) lattice gas
$D(\rho) = \mathcal{J}(\rho)/\chi(\rho),\ \sigma(\rho) = 2D(\rho)\chi(\rho)$

☑ Kipnis-Marchioro-Presutti (KMP) heat transport model
$D(\rho) = 1/2,\ \sigma(\rho) = \rho^2$

☑ Weakly asymmetric simple exclusions process (WASEP)
$D(\rho) = 1/2,\ \sigma(\rho) = \rho(1-\rho)$



EXAMPLES

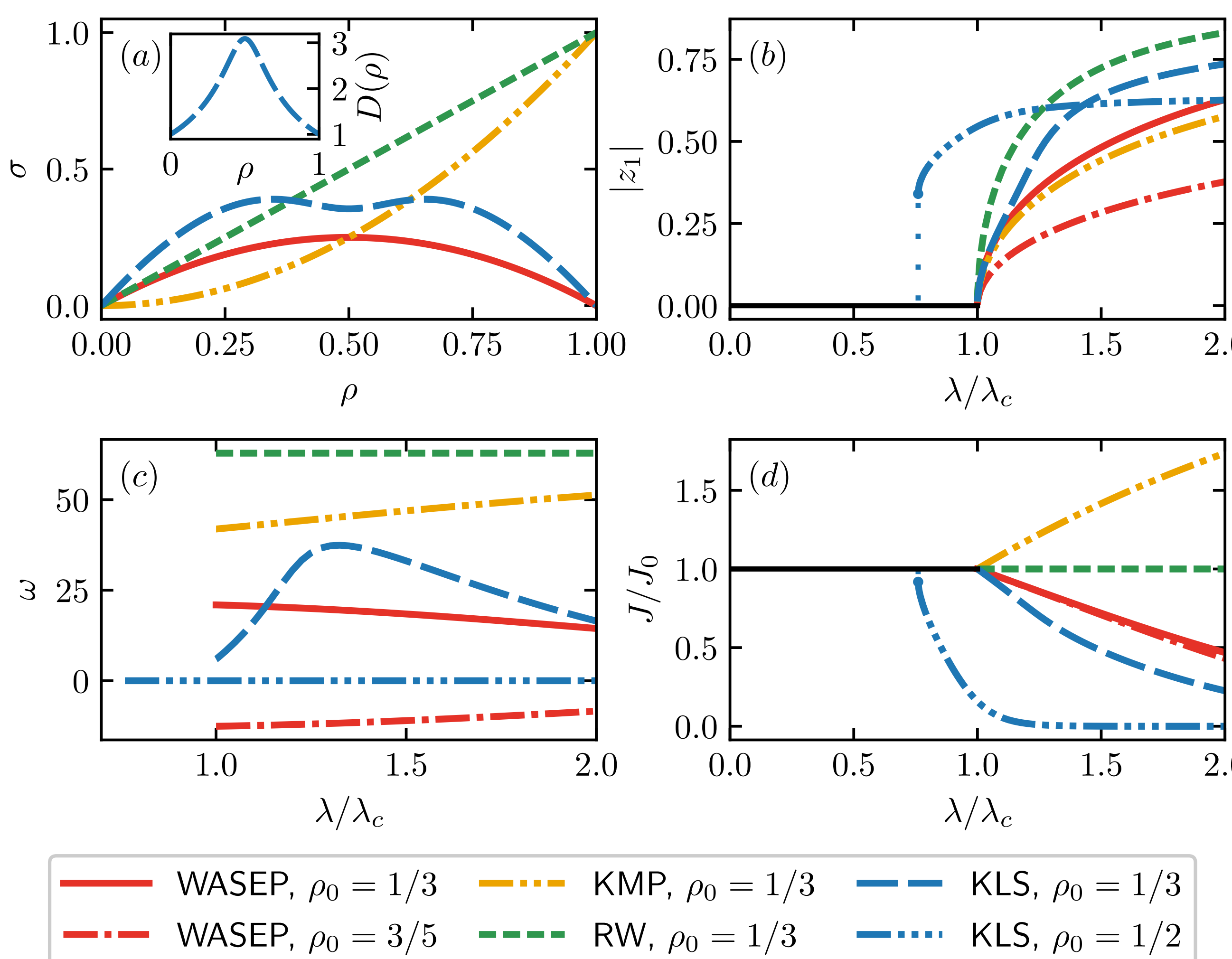
$$\mathcal{J}(\rho) = \frac{\nu[1+\delta(1-2\rho)] - \eta\sqrt{4\rho(1-\rho)}}{\nu^3} \qquad \nu \equiv \frac{1+\sqrt{(2\rho-1)^2+4\rho(1-\rho)e^{-4\beta}}}{\sqrt{4\rho(1-\rho)}}$$

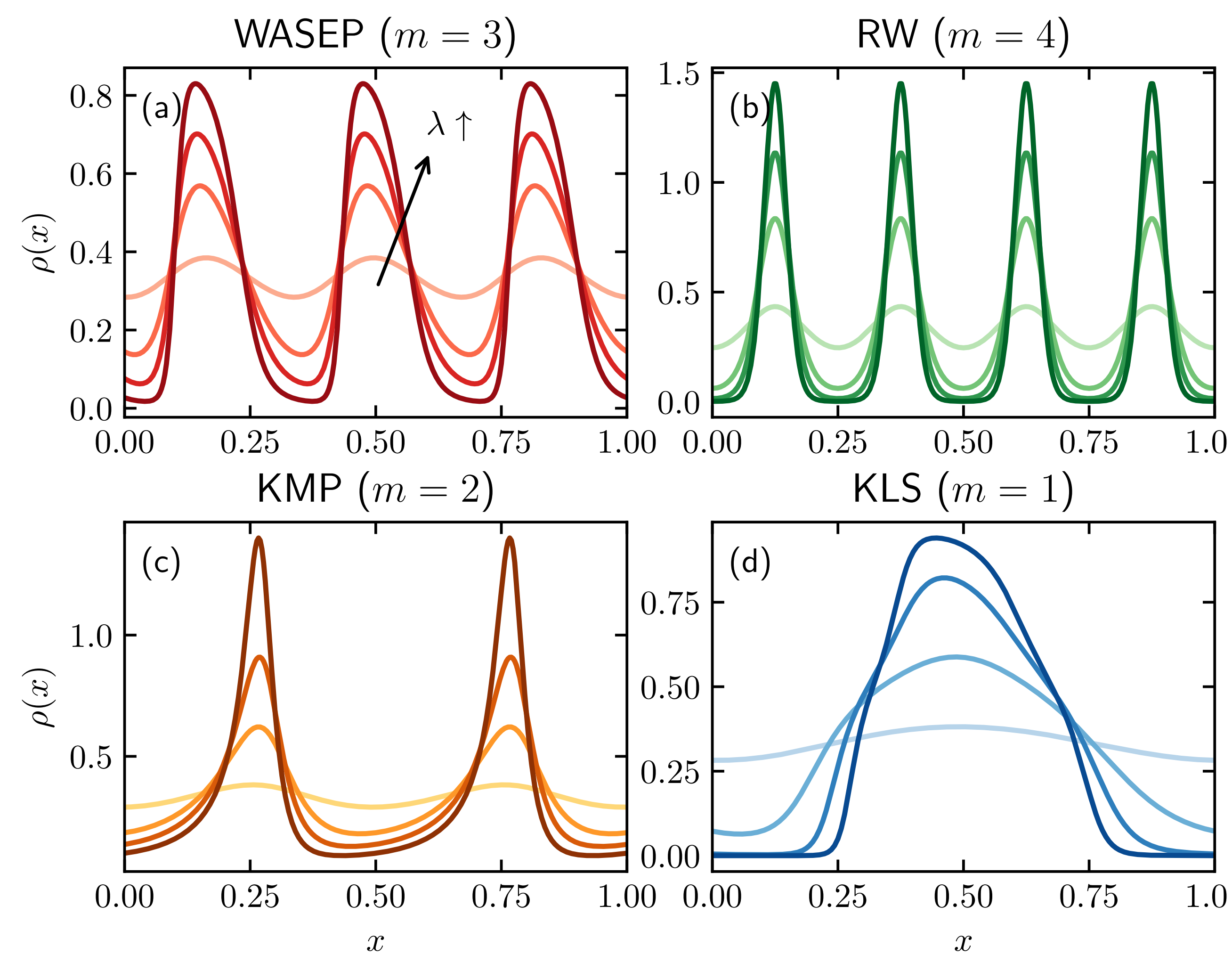
$$\chi(\rho) = \rho(1-\rho)\sqrt{(2\rho-1)^2+4\rho(1-\rho)e^{-4\beta}} \qquad e^{4\beta} \equiv \frac{1+\eta}{1-\eta}$$

- To illustrate these ideas, we apply them to **4 paradigmatic lattice gas models**

$$\partial_t\rho = -\Big(-D(\rho)\partial_x\rho + \sigma(\rho)E_\lambda^{(m)}(\rho;x)\Big) \qquad E_\lambda^{(m)}(\rho;x) = \epsilon + \lambda r_m(\rho)\sin(2\pi m x - \phi_m(\rho))$$

- ☑ Random walk (**RW**) fluid
  $D(\rho) = 1/2,\ \sigma(\rho) = \rho$
- ☑ Katz-Lebowitz-Spohn (**KLS**) lattice gas
  $D(\rho) = \mathcal{J}(\rho)/\chi(\rho),\ \sigma(\rho) = 2D(\rho)\chi(\rho)$
- ☑ Kipnis-Marchioro-Presutti (**KMP**) heat transport model
  $D(\rho) = 1/2,\ \sigma(\rho) = \rho^2$
- ☑ Weakly asymmetric simple exclusions process (**WASEP**)
  $D(\rho) = 1/2,\ \sigma(\rho) = \rho(1-\rho)$

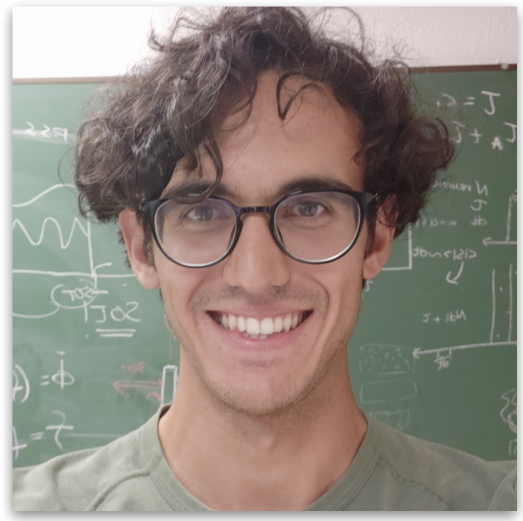




SUMMARY

- **Phase transitions forbidden in equilibrium** might be present at the level of **nonequilibrium fluctuations**
- **Time-crystal phases** have been identified for **unlikely currents** in simple diffusive systems (e.g. WASEP, KMP, etc.)
- These phenomena **can be made typical** (observed in the stationary state), and their origin can be traced back to an **instability triggered by a packing field mechanism**
- This leads to a systematic way of **'building'** these intriguing dynamical regimes
- We have shown how to **exploit the packing-field route to craft engineer and control on demand custom continuous time crystals** with m rotating condensates, which can be further enhanced with higher-order modes
- Overall, these results demonstrate the **versatility and broad possibilities of this promising route to time crystals**
- Similar approach could be exploited in **open quantum systems** (at last, $\hbar \neq 0$)

Phys. Rev. Lett. **125**, 160601 (2020)
Phys. Rev. E **108**, 014107 (2023)
arXiv 2404.xxxx (2024)



Carlos Pérez-Espigares
(Granada)

Rubén Hurtado
(Granada)

Federico Carollo
(Tübingen)

Thanks for your attention

ugr Universidad de Granada