SPECTRAL SIGNATURES OF DYNAMICAL PHASE TRANSITIONS

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- **Dynamical phase transitions (DPTs)** have been found in the **trajectory statistics** of **classical and quantum systems**

- DPTs appear **when conditioning a system to have a fixed value of some time-integrated observable**, such as, e.g., the **current** or the **activity**

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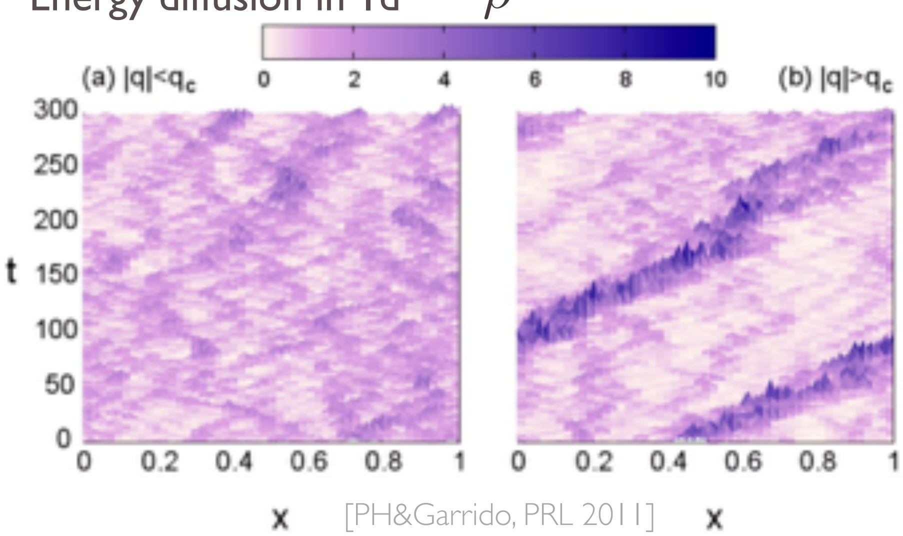

[PH&Garrido, PRL 2011]

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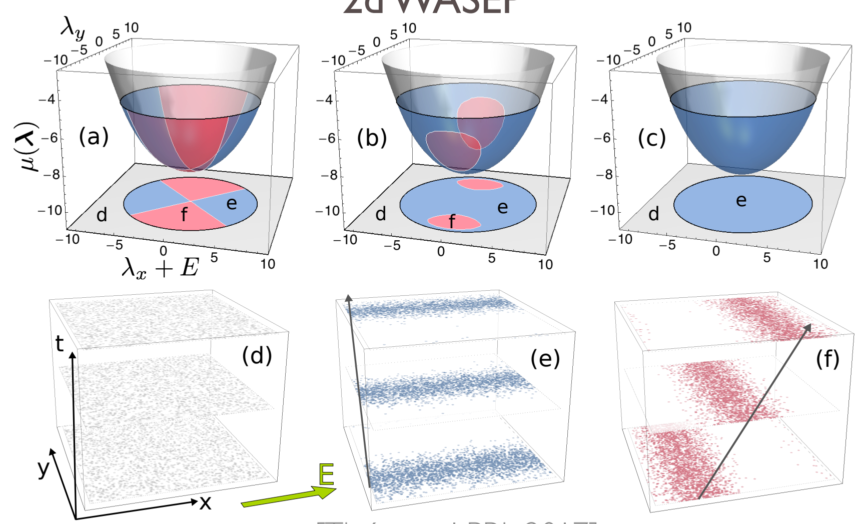

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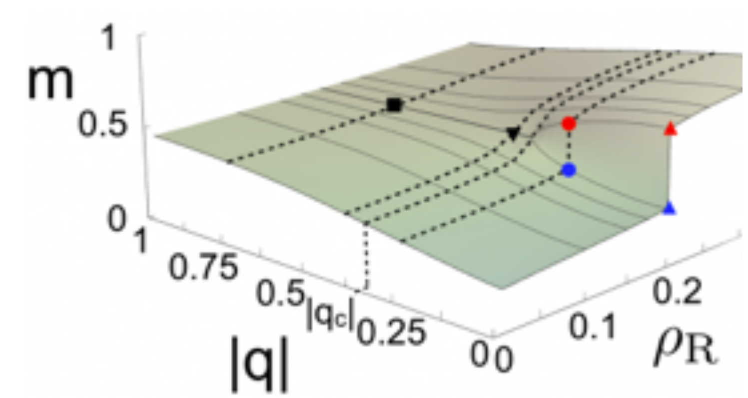

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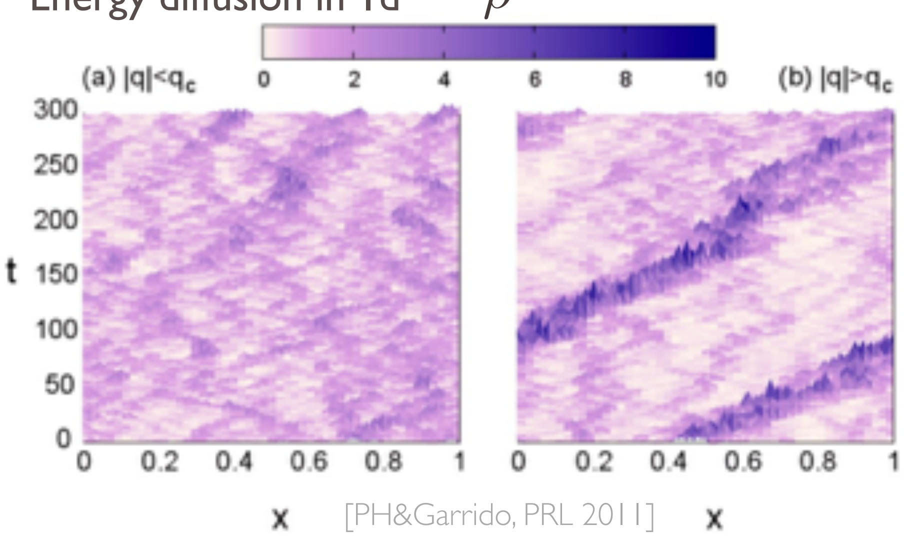

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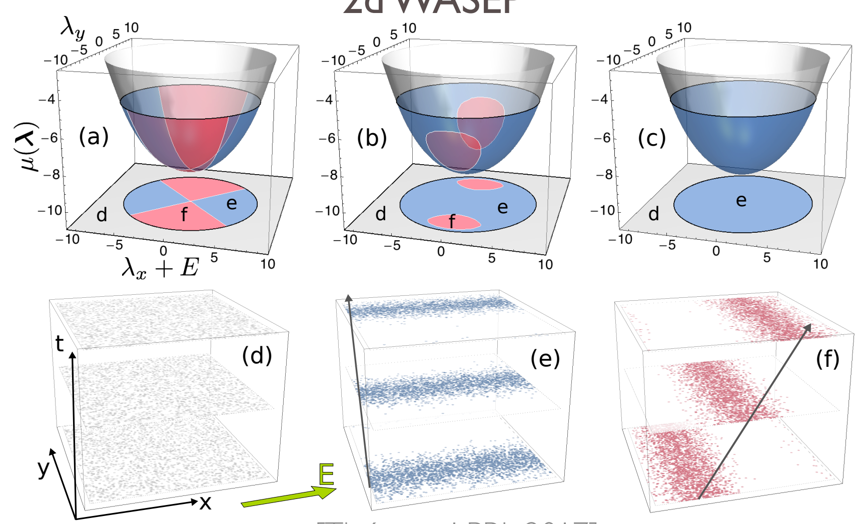

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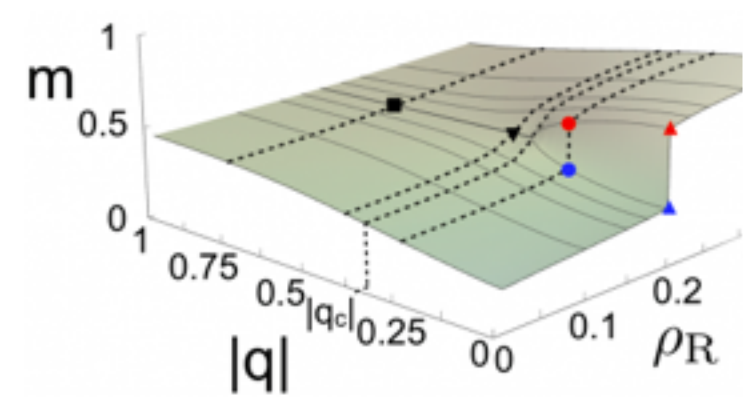

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- **Macroscopic fluctuation theory** has underpinned most progress on DPTs. **What about microscopic understanding?**

> Interestingly, many DPTs involve the spontaneous breaking of a Z_n symmetry

> How does spontaneous symmetry breaking appear at the microscopic level?

CURRENT FLUCTUATIONS FROM MICROSCOPICS

- **Quantum hamiltonian formalism** for the **master equation** $|P(t)\rangle = \sum_C P(C,t)\,|C\rangle$

$$\partial_t |P(t)\rangle = \mathbb{W}|P(t)\rangle$$

- **Markov generator** $\mathbb{W} = \sum_{C,C' \neq C} W_{C\to C'}\,|C'\rangle\,\langle C| - \sum_C R(C)\,|C\rangle\,\langle C|$

Exit rate

$$R(C) = \sum_{C'} W_{C\to C'}$$

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- **Ensemble of trajectories** conditioned on the **current** $Q = \sum_i q_{C_i C_{i-1}}$

$$P_t(Q) \sim \mathrm{e}^{-tG(Q/t)}$$

[Ruelle, Gartner&Ellis, Lebowitz&Spohn, Lecomte et al, and many others]

- **Dynamical partition function:**

$$Z_t(\lambda) = \sum_Q P_t(Q)\mathrm{e}^{\lambda Q} \sim \mathrm{e}^{t\theta(\lambda)}$$

- **Dynamical free energy** $\theta(\lambda) = -\min_q[G(q)-\lambda q]$ **largest eigenvalue of biased generator**

$$\mathbb{W}^\lambda = \sum_{C,C'\neq C} \mathrm{e}^{\lambda q_{C'C}} W_{C\to C'}\,|C'\rangle\,\langle C| - \sum_C R_C|C\rangle\langle C|$$

Biased jumps

No conservation of probability

- **Spectrum of** $\mathbb{W}^\lambda$:

$$\mathbb{W}^\lambda\,|R_i^\lambda\rangle = \theta_i(\lambda)\,|R_i^\lambda\rangle \qquad \langle L_i^\lambda|\,\mathbb{W}^\lambda = \theta_i(\lambda)\,\langle L_i^\lambda| \qquad \theta(\lambda) = \theta_0(\lambda)$$

MAKING RARE EVENTS TYPICAL

- $\mathbb{W}^\lambda$ generates **atypical trajectories** but $\langle -| \, \mathbb{W}^\lambda \neq 0$ **(non-physical!)**

- We can **make rare events TYPICAL** using **Doob's transform:** [Jack & Sollich 2010, Popkov et al 2010, Chetrite & Touchette 2015]

$$\mathbb{W}_{\rm D}^\lambda \equiv \mathbb{L}_0 \mathbb{W}^\lambda \mathbb{L}_0^{-1} - \theta_0(\lambda) \qquad \text{with} \quad (\mathbb{L}_0)_{ij} = \left(\langle L_0^\lambda | \right)_i \delta_{ij}$$

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- $\mathbb{W}_{\rm D}^\lambda$ **spectrum** is simply related to that of $\mathbb{W}^\lambda$

$$\theta_i^D(\lambda) = \theta_i(\lambda) - \theta_0(\lambda) \qquad \left| R_{i,\rm D}^\lambda \right\rangle = \mathbb{L}_0 \left| R_i^\lambda \right\rangle \qquad \left\langle L_{i,\rm D}^\lambda \right| = \left\langle L_i^\lambda \right| \mathbb{L}_0^{-1}$$

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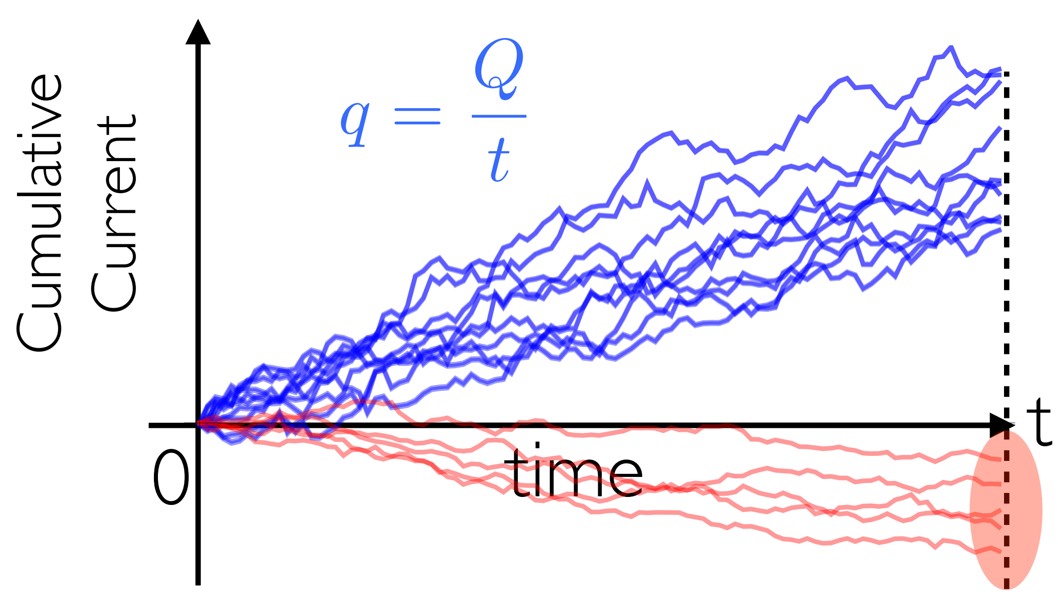

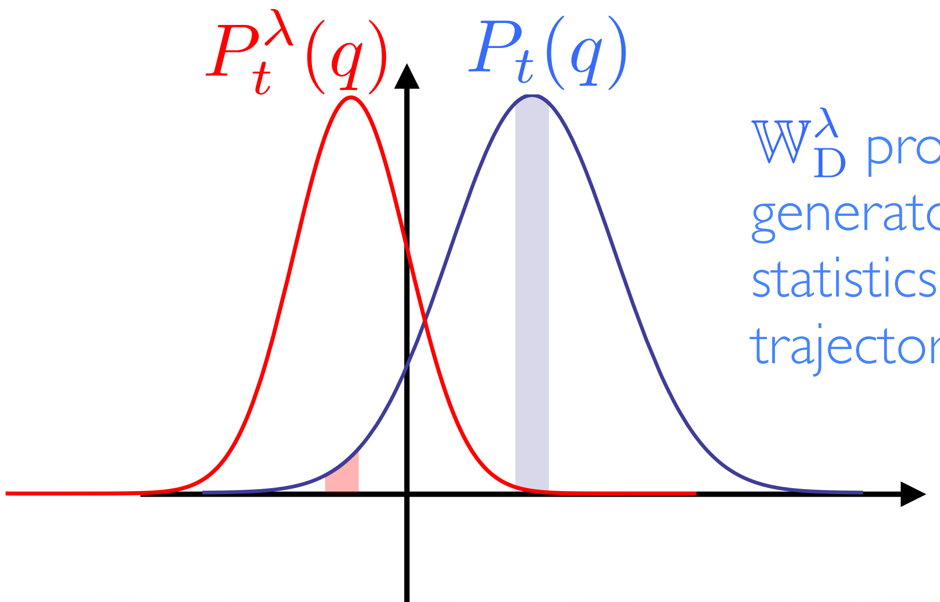

$\mathbb{W}_{\mathrm{D}}^\lambda$ proper stochastic generator for the statistics of atypical trajectories

DPTs hence appear as standard critical phenomena for the Doob's dynamics

DPTS, SYMMETRY AND DEGENERACY

- **Many DPTs** are accompanied by a **spontaneous Z_n symmetry breaking phenomenon**

- **Z_n group:** cyclic group of order n, generated by unitary operator $\hat{S} \in \mathbb{Z}_n$, with $\hat{S}^n = \mathbb{I}$

- $\hat{S}$ **symmetry of the stochastic process** iff $[\mathbb{W}, \hat{S}] = 0$. Moreover, if $\hat{S}$ leaves invariant then current, then $[\mathbb{W}_{\mathrm{D}}^{\lambda}, \hat{S}] = 0$ and **there is a common eigenbasis**

$$\hat{S}\,|R_{j,\mathrm{D}}^{\lambda}\rangle = \phi_j\,|R_{j,\mathrm{D}}^{\lambda}\rangle \qquad \langle L_{j,\mathrm{D}}^{\lambda}|\,\hat{S} = \phi_j\,\langle L_{j,\mathrm{D}}^{\lambda}| \qquad \phi_j = \mathrm{e}^{i2\pi k_j/n} \qquad k_j = 0, 1, \ldots, n-1$$

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- **Solution of Doob master equation:** $\mathrm{Re}(\theta_{j,\mathrm{D}}^{\lambda}) \le 0\ \forall j$ (assume $\mathrm{Im}(\theta_{j,\mathrm{D}}^{\lambda}) = 0$)

steady state

$$|P_{t,P_0}^{\lambda}\rangle = \mathrm{e}^{+t\hat{\mathbb{W}}_{\mathrm{D}}^{\lambda}}|P_0\rangle = |R_{0,\mathrm{D}}^{\lambda}\rangle + \sum_{j>0} \mathrm{e}^{t\theta_{j,\mathrm{D}}^{\lambda}}|R_{j,\mathrm{D}}^{\lambda}\rangle\langle L_{j,\mathrm{D}}^{\lambda}|P_0\rangle \xrightarrow{t\to\infty} |P_{\mathrm{ss},P_0}^{\lambda}\rangle$$

- **If** $\mathbb{W}_{\mathrm{D}}^{\lambda}$ **is gapped:**

$$|P_{\mathrm{ss},P_0}^{\lambda}\rangle = |R_{0,\mathrm{D}}^{\lambda}\rangle$$

All probability

No symmetry breaking for gapped spectrum

$|R_{0,\mathrm{D}}^{\lambda}\rangle$ is invariant under $\hat{S}$

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- **For gapless, n-fold degenerate $\mathbb{W}_\mathrm{D}^\lambda$**

$$|P_{\mathrm{ss},P_0}^\lambda\rangle = |R_{0,\mathrm{D}}^\lambda\rangle + \sum_{j=1}^{n-1} |R_{j,\mathrm{D}}^\lambda\rangle\,\langle L_{j,\mathrm{D}}^\lambda|P_0\rangle$$

Probability redistribution

Symmetry is broken in the gapless phase

$$\hat{S}|P_{\mathrm{ss},P_0}^\lambda\rangle \neq |P_{\mathrm{ss},P_0}^\lambda\rangle$$

PHASE PROBABILITY VECTORS

- **Degeneracy** implies the appearance of **different steady states**.

- Define **n independent phase probability vectors** $|\Pi_l^\lambda\rangle$, with $l \in [0 \,..\, n-1]$, normalized $\langle -|\Pi_l^\lambda\rangle = 1$, and related via the symmetry operator, $|\Pi_{l+1}^\lambda\rangle = \hat{S}\,|\Pi_l^\lambda\rangle$

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- The **phase probability vectors** associated to each **symmetry broken sector** are thus

$$|\Pi_l^\lambda\rangle = \sum_{j=0}^{n-1} (\phi_j)^l |R_{j,\mathrm{D}}^\lambda\rangle \qquad \longrightarrow \qquad |R_{j,\mathrm{D}}^\lambda\rangle = \frac{1}{n}\sum_{l=0}^{n-1} (\phi_j)^{-l}\,|\Pi_l^\lambda\rangle$$

- **Doob steady state** as **weighted sum of phase probability vectors**

$$|P_{\mathrm{ss},P_0}^\lambda\rangle = \sum_{l=0}^{n-1} w_l |\Pi_l^\lambda\rangle \qquad \longrightarrow \qquad w_l = \frac{1}{n} + \frac{1}{n}\sum_{j=1}^{n-1} (\phi_j)^{-l} \langle L_{j,\mathrm{D}}^\lambda | P_0\rangle$$

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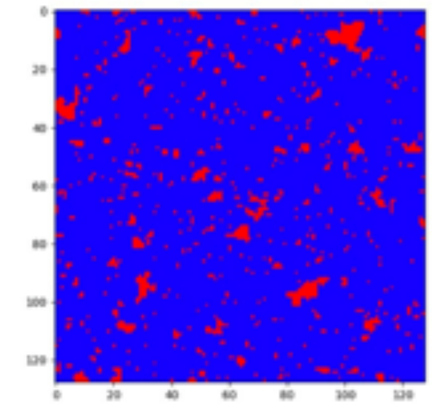

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- Once a **symmetry-breaking DPT** kicks in, **statistically-relevant configurations fall into well-defined symmetry classes** ℓ_C

$$\frac{\langle C|\Pi_l^\lambda\rangle}{\langle C|\Pi_{\ell_C}^\lambda\rangle} \approx 0\,, \qquad \forall l \neq \ell_C$$

- This observation implies a **hidden spectral structure in the degenerate subspace**

$$\langle C|R_{j,\mathrm{D}}^\lambda\rangle \approx (\phi_j)^{-\ell_C} \langle C|R_{0,\mathrm{D}}^\lambda\rangle$$

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- **Order parameter space inherits structure:**

$$||P^\lambda_{\mathrm{ss},P_0}\rangle\rangle = ||R^\lambda_{0,\mathrm{D}}\rangle\rangle + \sum_{j=1}^{n-1} ||R^\lambda_{j,\mathrm{D}}\rangle\rangle\, \langle L^\lambda_{j,\mathrm{D}}|P_0\rangle$$

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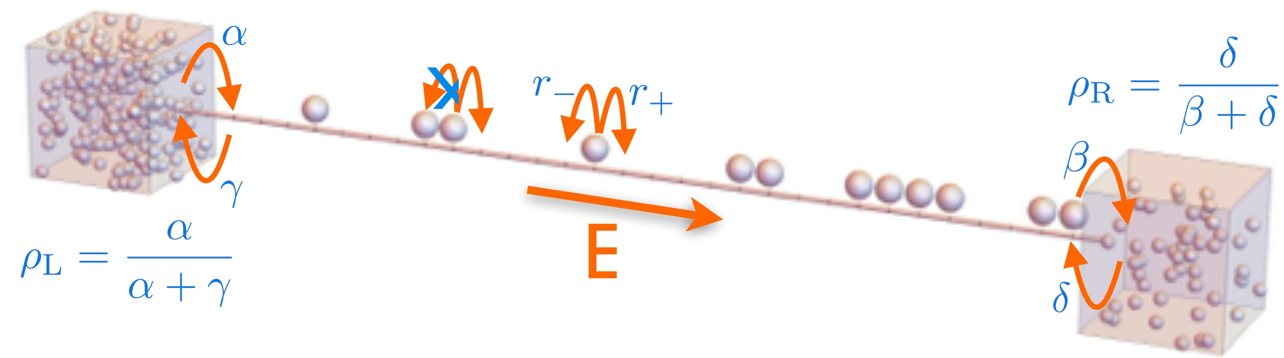

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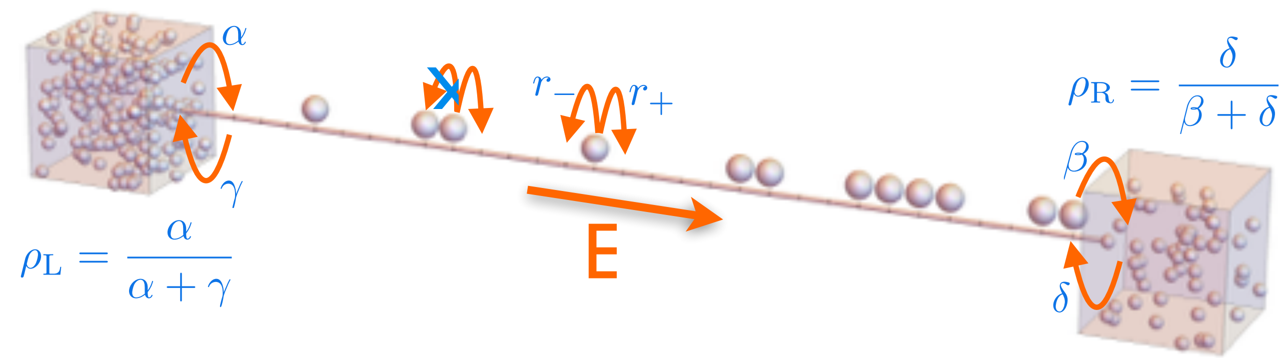

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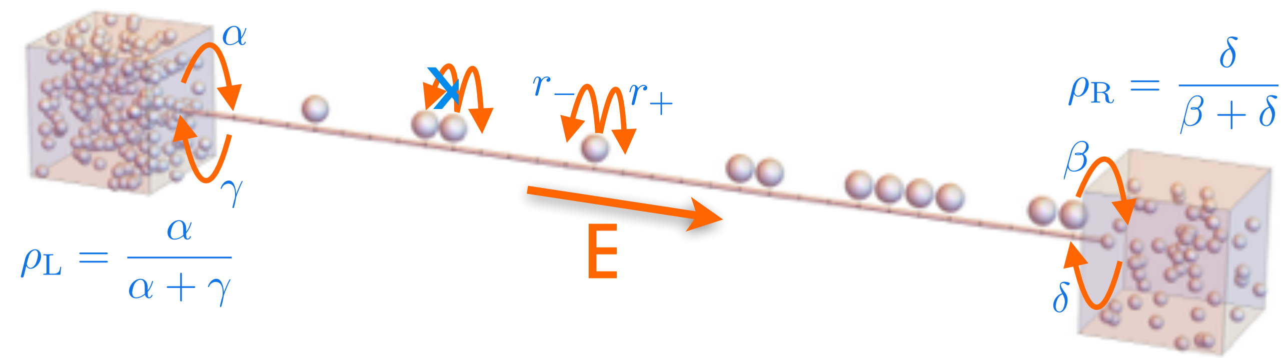

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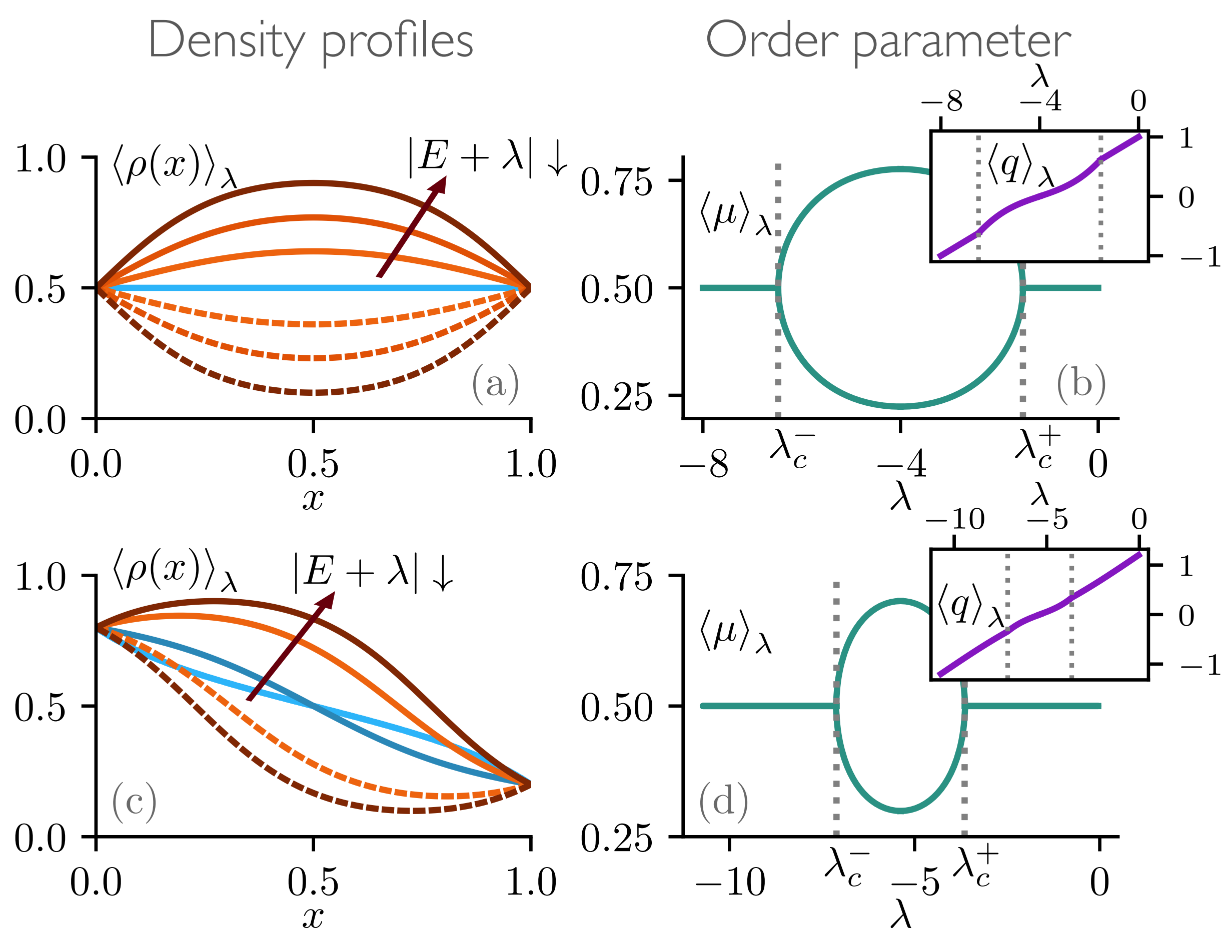

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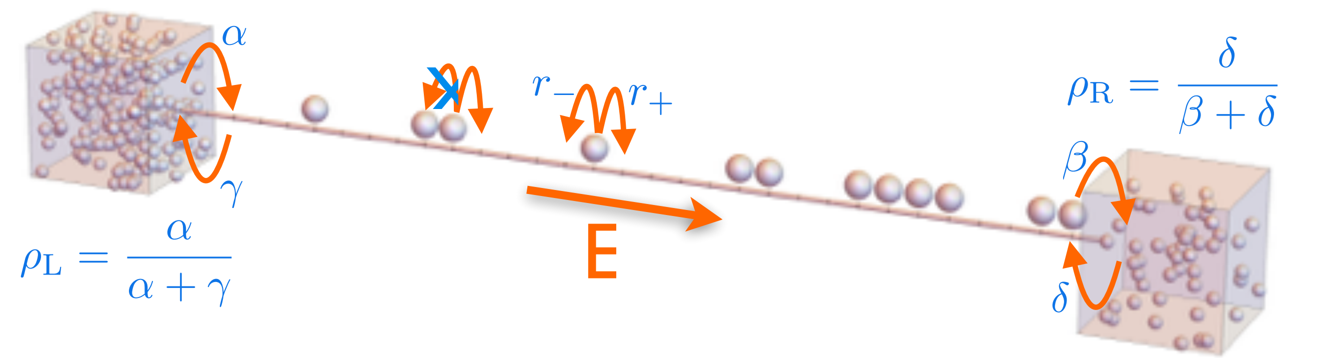

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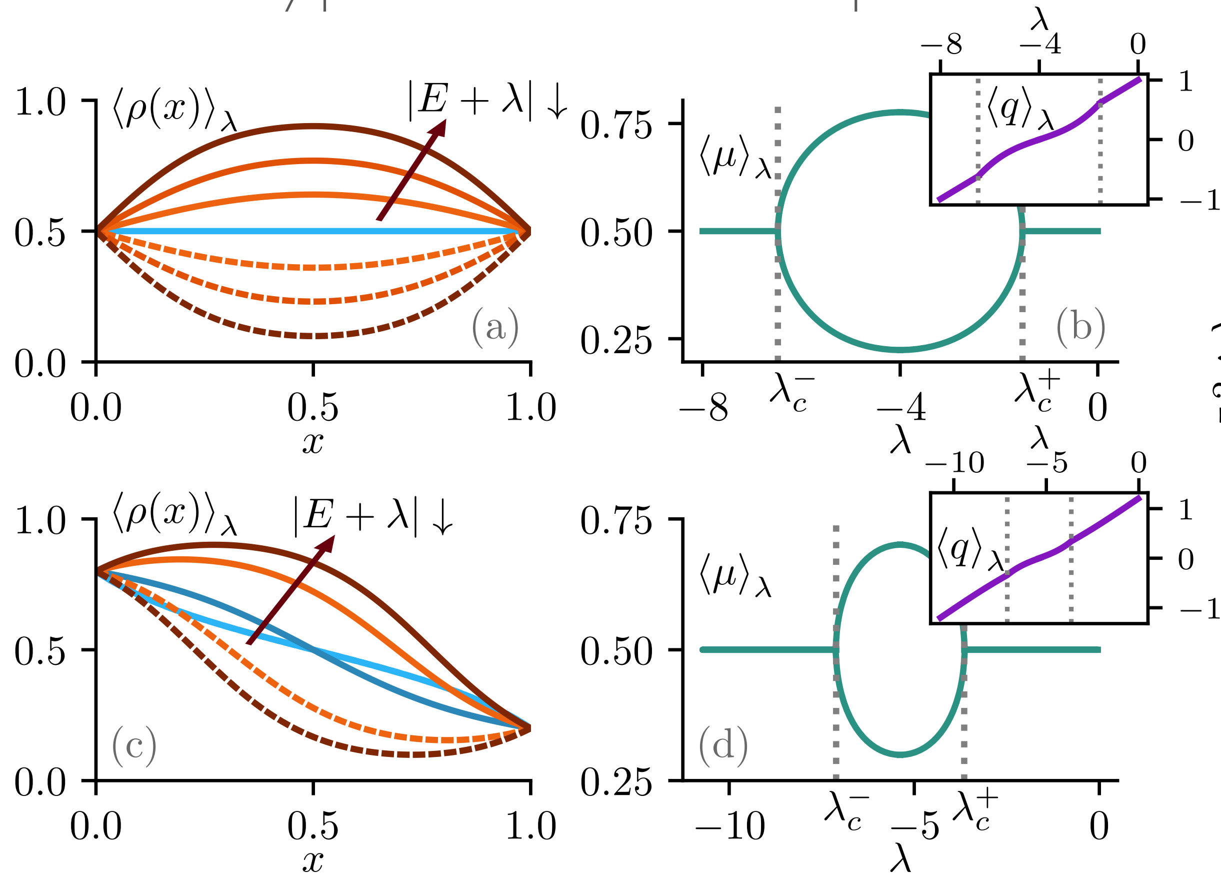
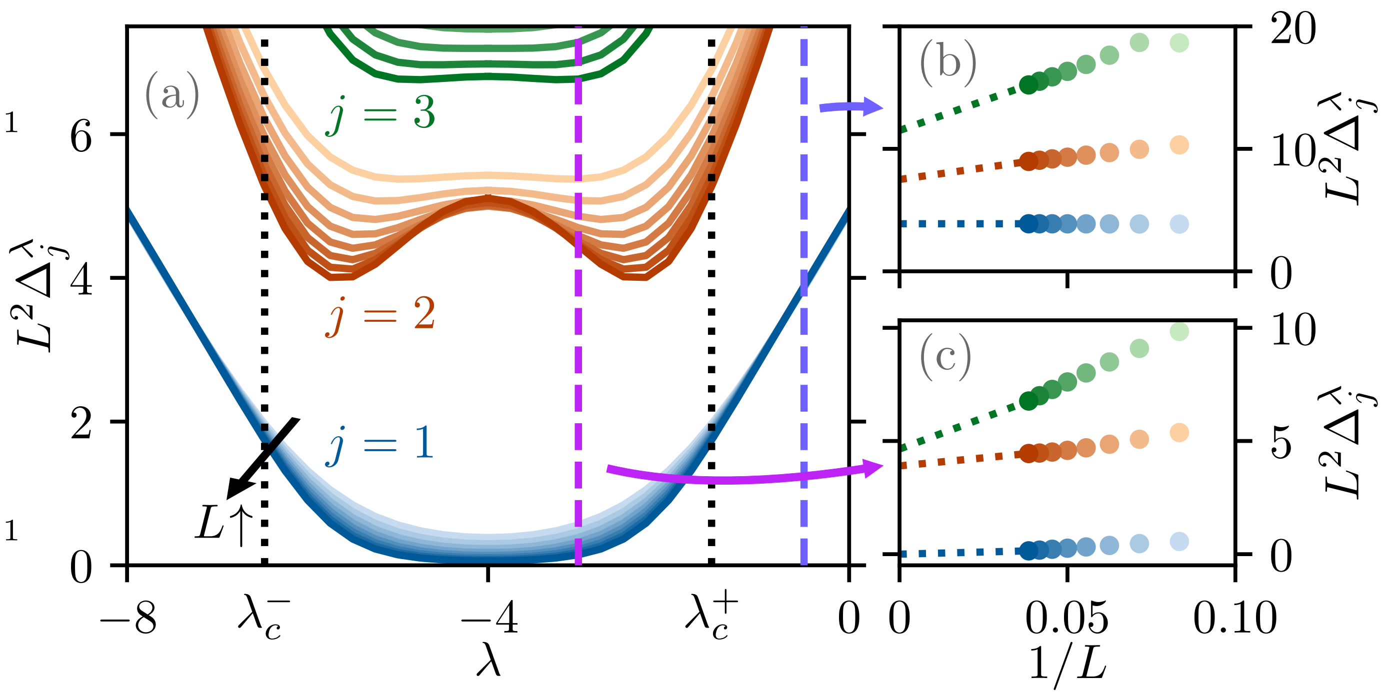
Density profiles

Order parameter





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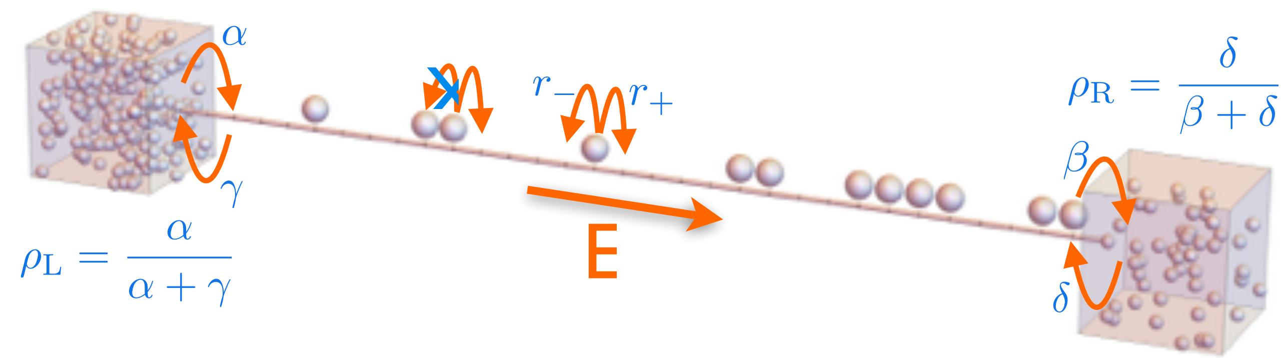

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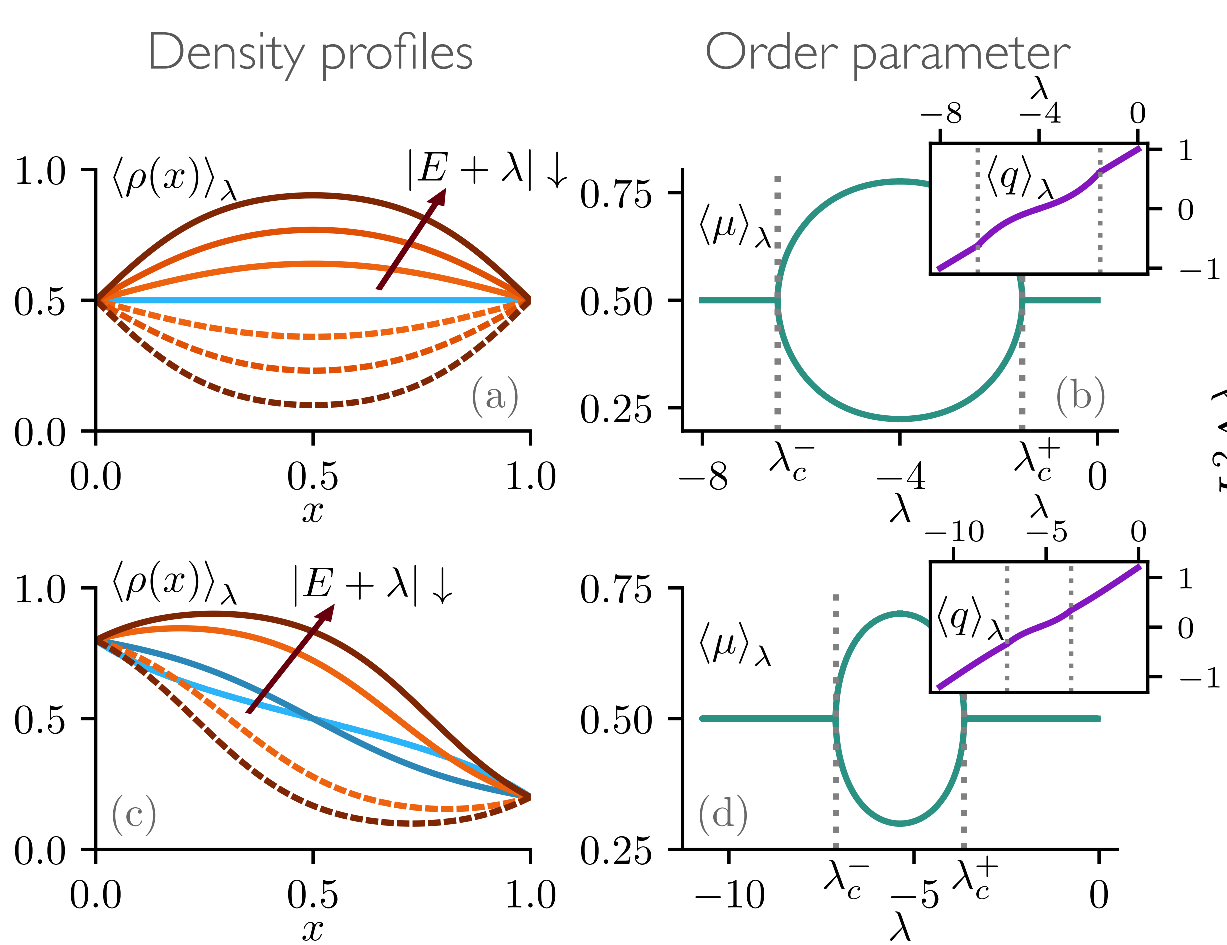
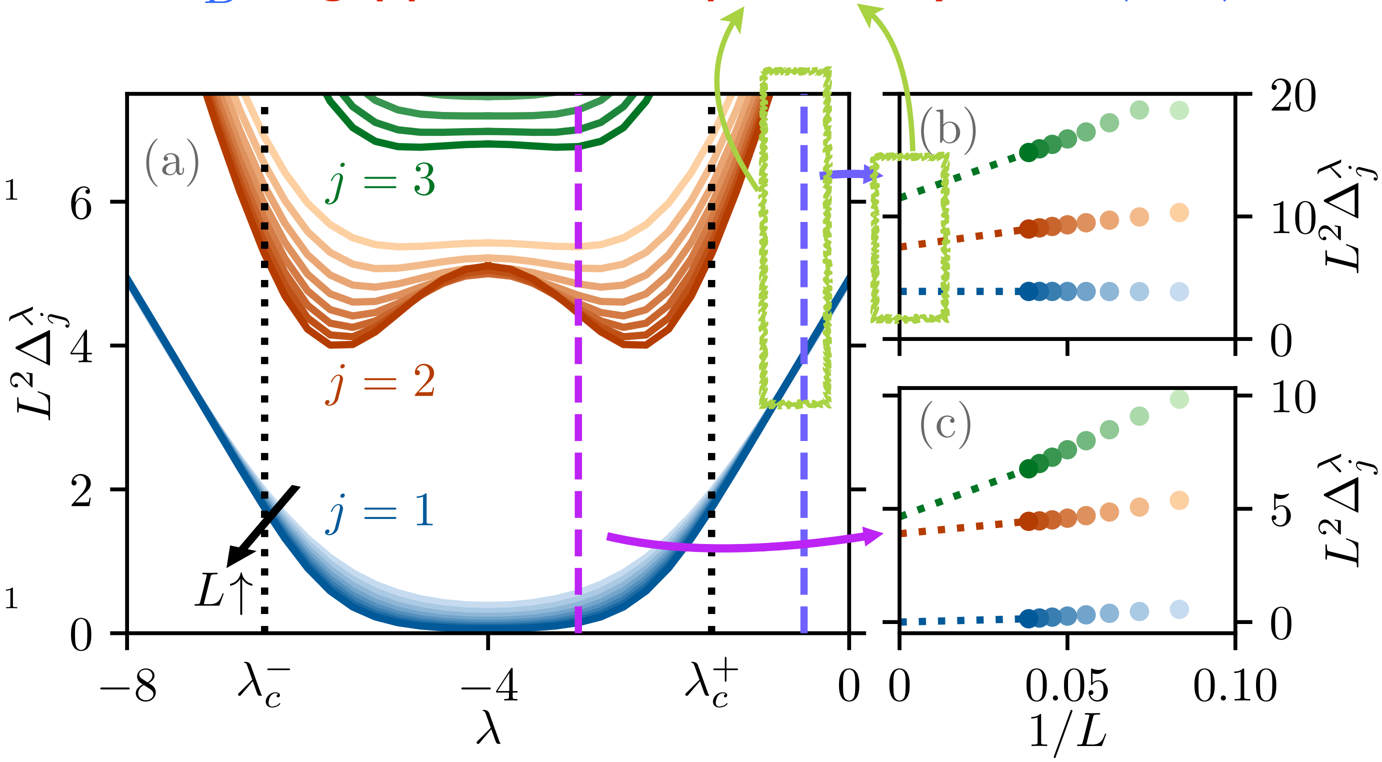
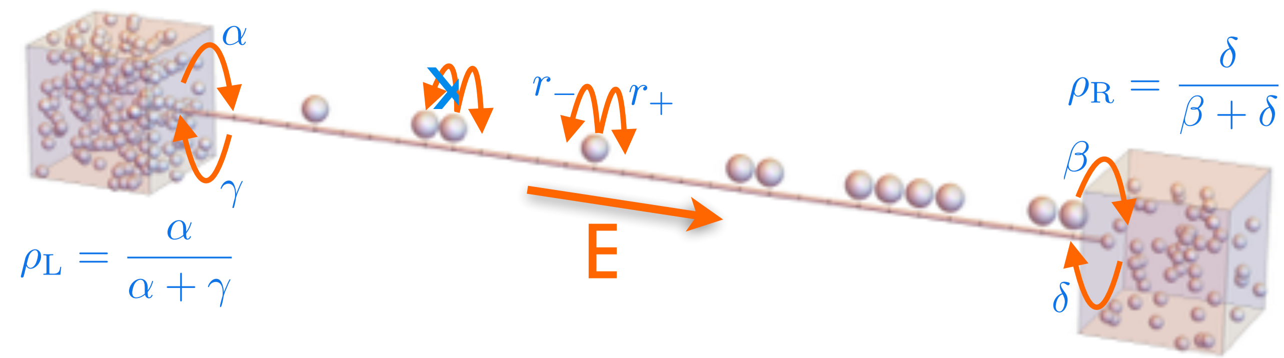


$\mathbb{W}_D^\lambda$ **is gapped** $\Rightarrow$ **unique steady state** $|P_{st}\rangle$



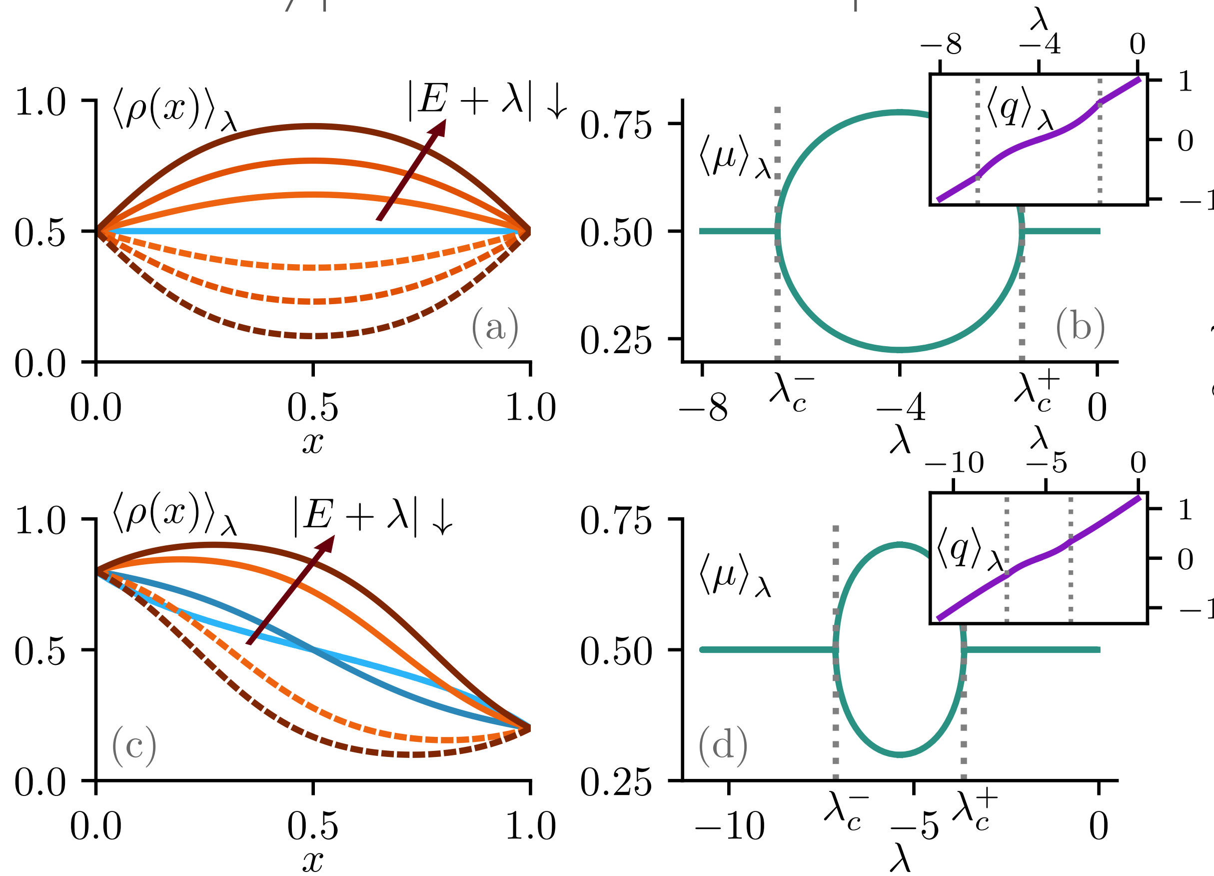
WEAKLY ASYMMETRIC SIMPLE EXCLUSION PROCESS

- **WASEP:** 1d lattice with occupation numbers $n_i=0,1$ + particle jumps to empty neighbors with rates $r_\pm = \exp(\pm E/L)/2$



- For $\rho_R = 1 - \rho_L$ the dynamics exhibits a **particle-hole (PH) symmetry**
  $n_k \to 1 - n_k \quad , \quad k \to L - k + 1$
- **DPT in current fluctuations**: for $|q| < q_c$, the system either **crowds or depletes of particles** to sustain such current fluctuation. $\mathbb{Z}_2$ **symmetry-breaking DPT**

Density profiles   Order parameter



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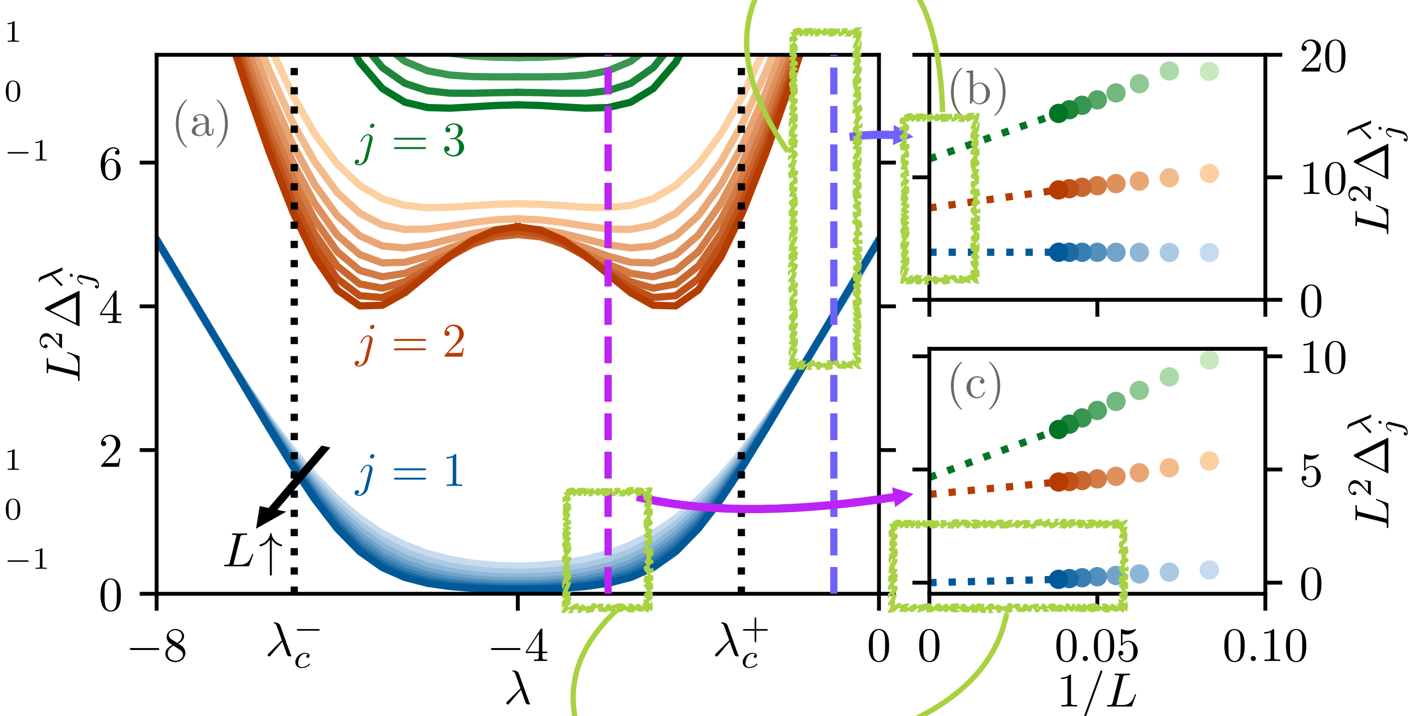
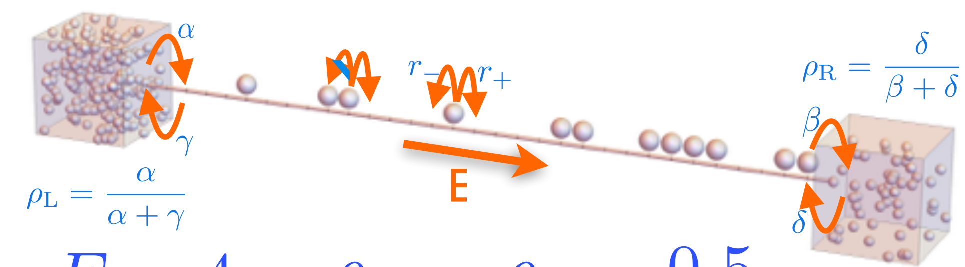


Vanishing spectral gap of $\mathbb{W}_D^\lambda$ **as** $L \to \infty$

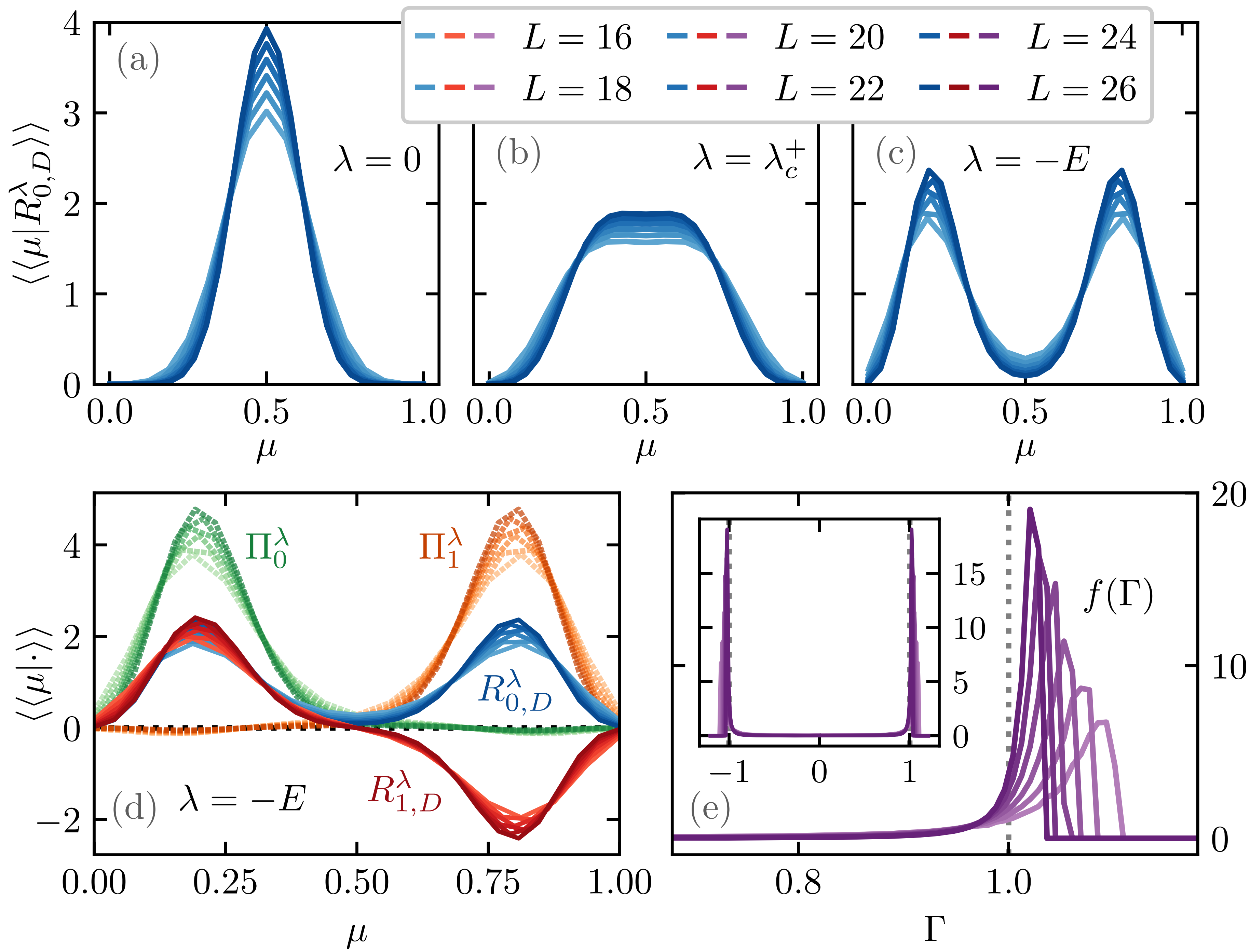
$$|P^\lambda_{ss,P_0}\rangle = |R^\lambda_{0,D}\rangle + |R^\lambda_{1,D}\rangle \langle L^\lambda_{1,D}|P_0\rangle$$

WEAKLY ASYMMETRIC SIMPLE EXCLUSION PROCESS

- **Order parameter**: mean occupation $\mu = L^{-1} \sum_{k=1}^{L} n_k$



$E = 4 \quad \rho_R = \rho_L = 0.5$



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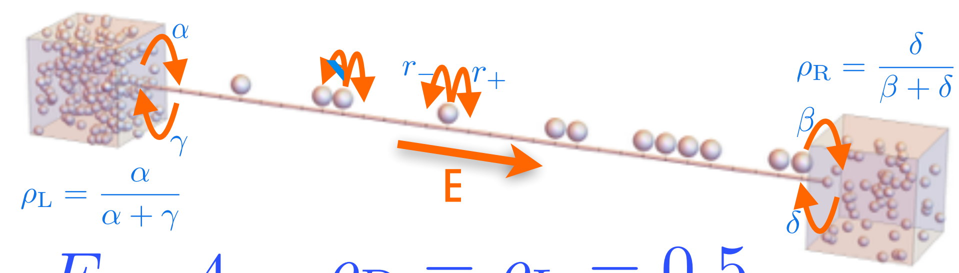
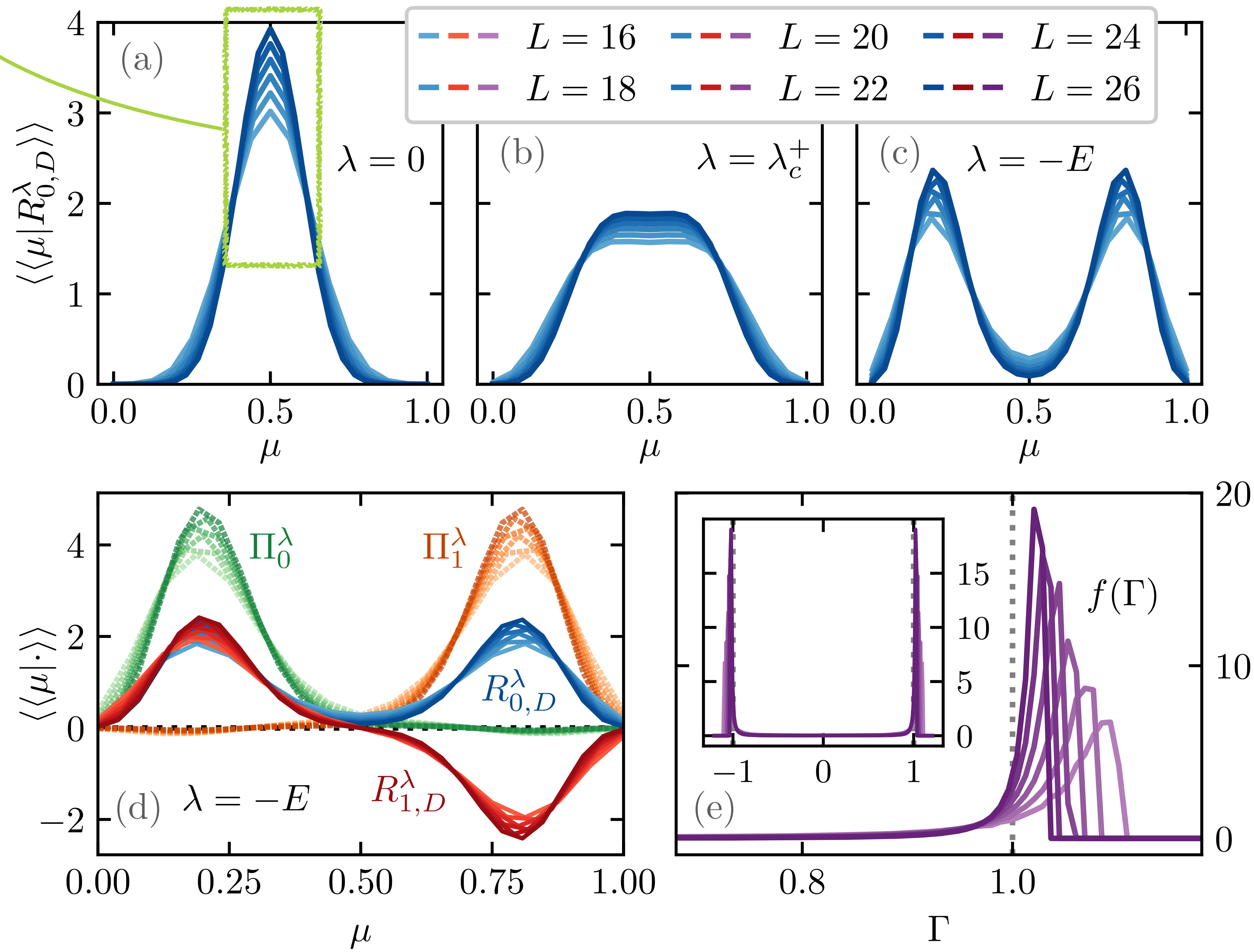


$E = 4 \qquad \rho_{\rm R} = \rho_{\rm L} = 0.5$

Unimodal $\langle\langle \mu || R^{\lambda}_{0,\rm D} \rangle\rangle$ before the DPT



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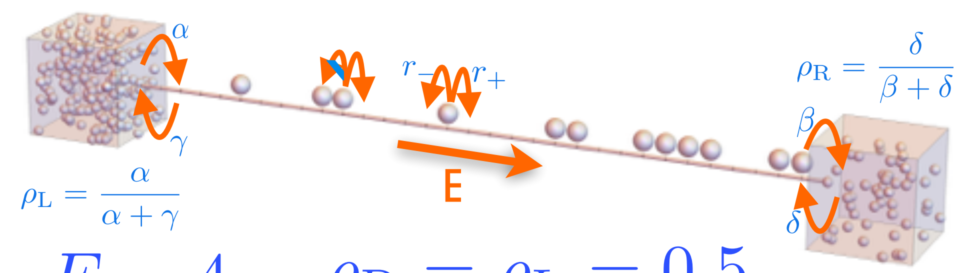

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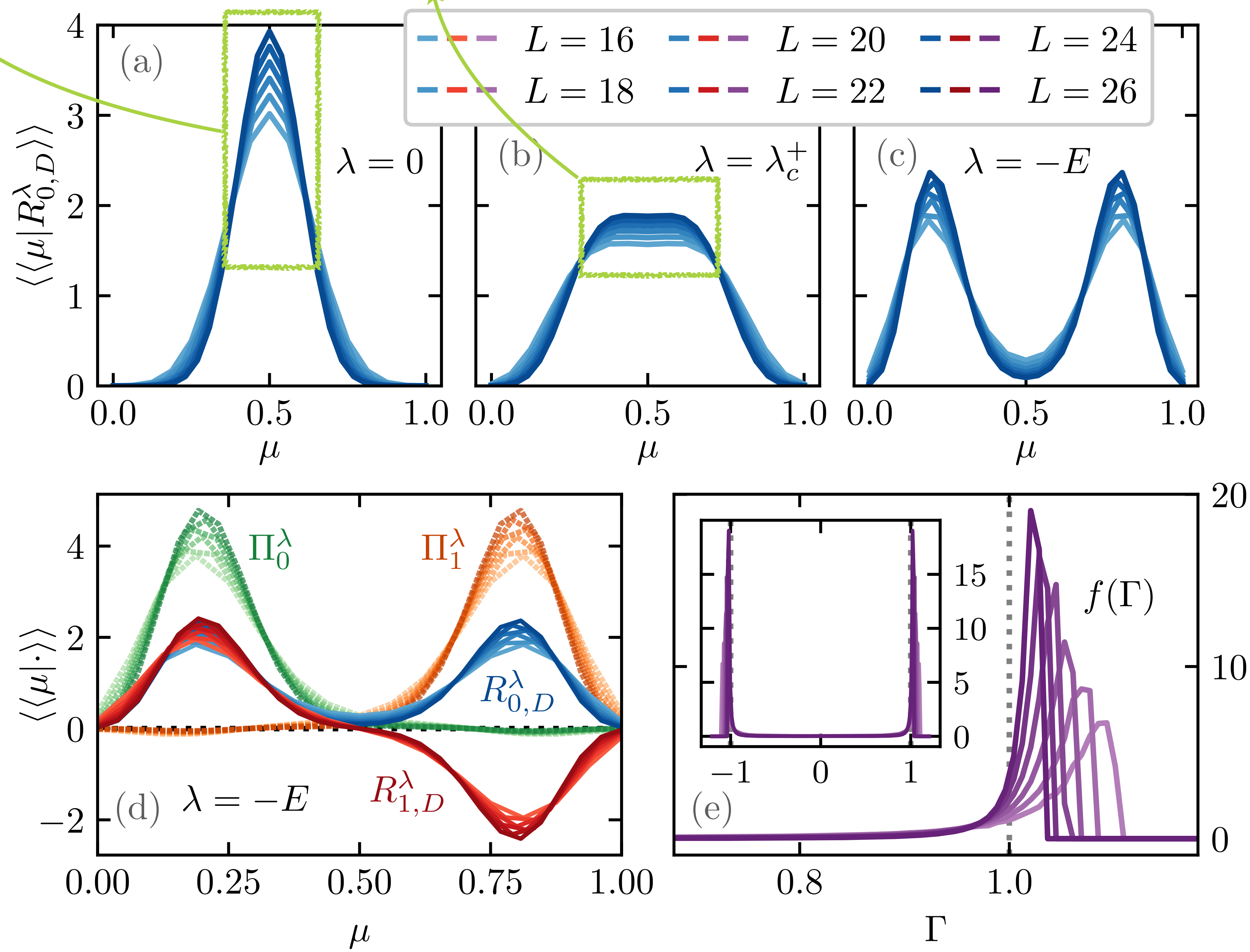
Flat $\langle\langle \mu || R^{\lambda}_{0,D} \rangle\rangle$ at the DPT



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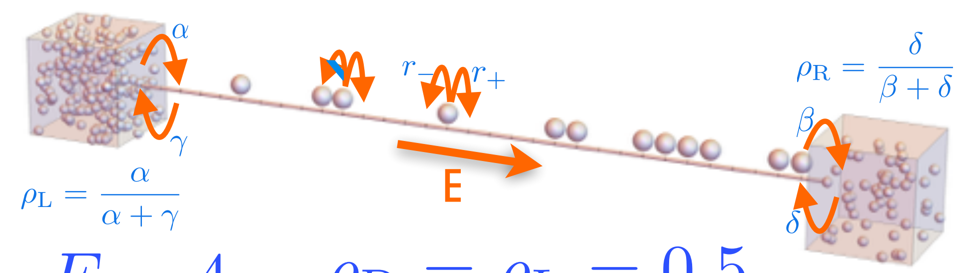

Unimodal $\langle\langle\mu||R^{\lambda}_{0,\mathrm{D}}\rangle\rangle$ before the DPT

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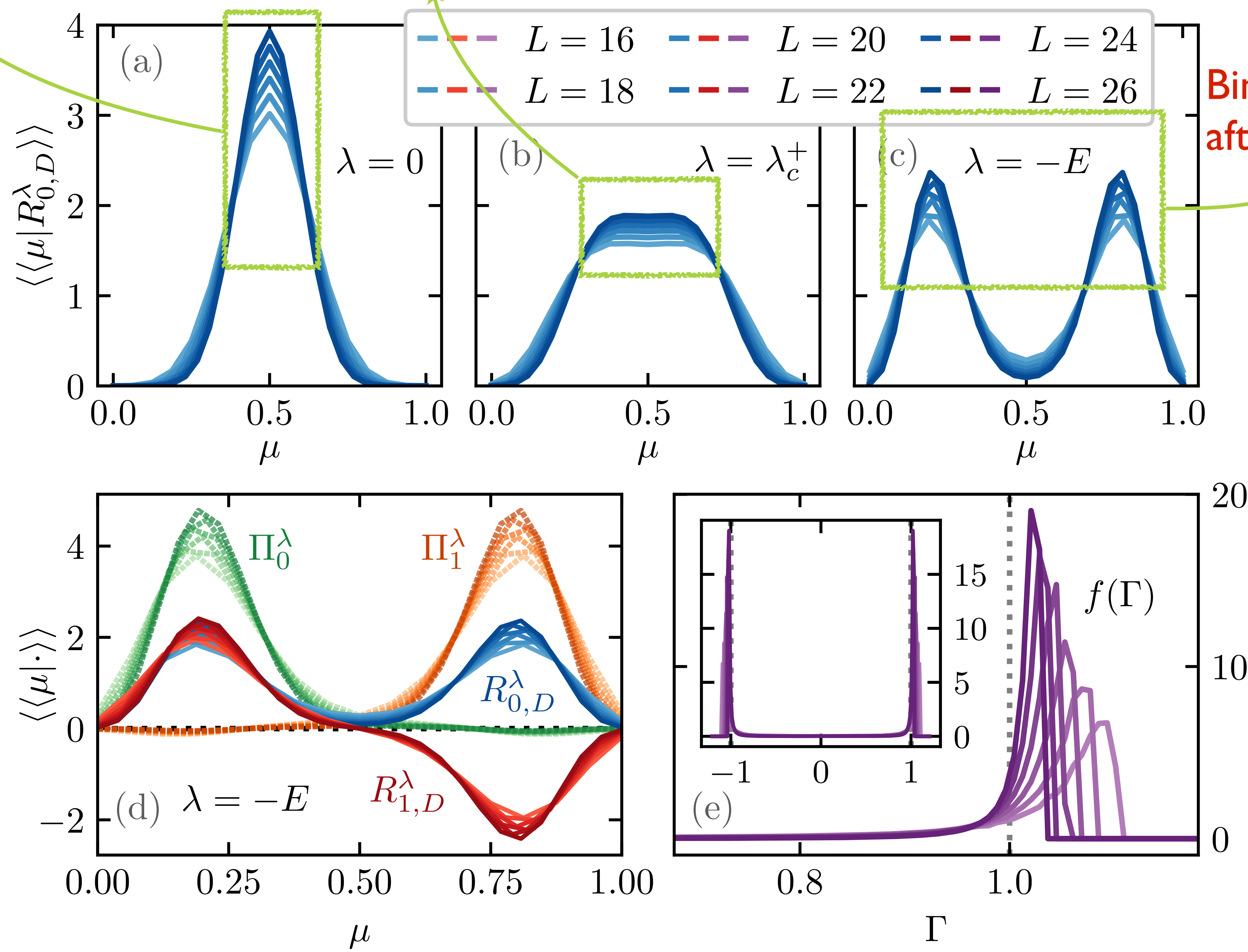

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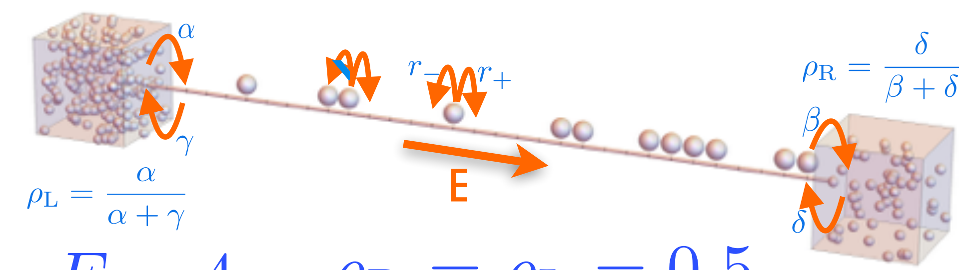

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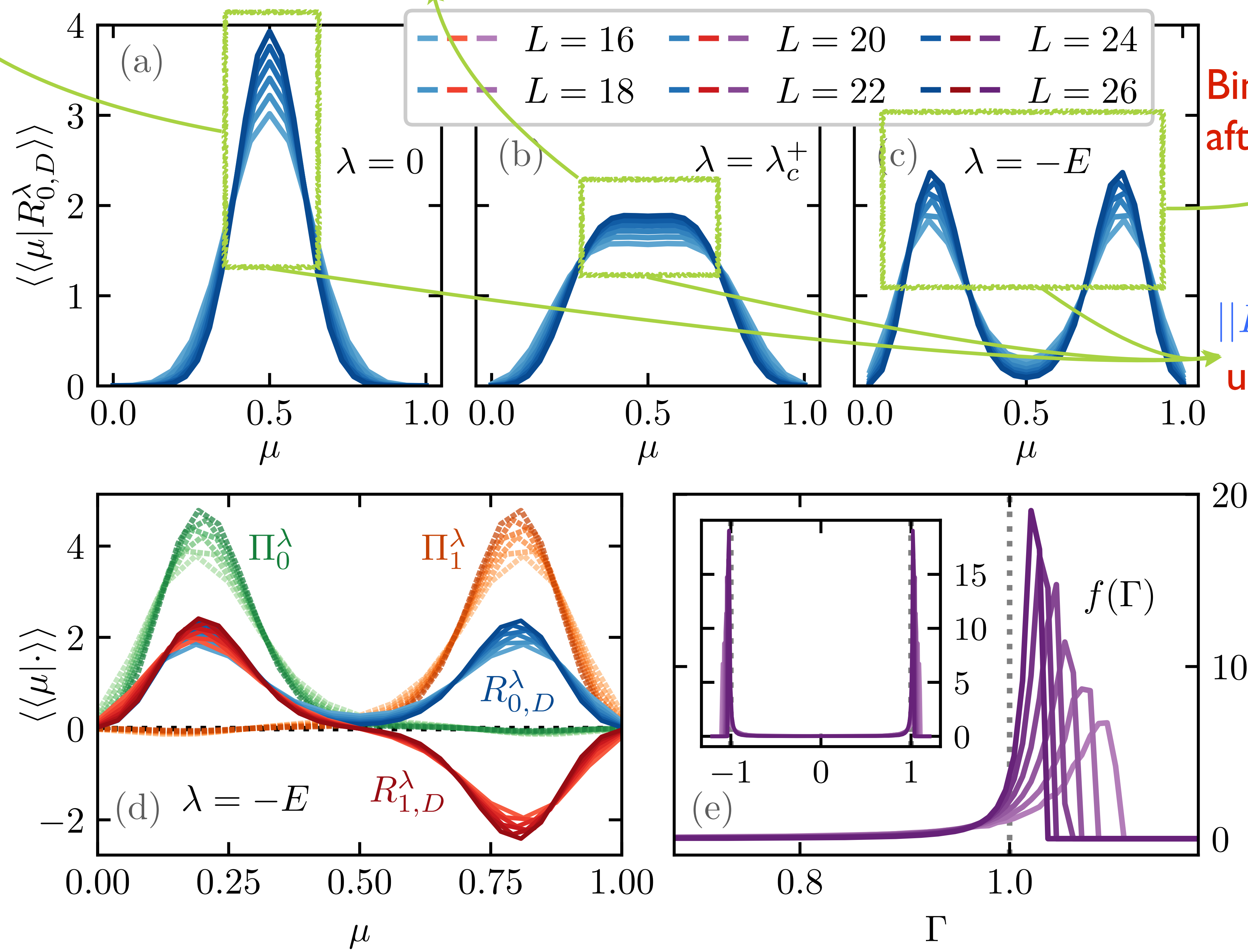
$|| R^{\lambda}_{0,D} \rangle\rangle$ is invariant under $\mu \to 1-\mu$



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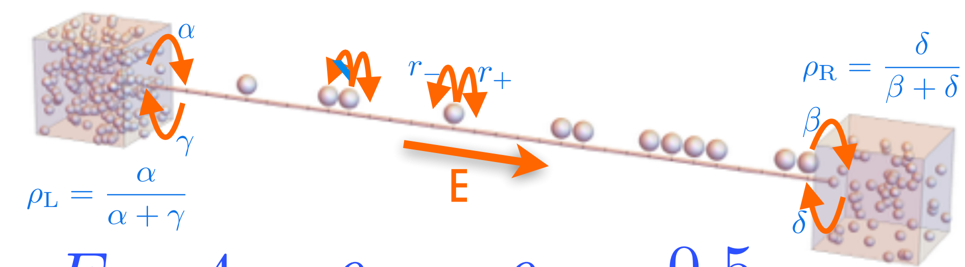

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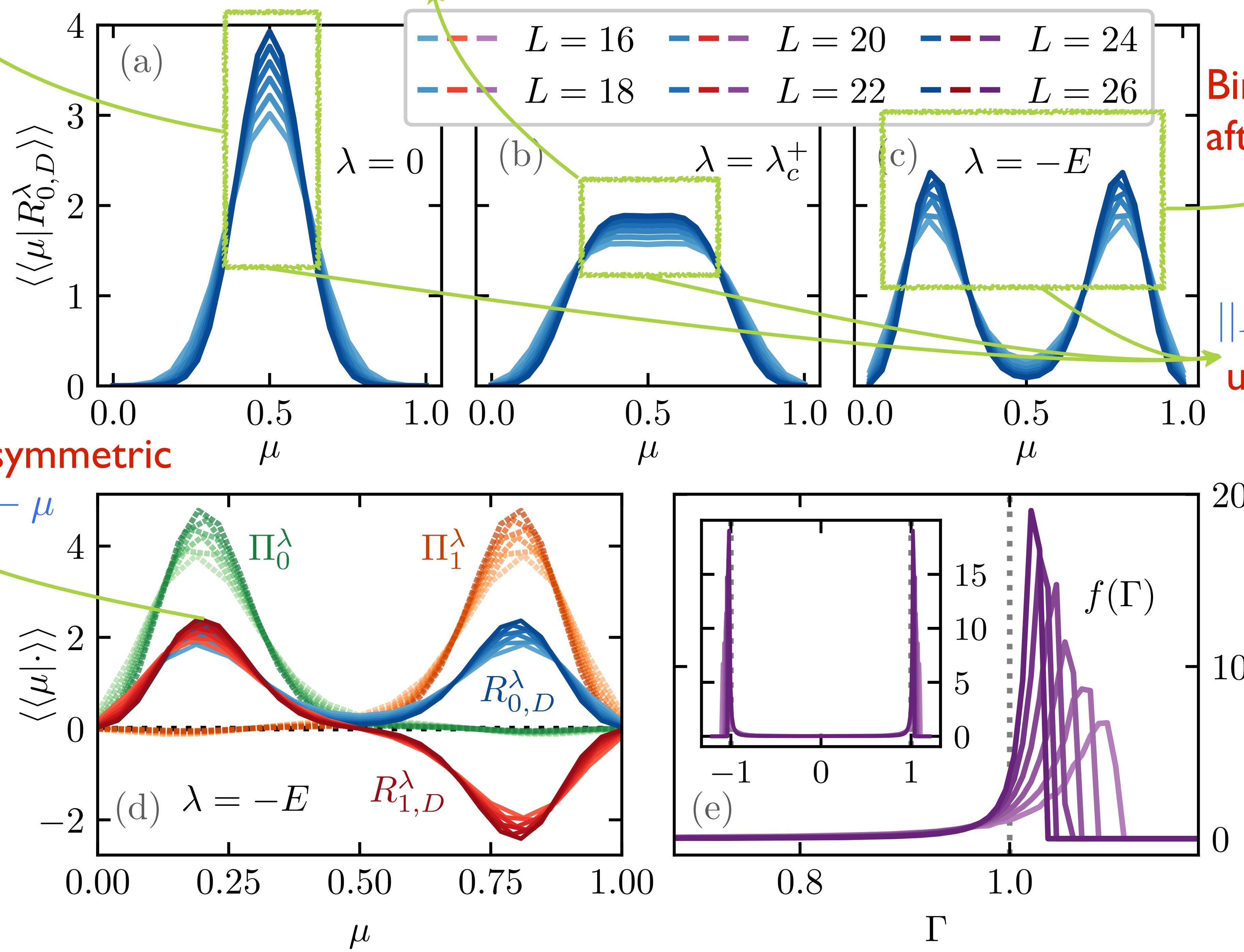

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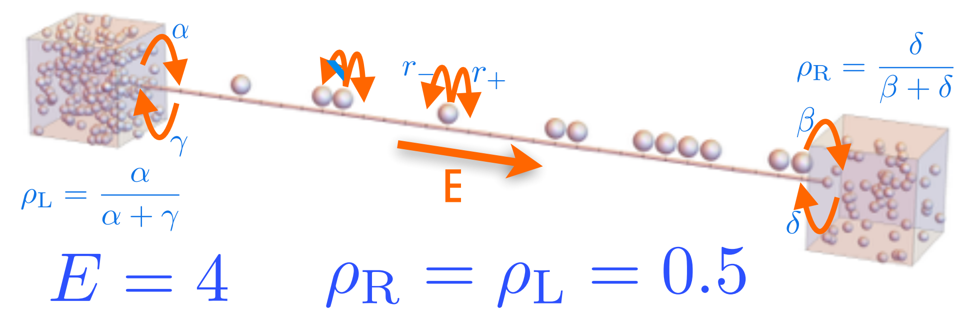

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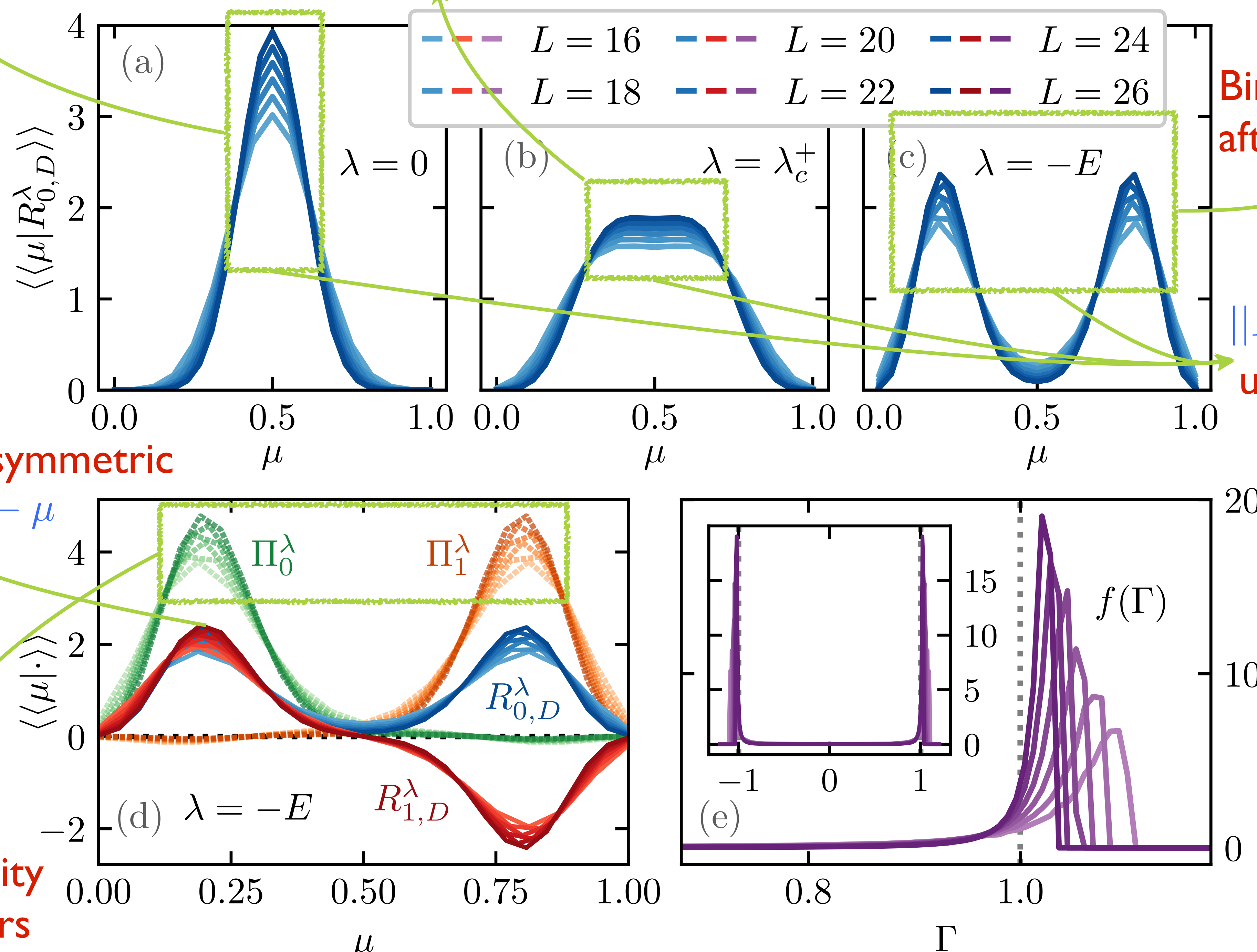

Phase probability reduced vectors

$||\Pi^{\lambda}_{l}\rangle\rangle = ||R^{\lambda}_{0,D}\rangle\rangle + (-1)^l ||R^{\lambda}_{1,D}\rangle\rangle$

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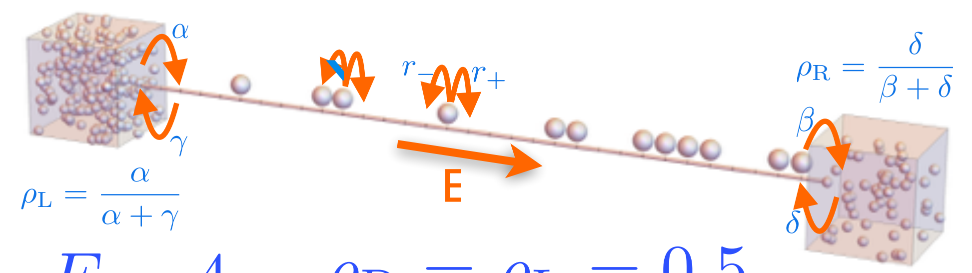

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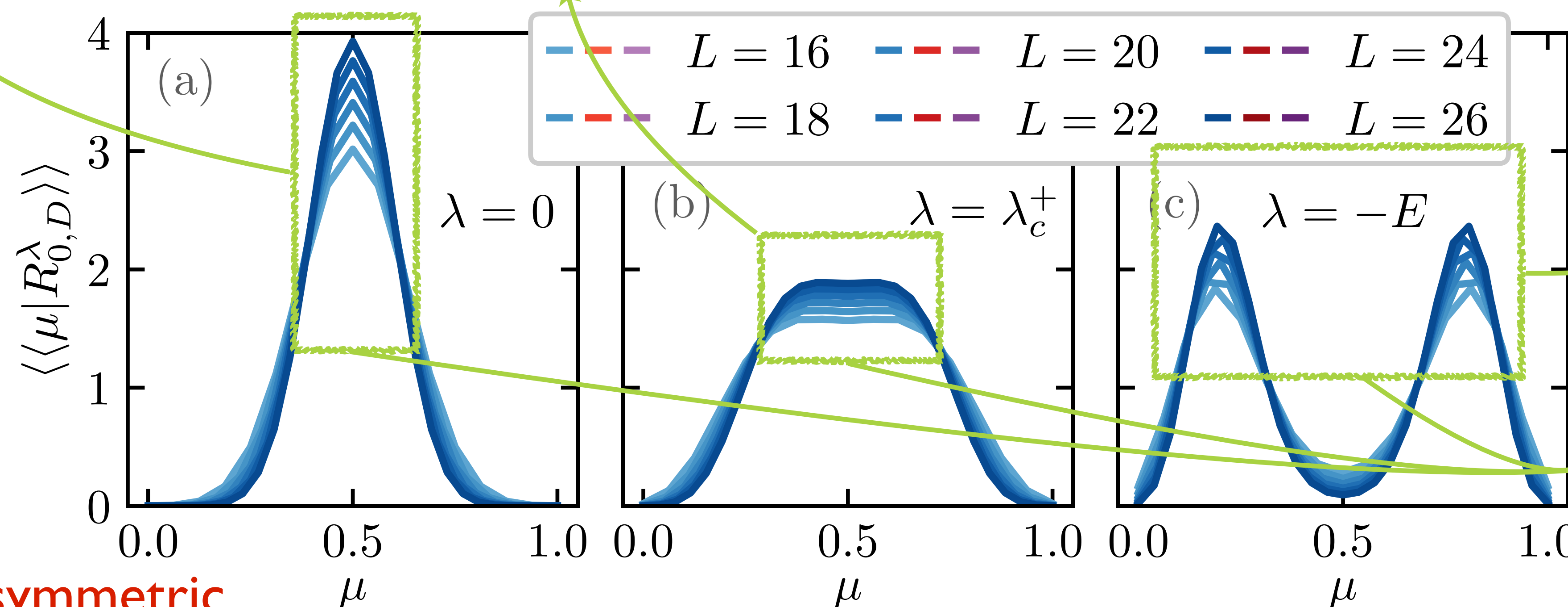

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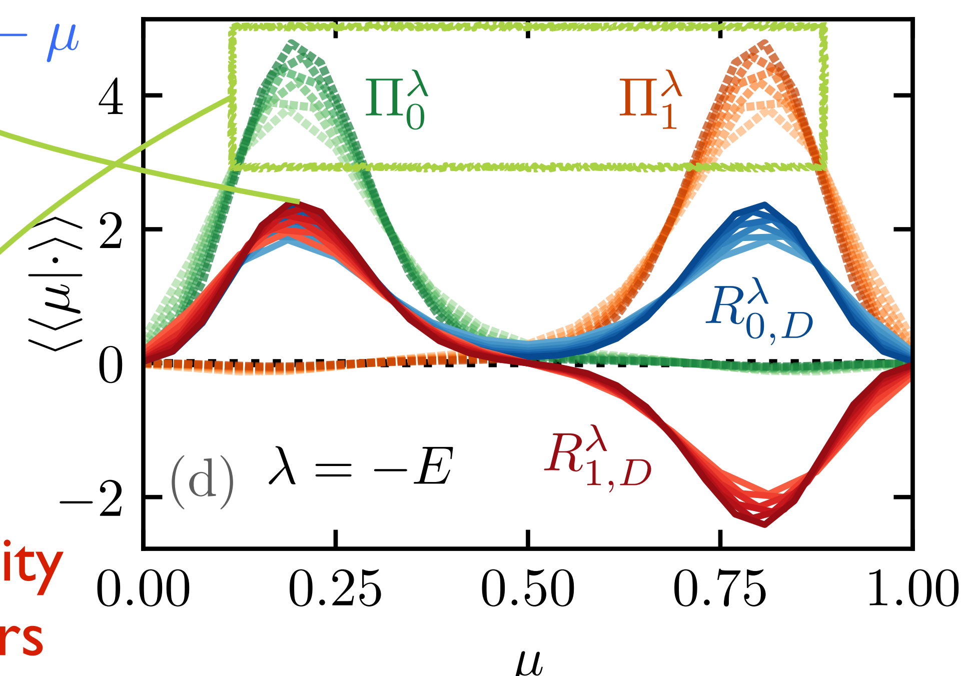
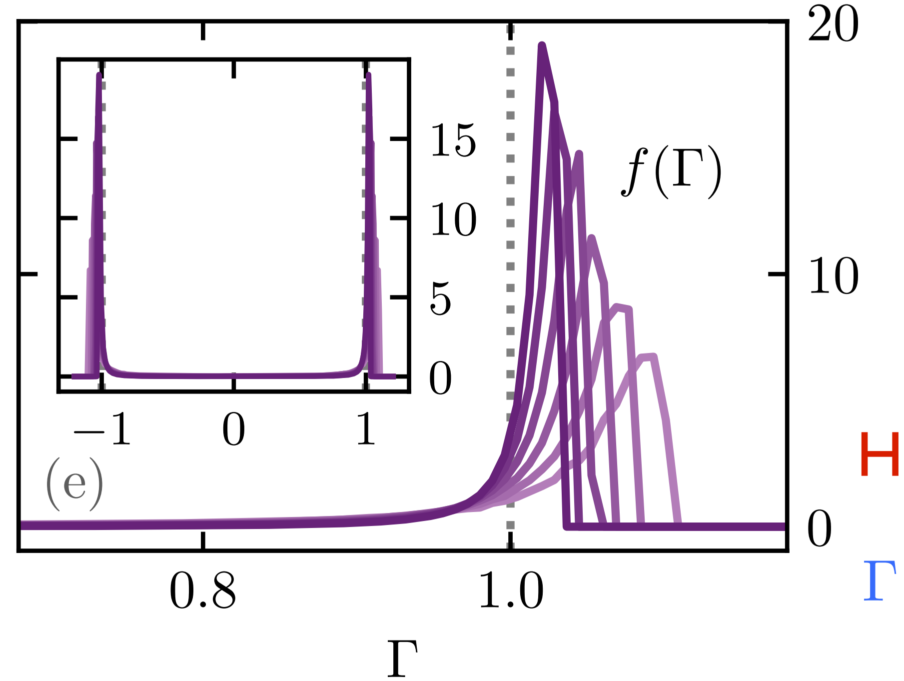


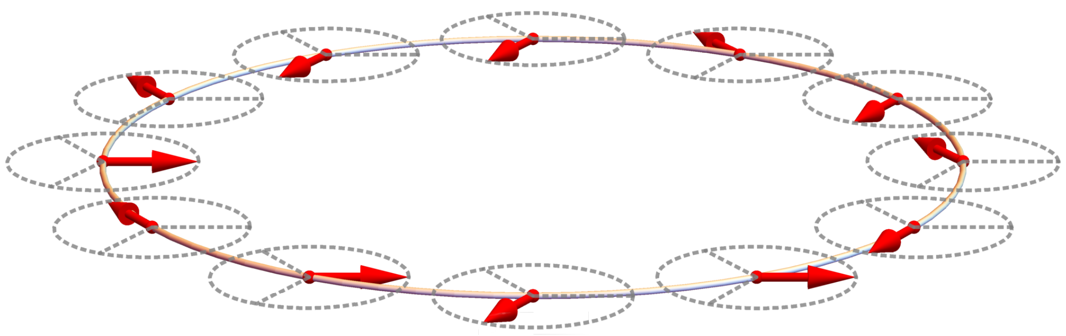


Phase probability reduced vectors $||\Pi_l^{\lambda}\rangle\rangle = ||R^{\lambda}_{0,D}\rangle\rangle + (-1)^l||R^{\lambda}_{1,D}\rangle\rangle$

Histogram for $\Gamma(C) = \frac{\langle C|R^{\lambda}_{1,D}\rangle}{\langle C|R^{\lambda}_{0,D}\rangle}$

$\langle C|R^{\lambda}_{1,D}\rangle \approx (-1)^{-\ell_C}\langle C|R^{\lambda}_{0,D}\rangle$

ENERGY FLUCTUATIONS IN SPIN SYSTEMS



- **Potts model:** 1d periodic lattice with spins $s_k \in \{0, 1, ..., r-1\}$ distributed in unit circle with angles $\varphi_k = 2\pi s_k / r$. Glauber spin-flip dynamics and Hamiltonian:

$$H = -J \sum_{k=1}^{L} \cos(\varphi_{k+1} - \varphi_k)$$

- $\mathbb{Z}_r$ **symmetry:** Hamiltonian invariant under global rotations multiple of $2\pi/r$

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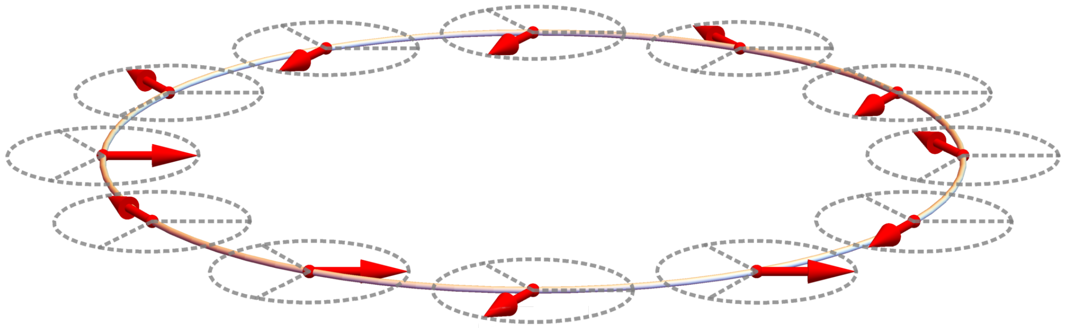

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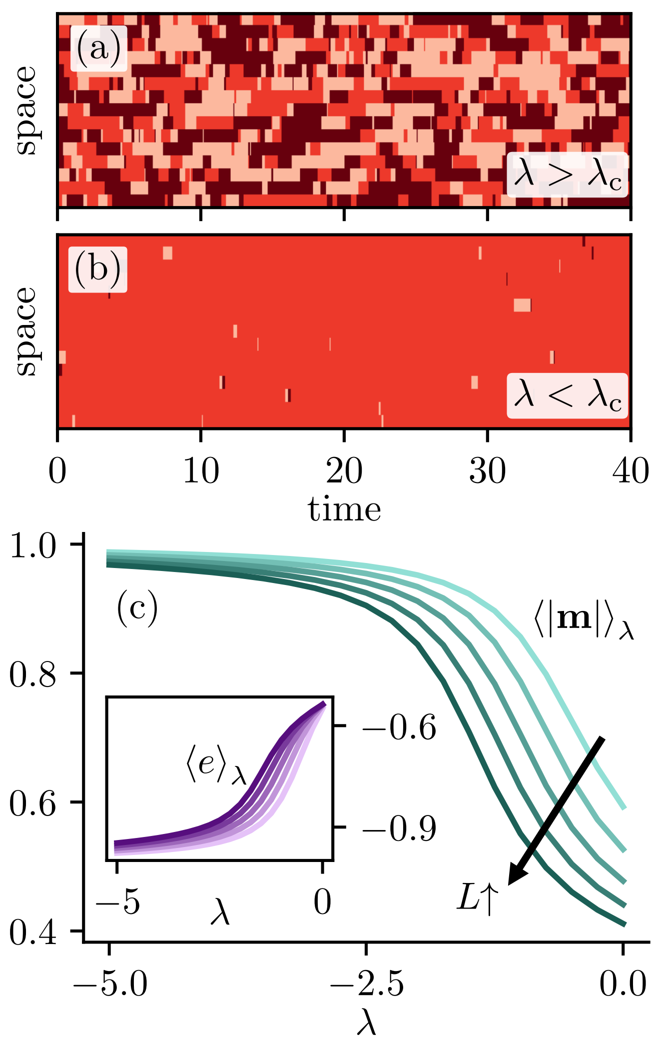

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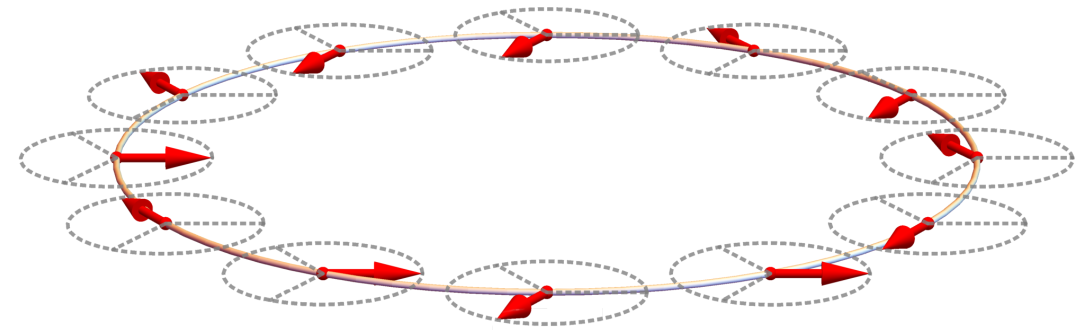

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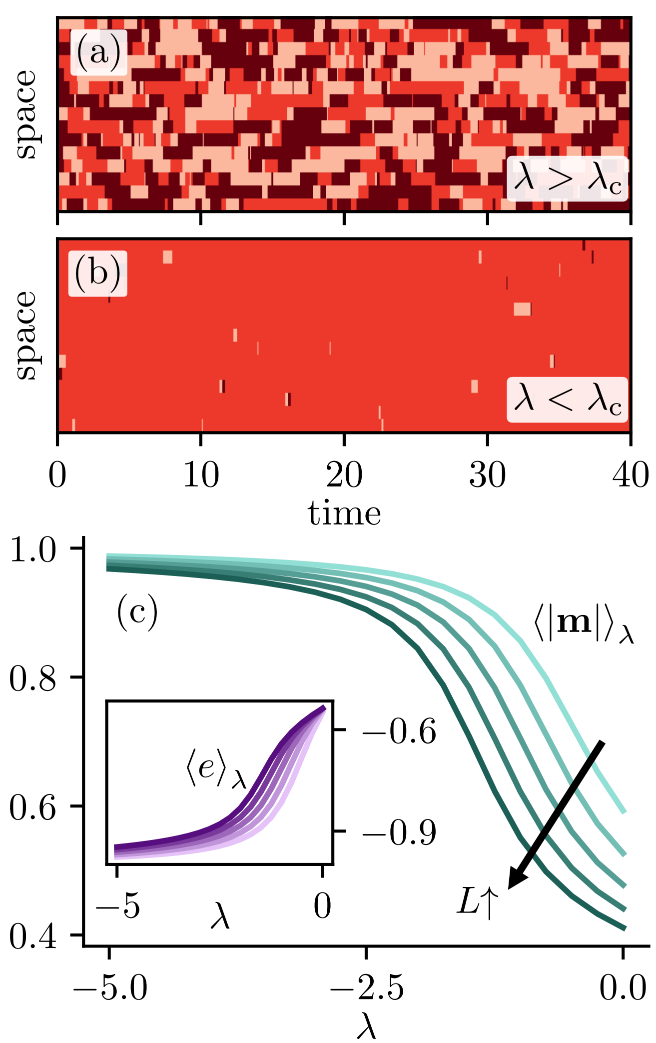

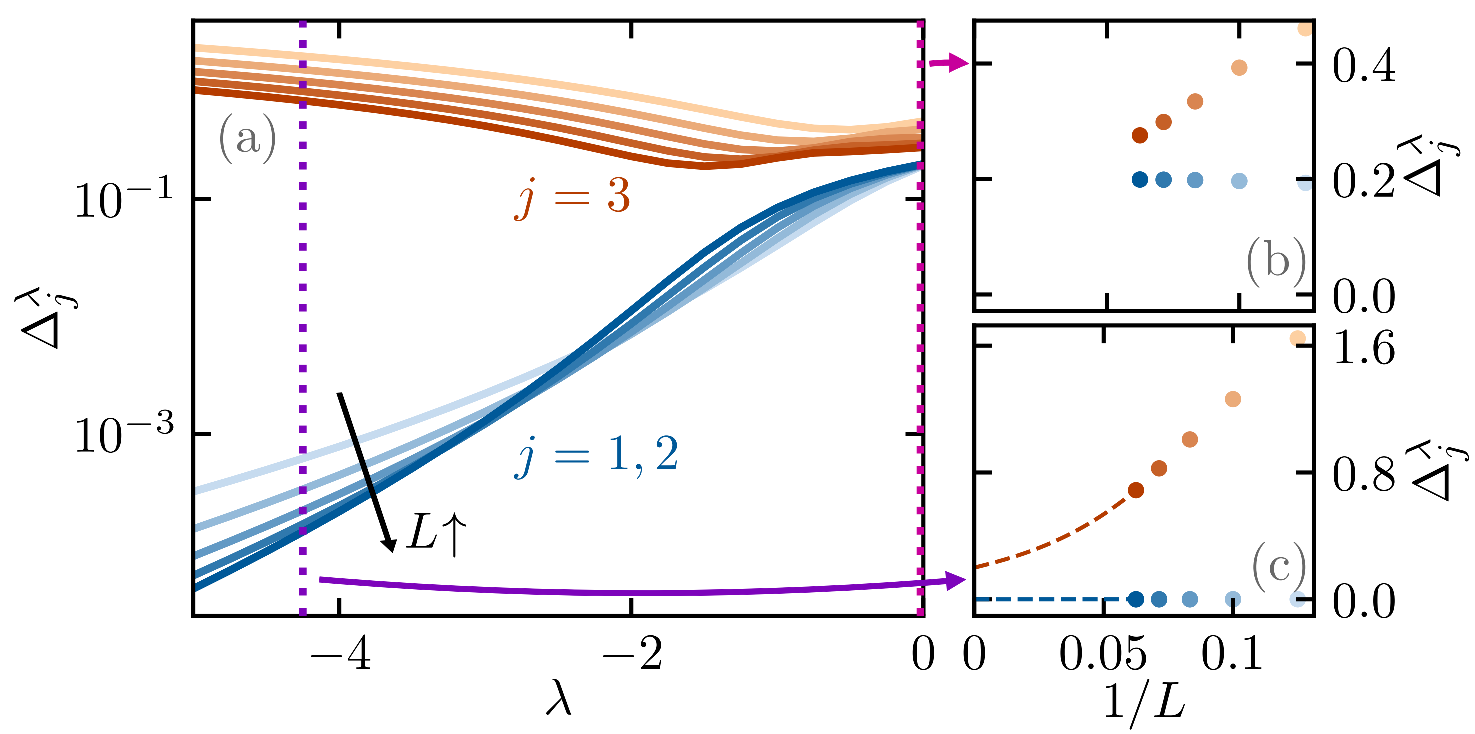

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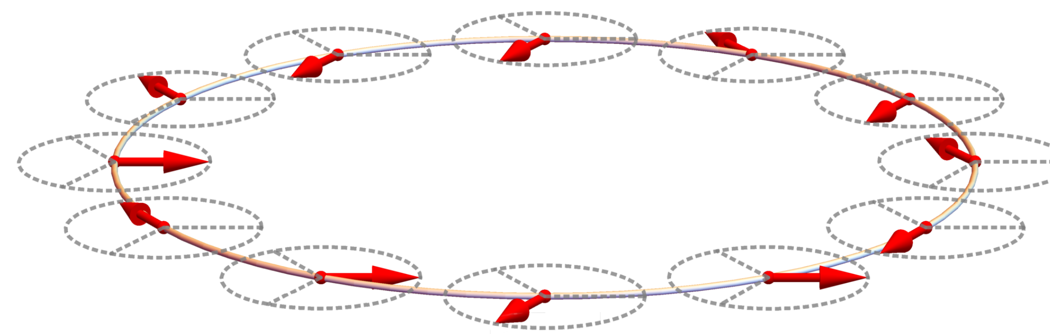

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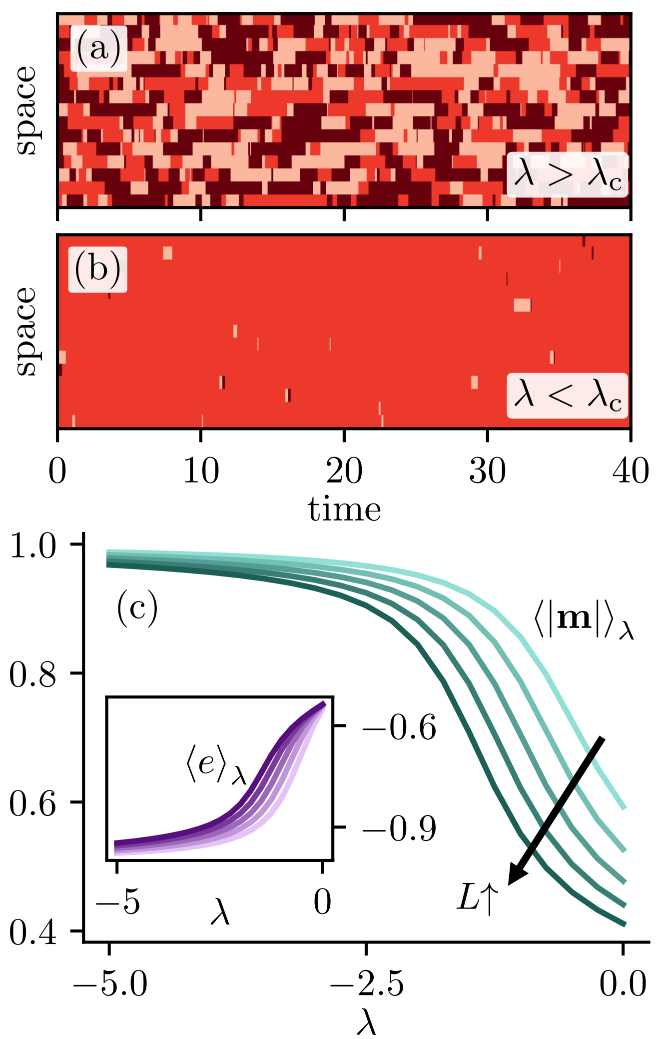

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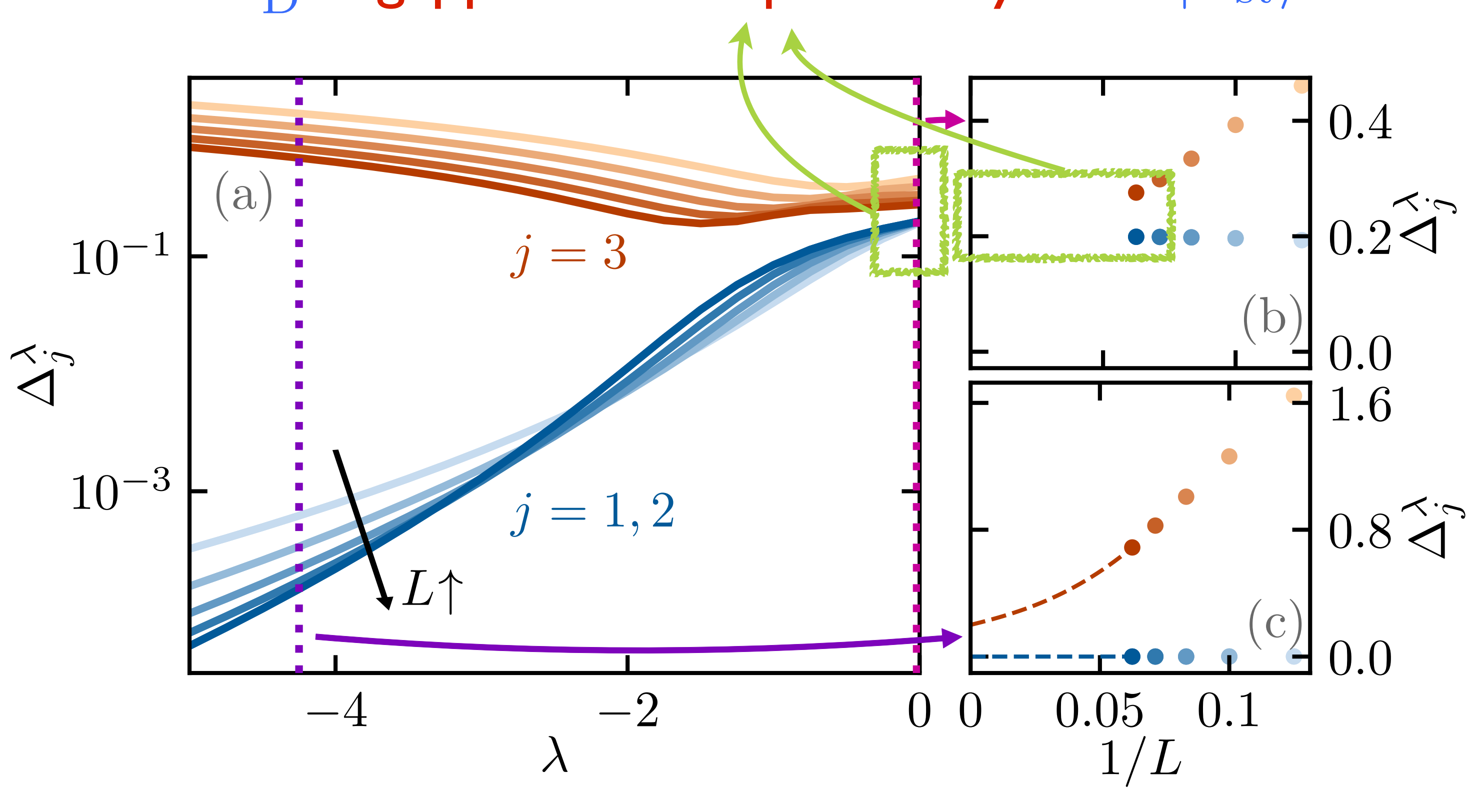

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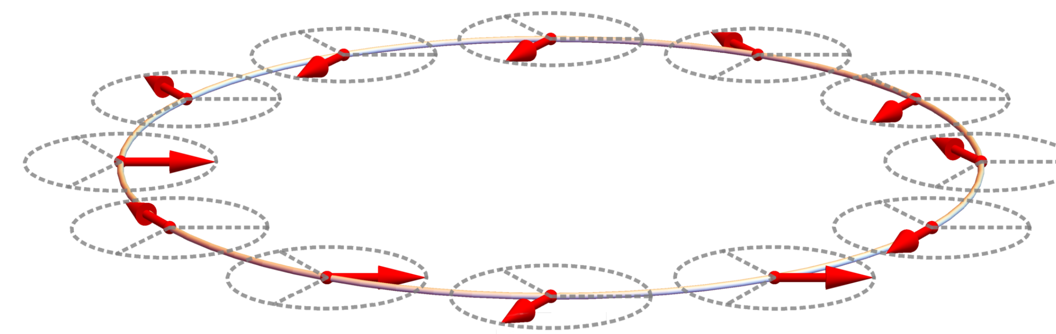

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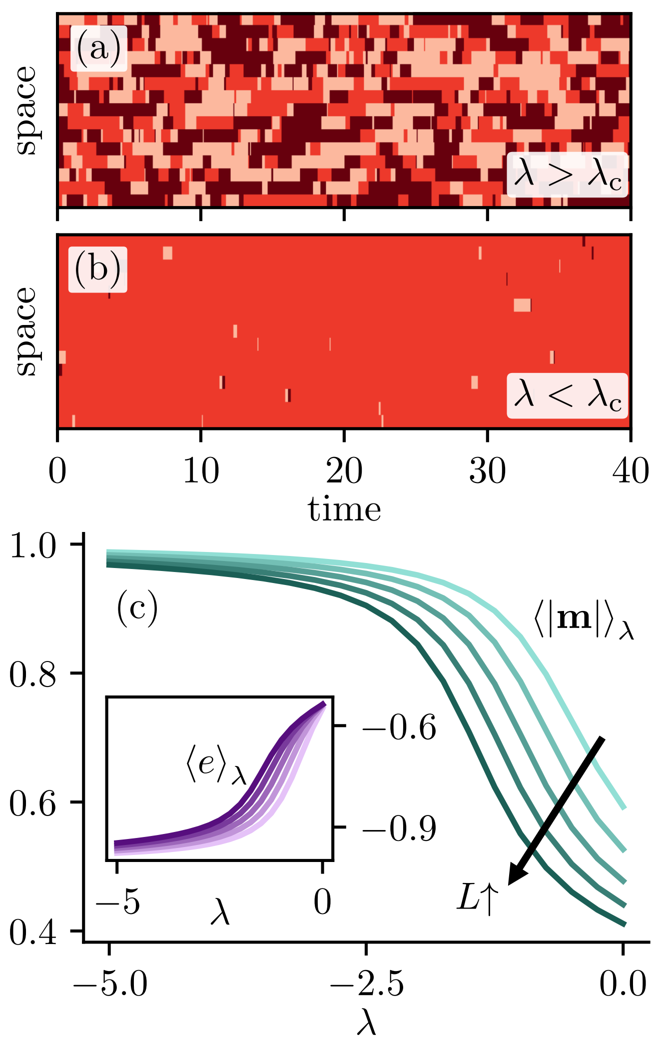

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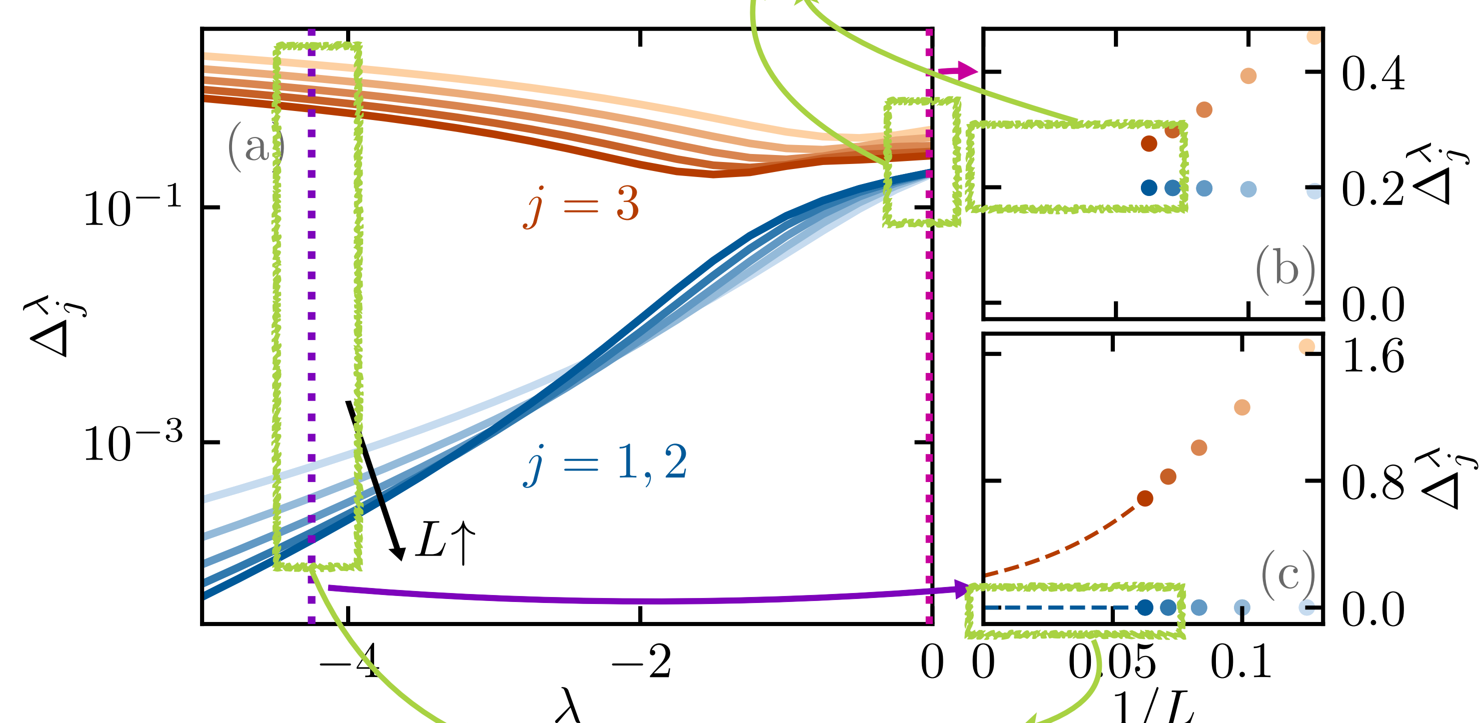

Vanishing spectral gap of $\mathbb{W}_D^\lambda$ as $L \to \infty$

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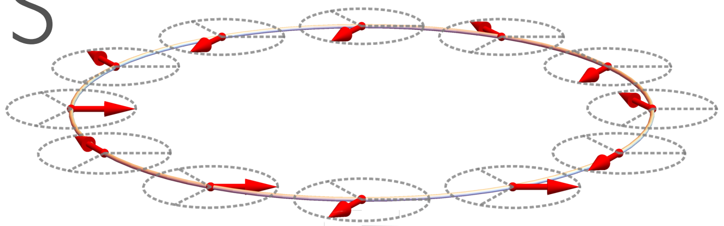

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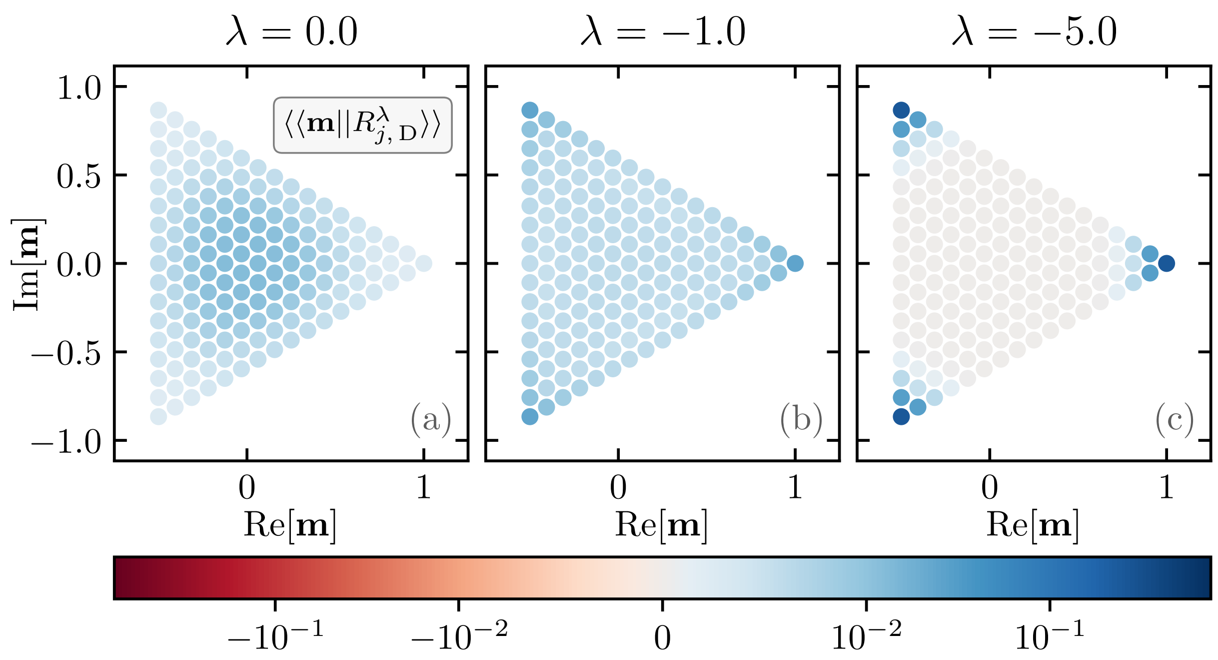

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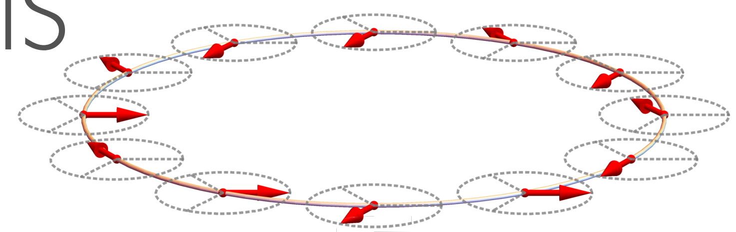

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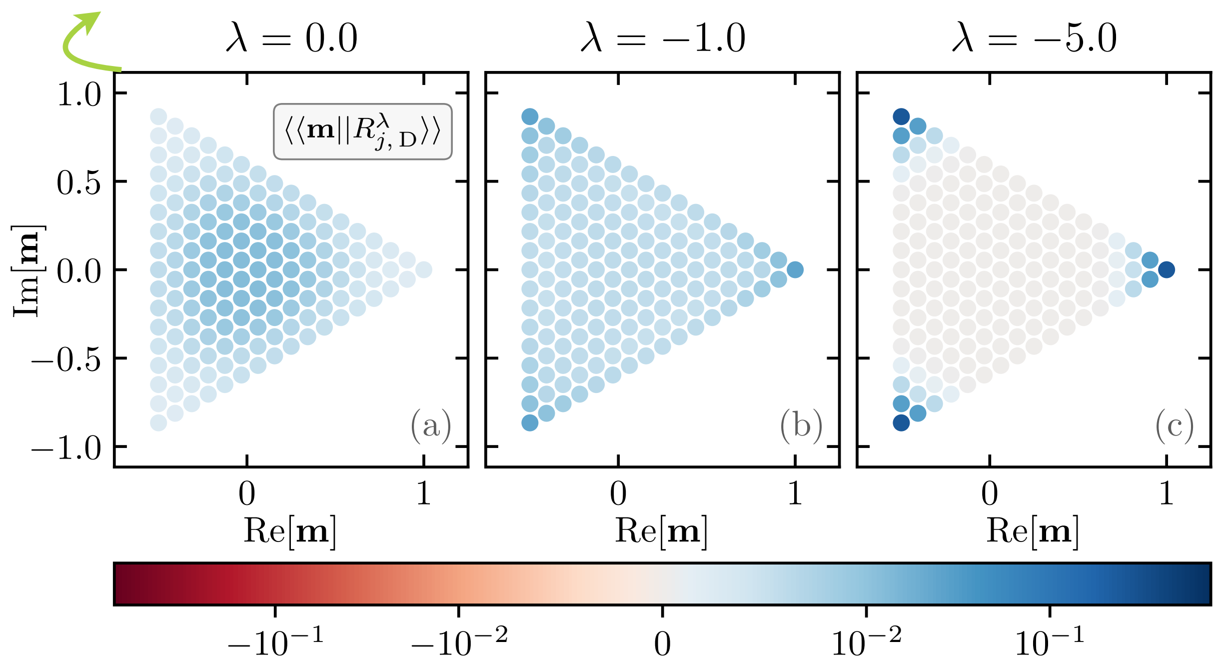
$\langle\langle \mathbf{m} || R^{\lambda}_{0,\mathrm{D}} \rangle\rangle$ peaked around $|\mathbf{m}| = 0$ before the DPT



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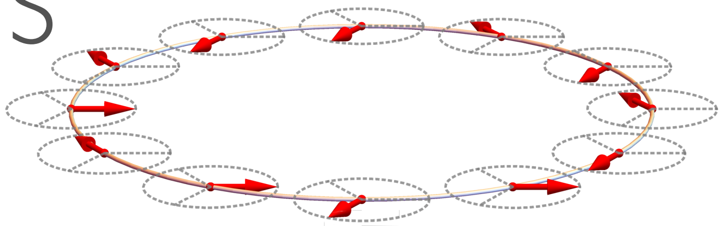

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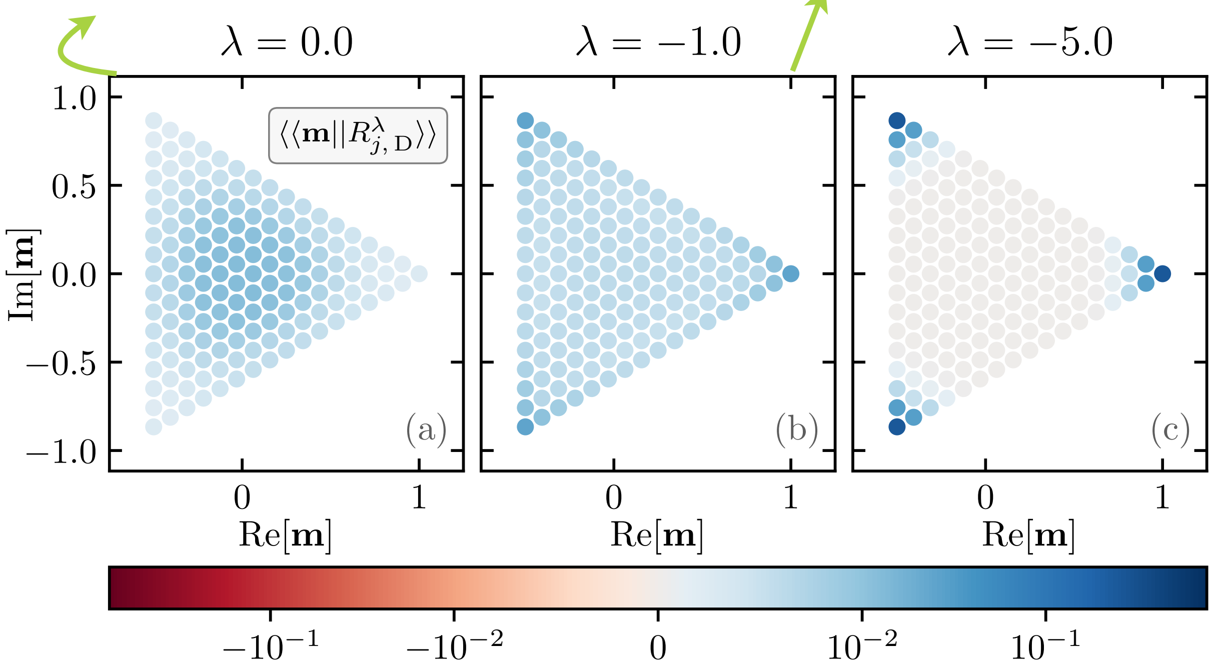

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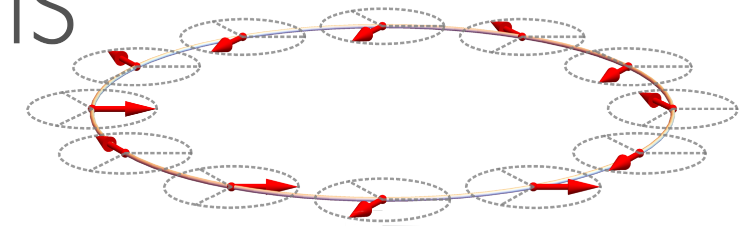

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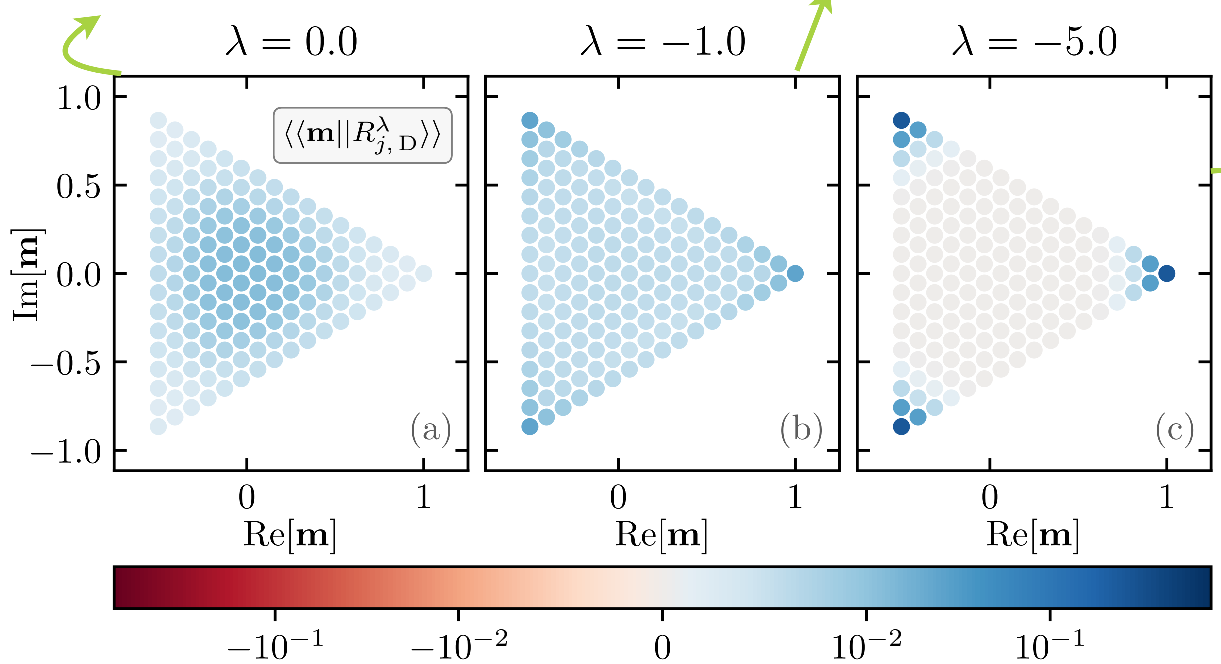

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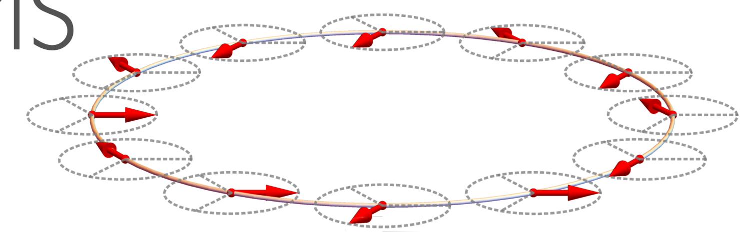

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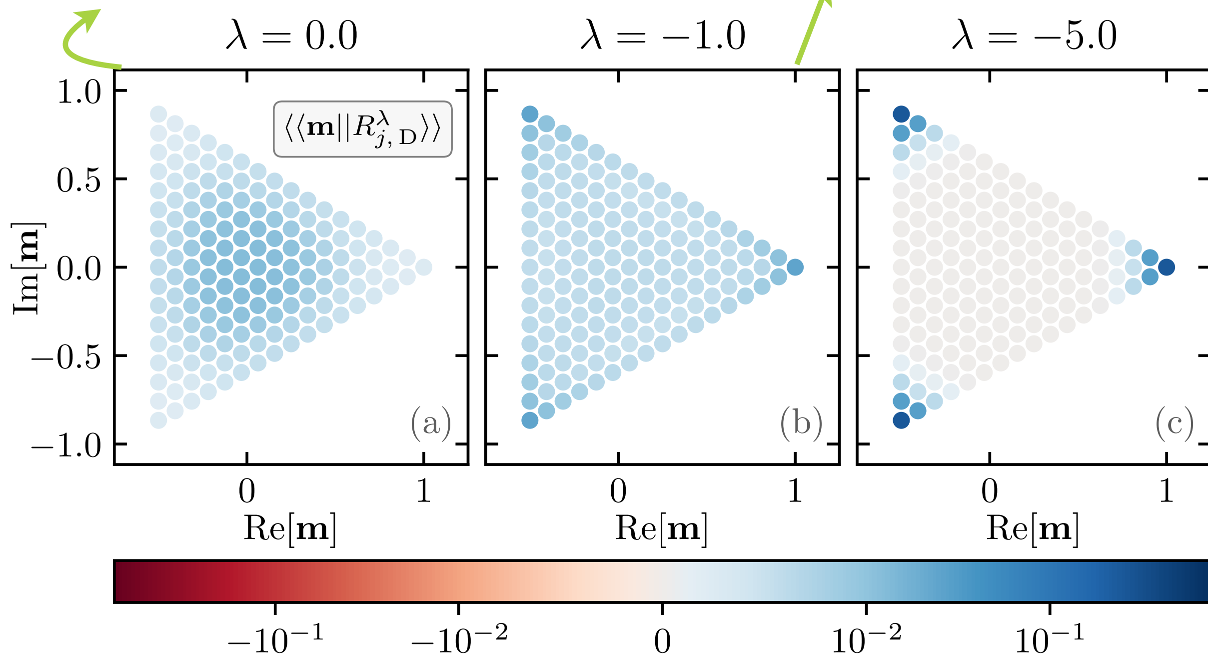
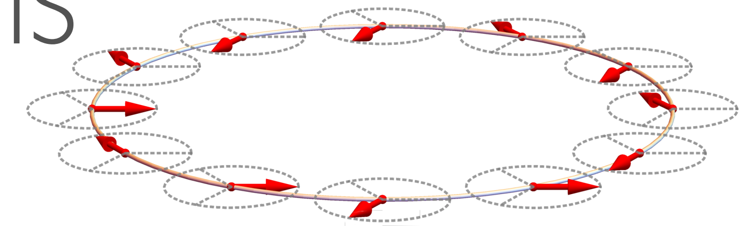
$\langle\langle || R^{\lambda}_{0,\mathrm{D}} \rangle\rangle$ is invariant under rotations of $2\pi n/3$



ENERGY FLUCTUATIONS IN SPIN SYSTEMS



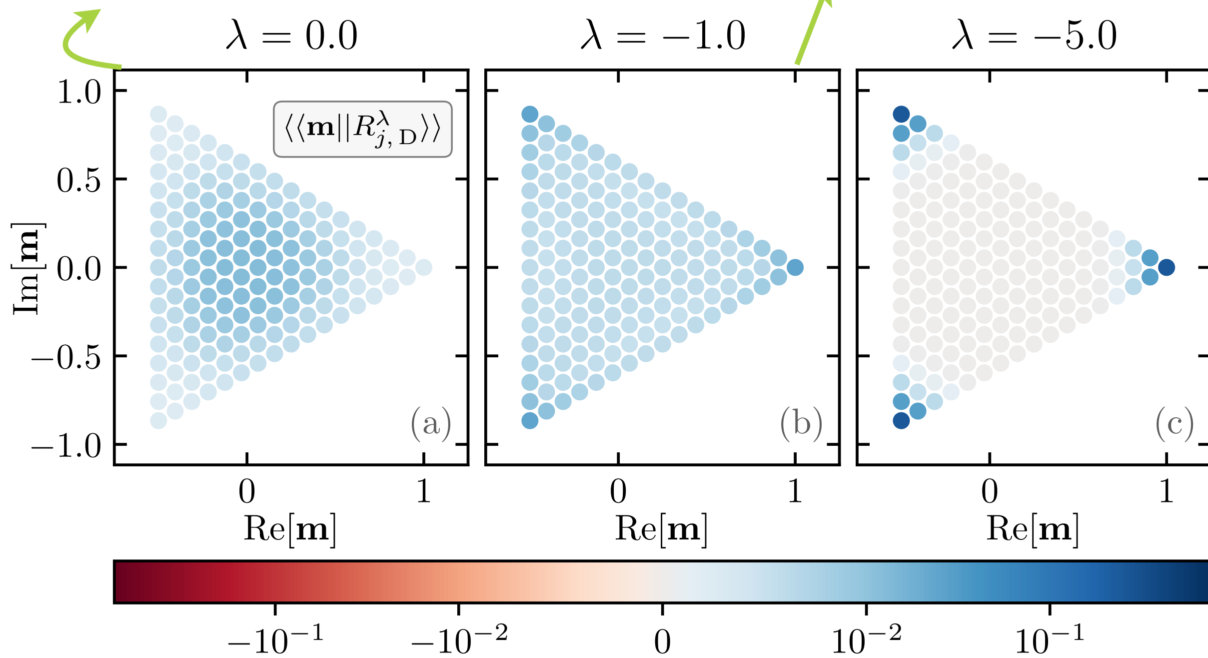
- **Order parameter**: magnetization $\mathbf{m} = L^{-1} \sum_{k=1}^{L} \mathrm{e}^{i\varphi_k}$

$\langle\langle \mathbf{m} || R^\lambda_{0,\mathrm{D}} \rangle\rangle$ peaked around $|\mathbf{m}| = 0$ before the DPT

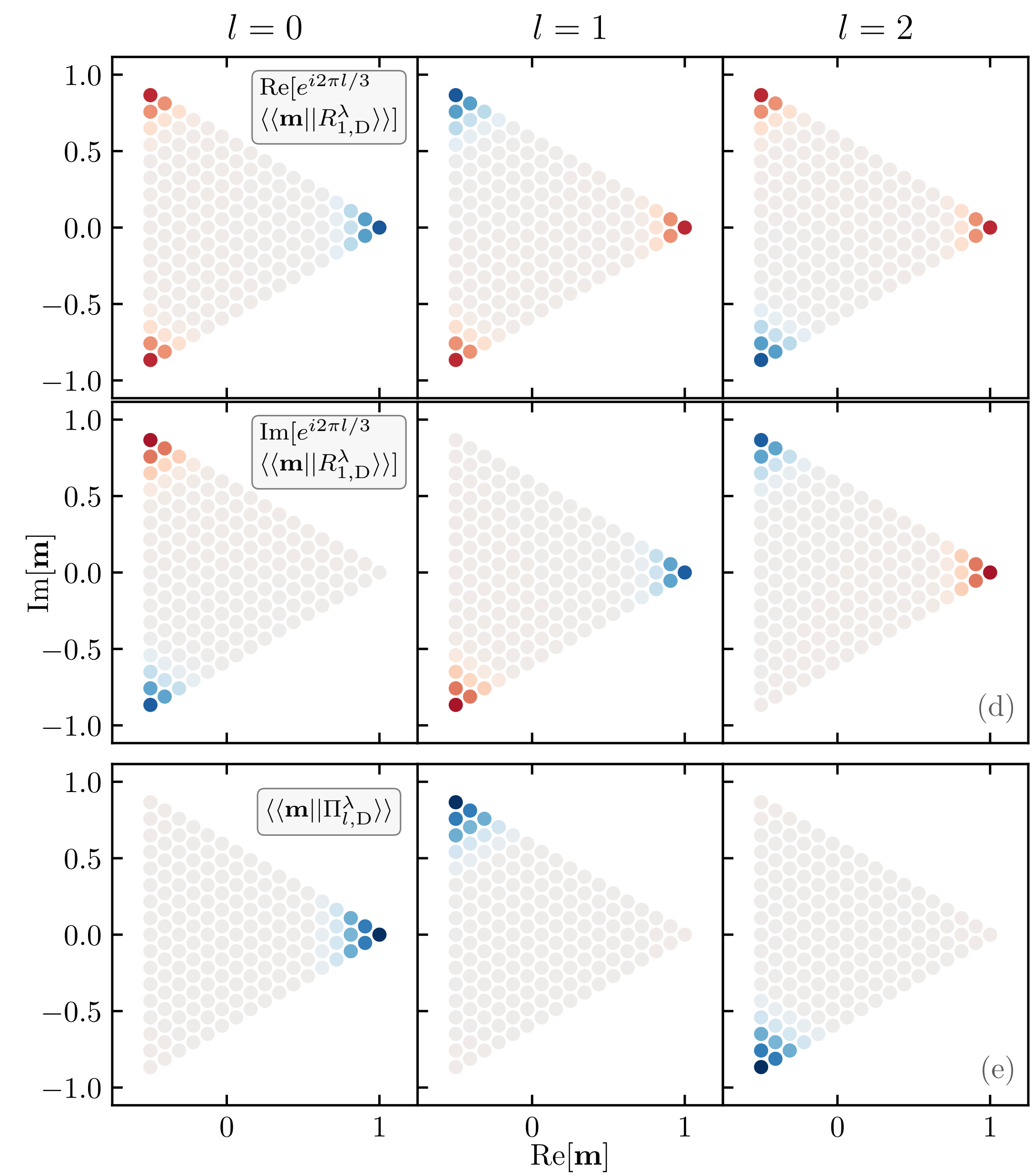
Flat $\langle\langle \mathbf{m} || R^\lambda_{0,\mathrm{D}} \rangle\rangle$ at the DPT

$\langle\langle \mathbf{m} || R^\lambda_{0,\mathrm{D}} \rangle\rangle$ peaks at $|\mathbf{m}| = 1$ and $\varphi = 0, 2\pi/3, 4\pi/3$ after the DPT

$||R^\lambda_{0,\mathrm{D}}\rangle\rangle$ is invariant under rotations of $2\pi n/3$

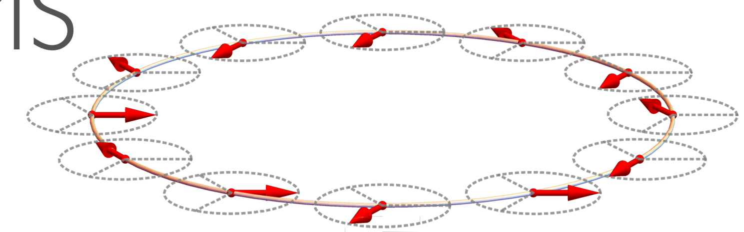


$||R^\lambda_{2,\mathrm{D}}\rangle\rangle = ||R^\lambda_{1,\mathrm{D}}\rangle\rangle^*$



$||\Pi^\lambda_l\rangle\rangle = ||R^\lambda_{0,\mathrm{D}}\rangle\rangle + 2\mathrm{Re}[\mathrm{e}^{i2\pi l/3} ||R^\lambda_{1,\mathrm{D}}\rangle\rangle]$

ENERGY FLUCTUATIONS IN SPIN SYSTEMS



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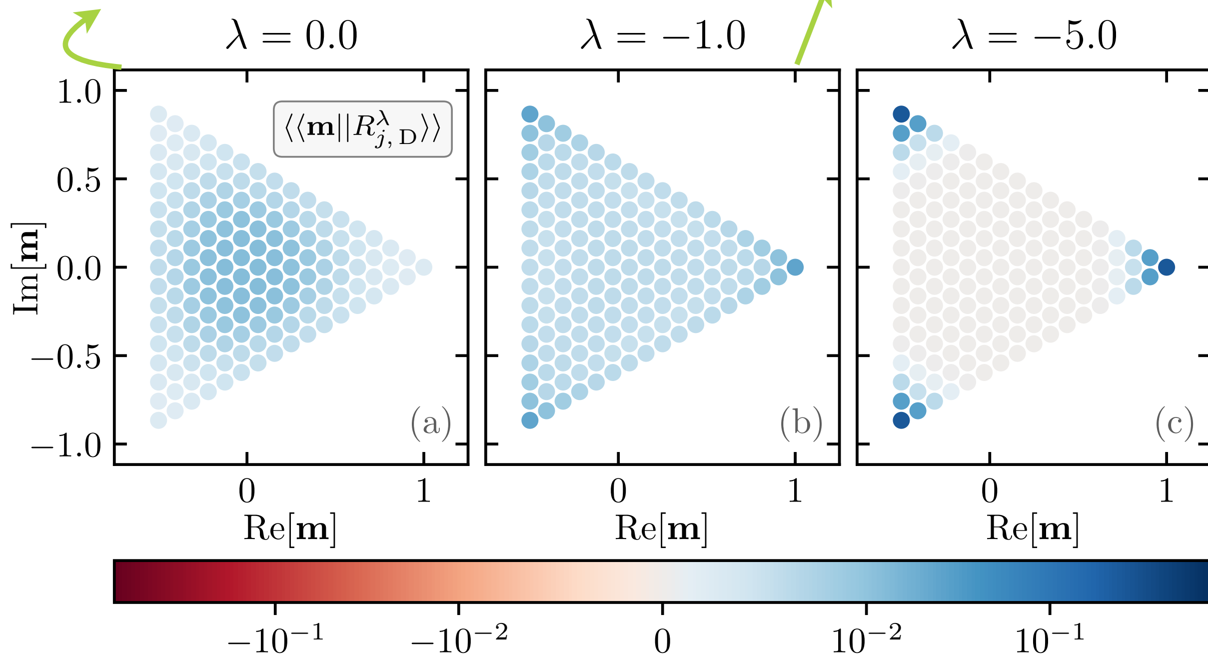

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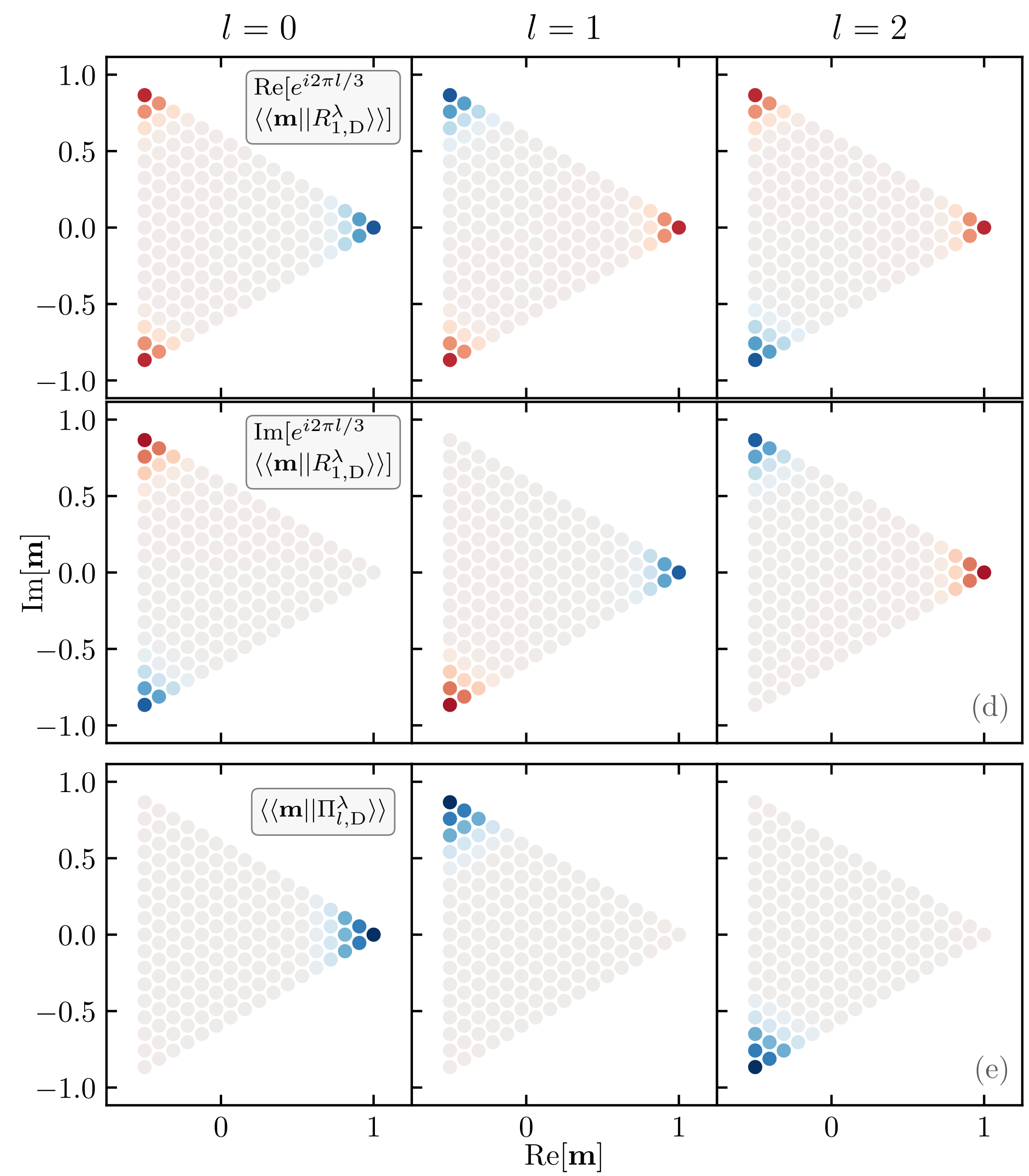
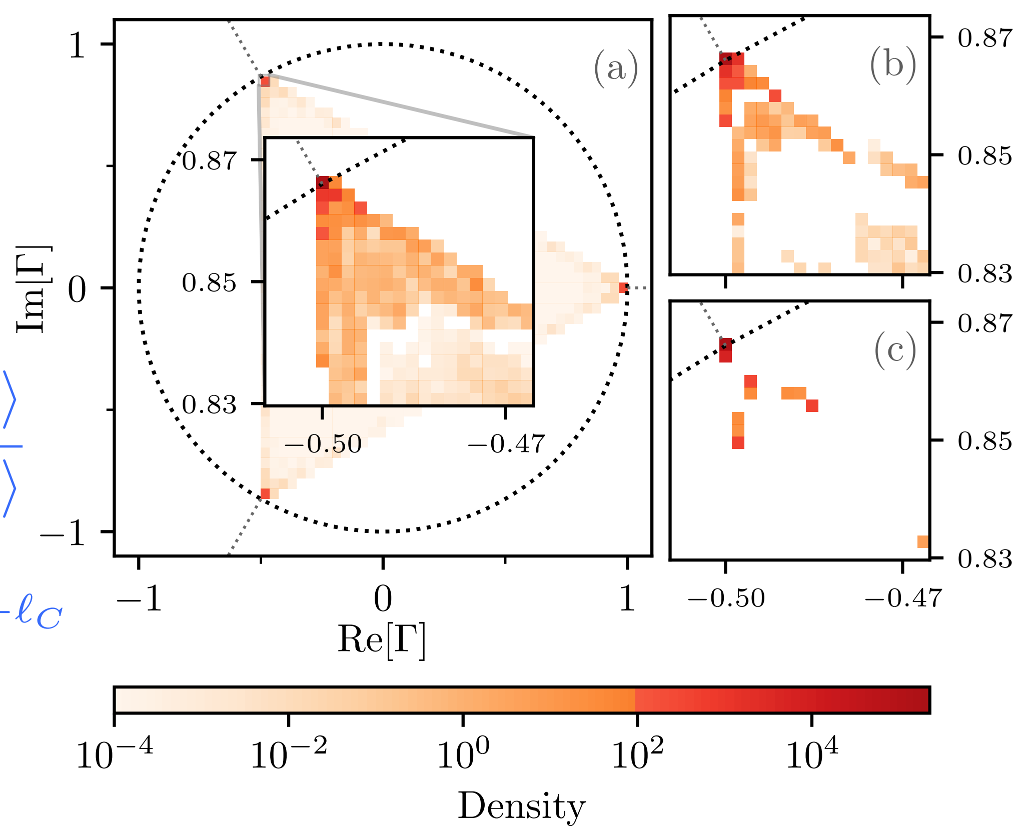


$$\Gamma(C) = \frac{\langle C|R^{\lambda}_{1,\mathrm{D}}\rangle}{\langle C|R^{\lambda}_{0,\mathrm{D}}\rangle}$$

$$\Gamma(C) \approx (\mathrm{e}^{i2\pi/3})^{-\ell_C}$$



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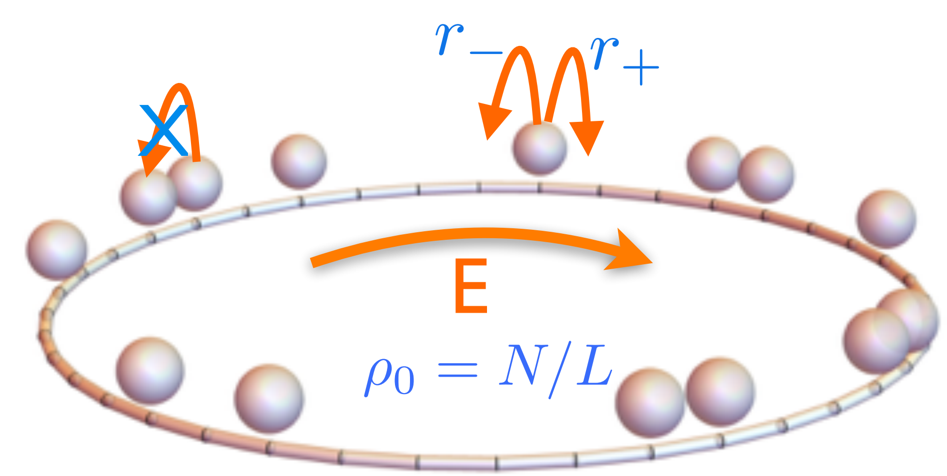
A TIME CRYSTAL DPT

- **WASEP:** 1d lattice with occupation numbers n_i=0,1 + particle jumps with rates $r_\pm = \exp(\pm E/L)/2$
- The closed WASEP exhibits **translation symmetry**



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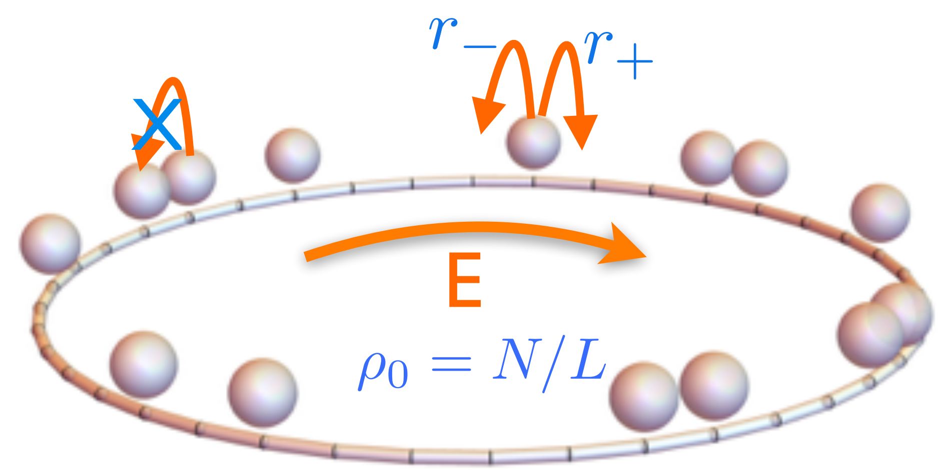

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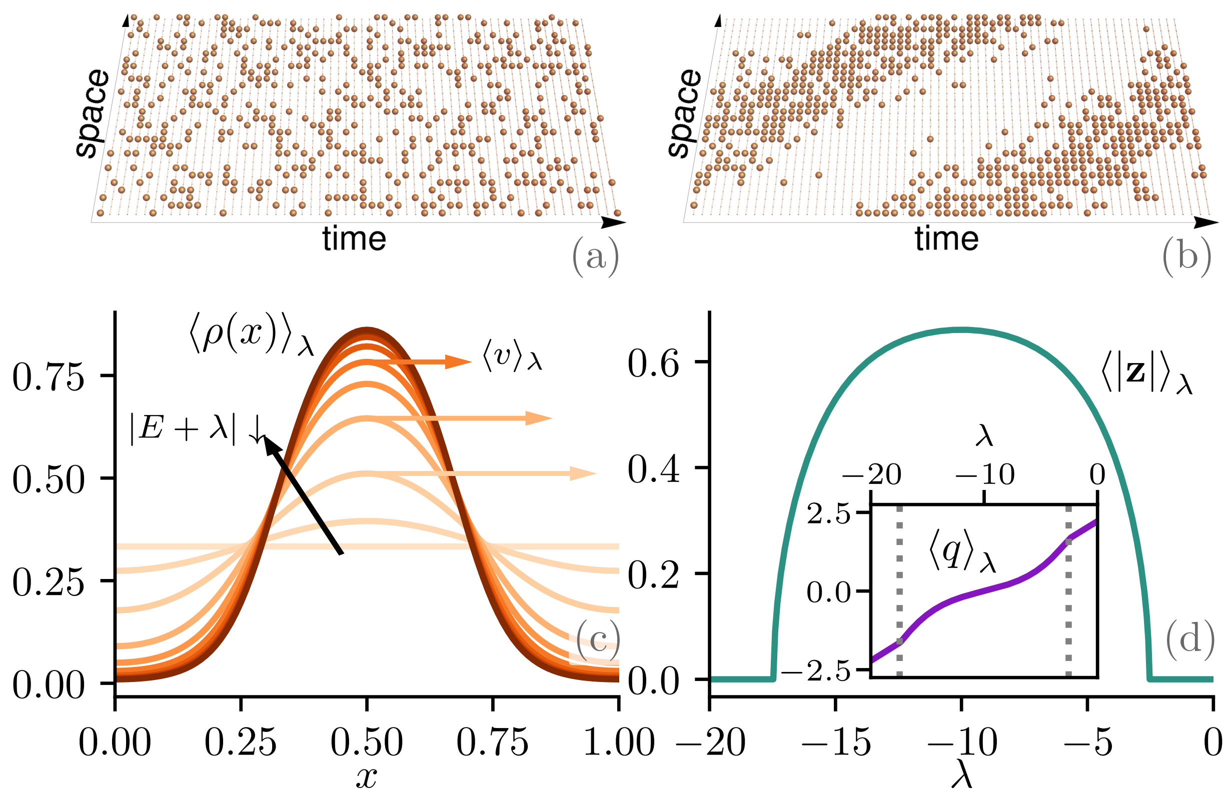

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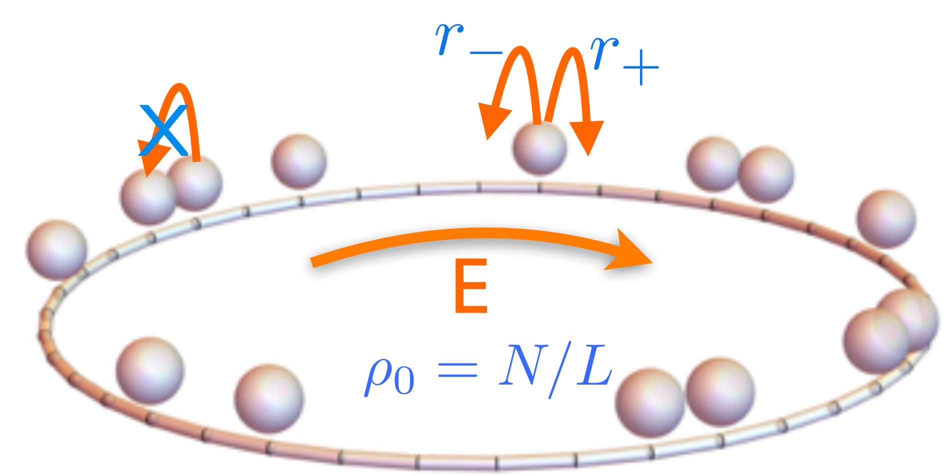

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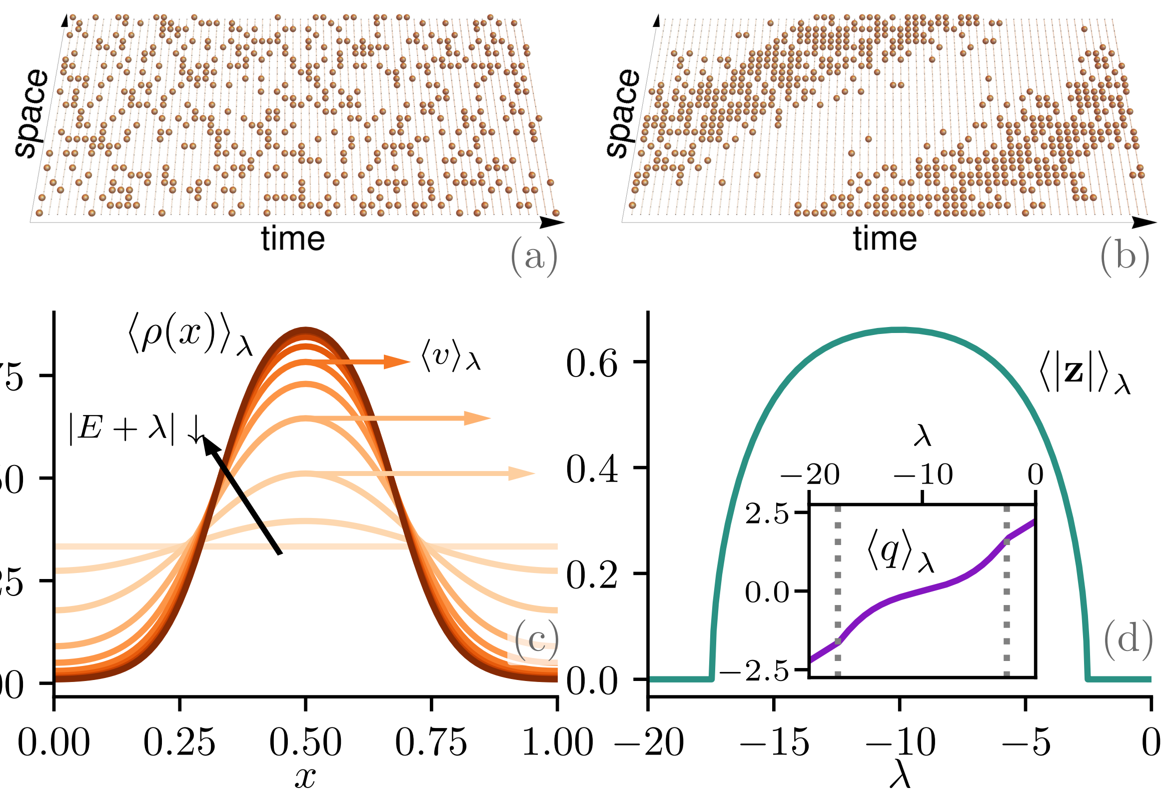

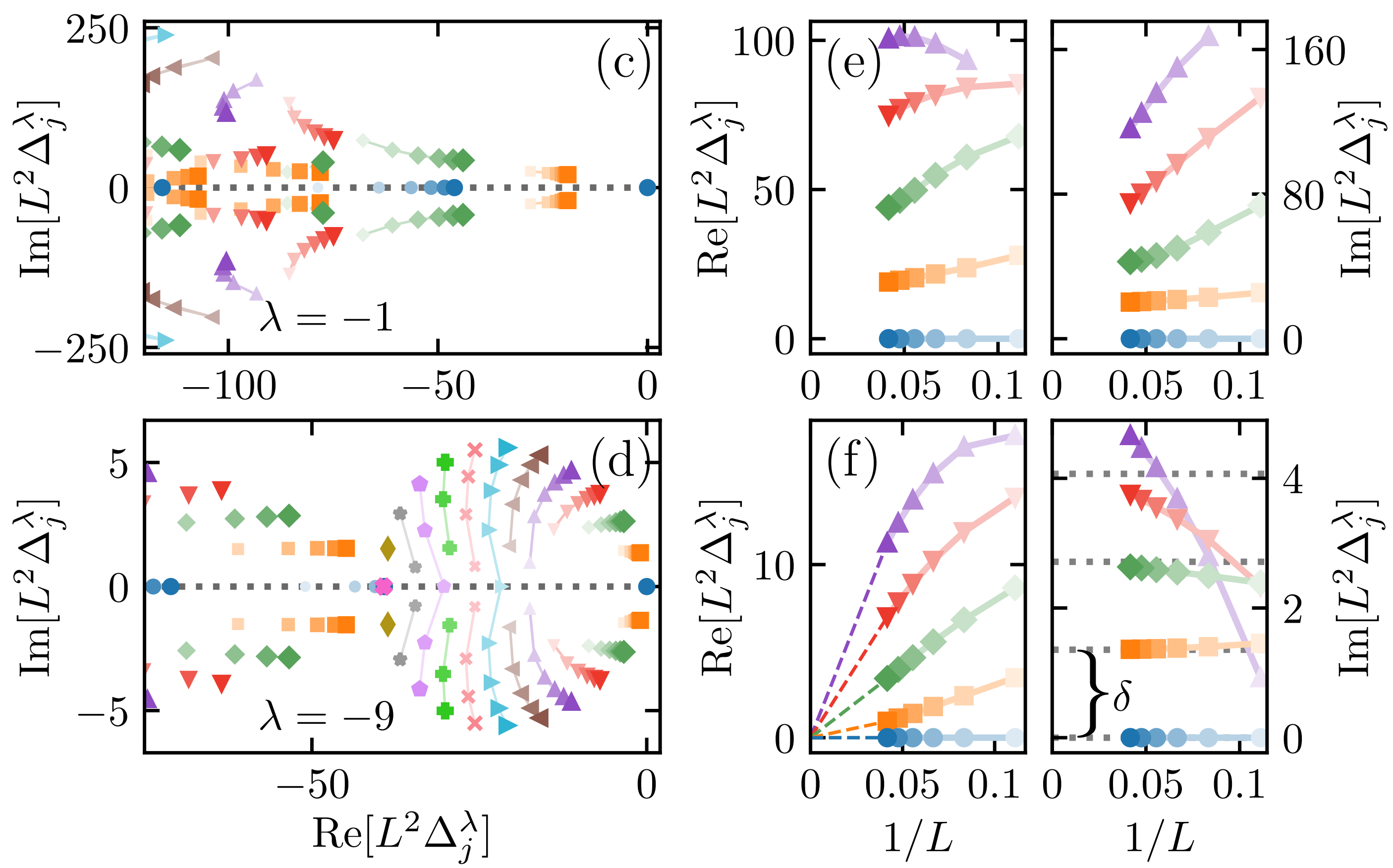

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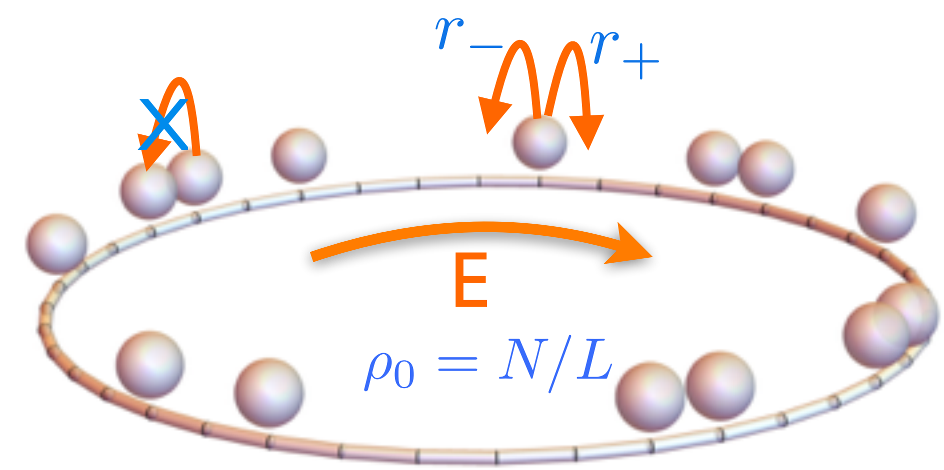

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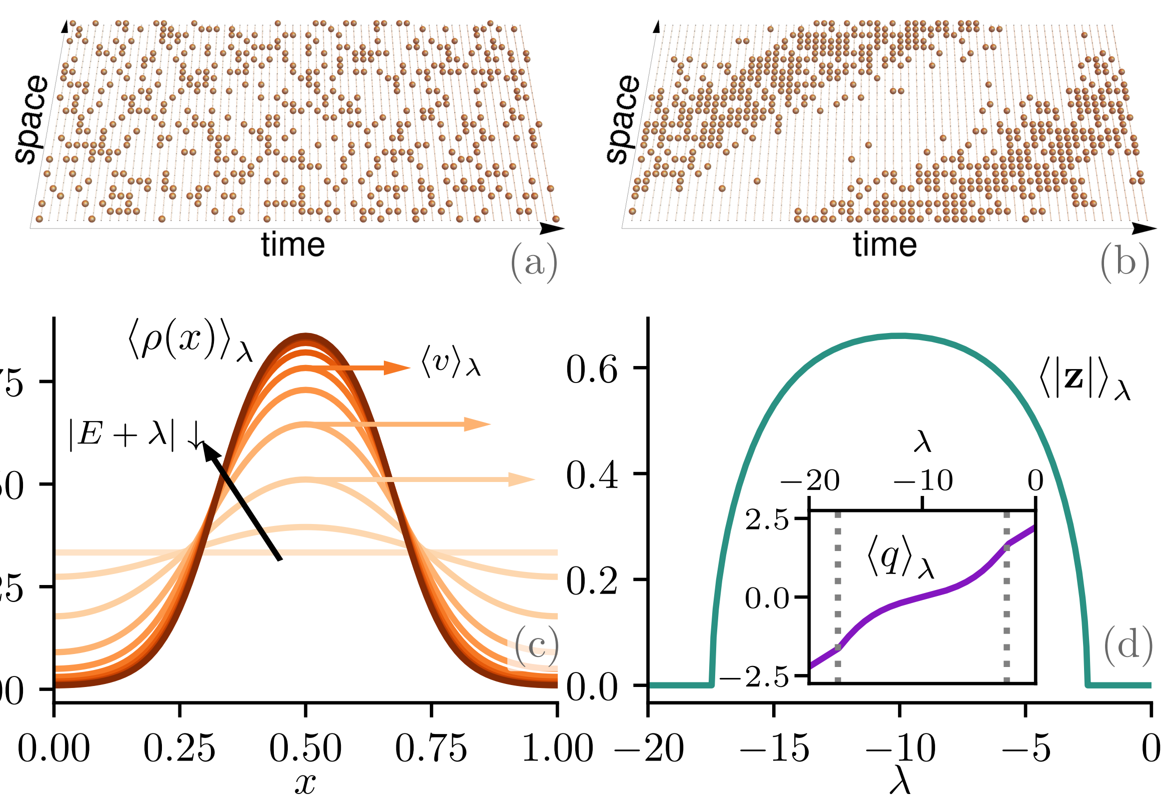
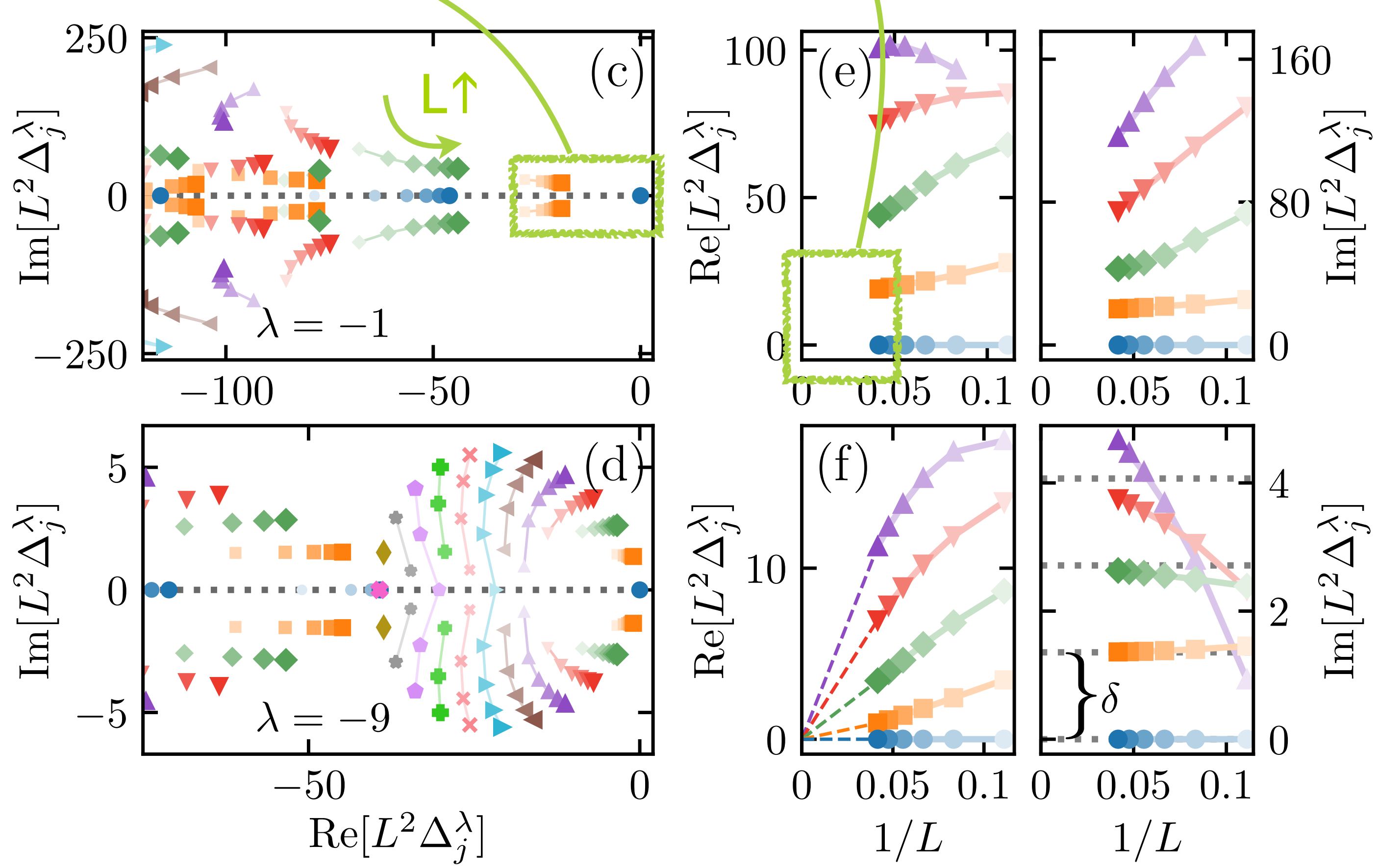


$\mathbb{W}_D^\lambda$ **is gapped** $\Rightarrow$ **unique steady state** $|P_{st}\rangle$



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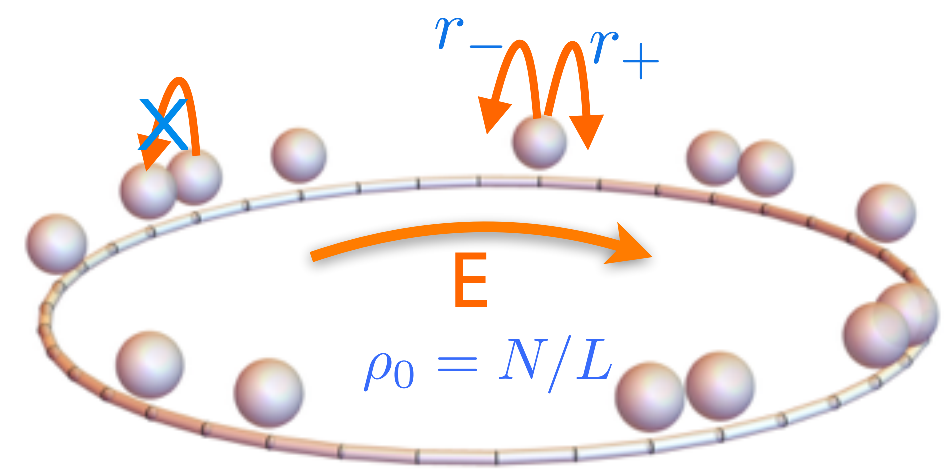

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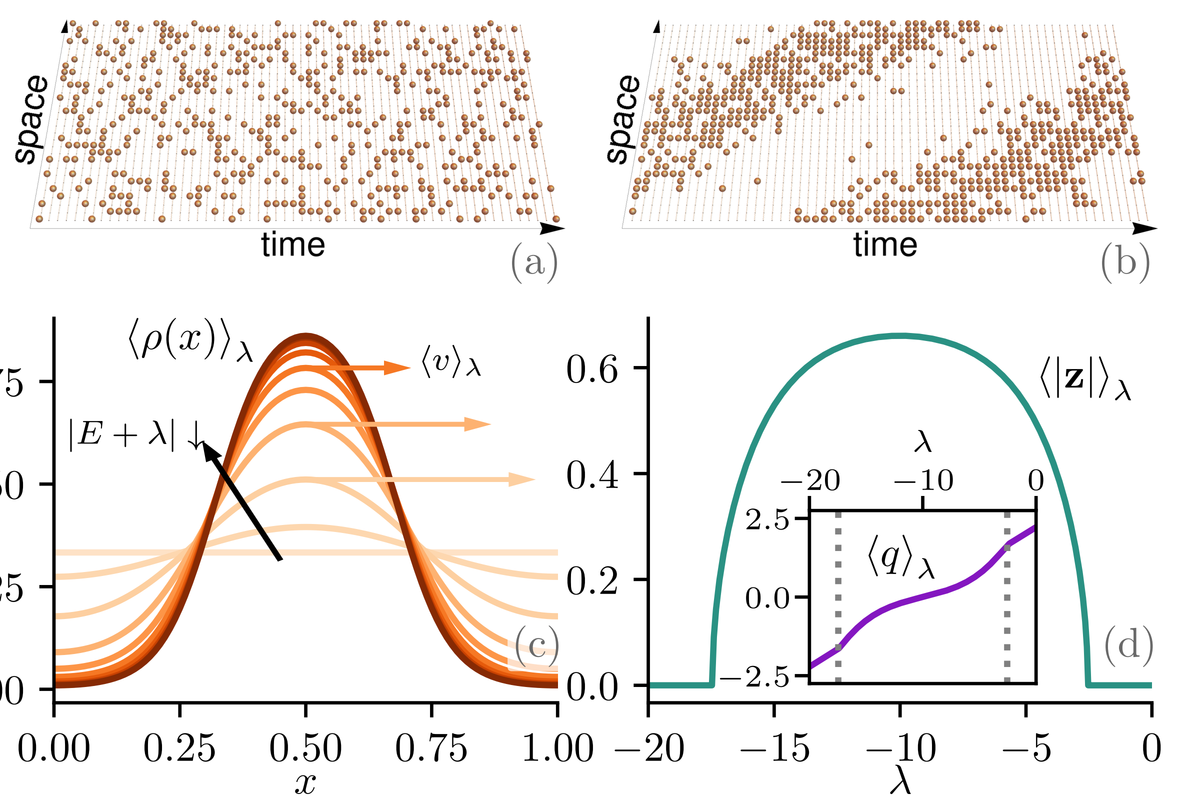
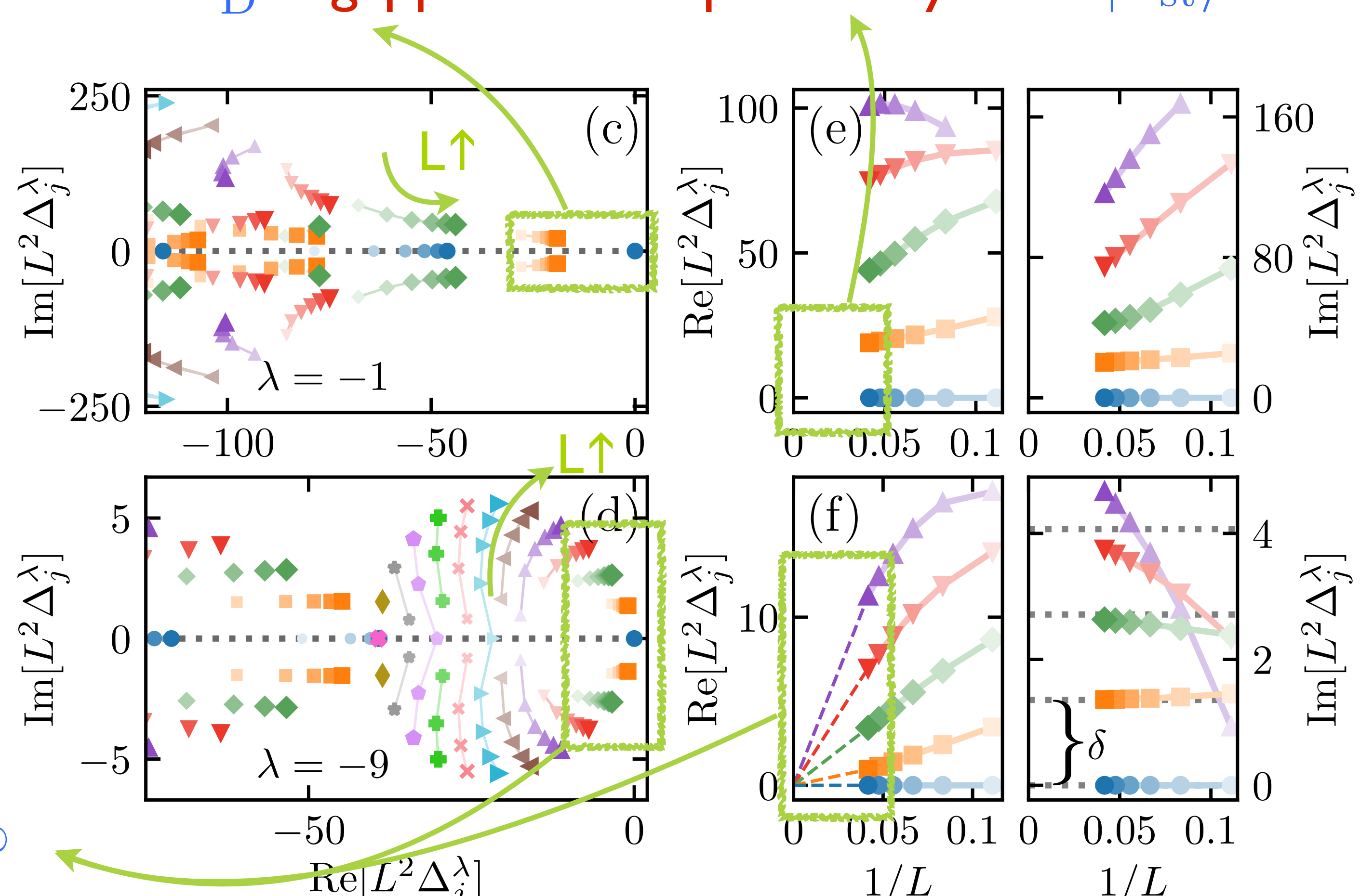
$\mathbb{W}_{\text{D}}^{\lambda}$ is gapped $\Rightarrow$ unique steady state $|P_{\text{st}}\rangle$

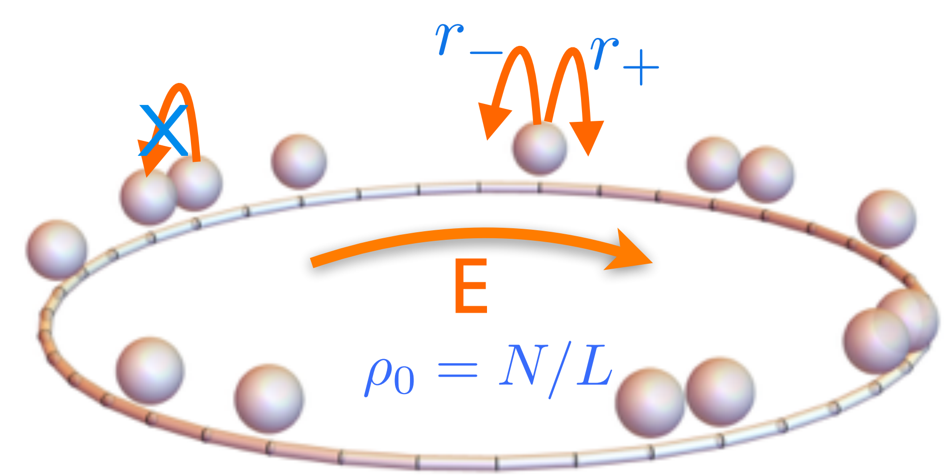




Vanishing spectral gap of $\mathbb{W}_{\text{D}}^{\lambda}$ as $L \to \infty$

$$|P_{\text{ss},P_0}^{\lambda}\rangle \approx |R_{0,\text{D}}^{\lambda}\rangle + 2 \sum_{\substack{j=2 \\ j \text{ even}}}^{L-1} \text{Re}\left[\text{e}^{+it\frac{j\delta}{2}} |R_{j,\text{D}}^{\lambda}\rangle \langle L_{j,\text{D}}^{\lambda}|P_0\rangle \right]$$

A TIME CRYSTAL DPT



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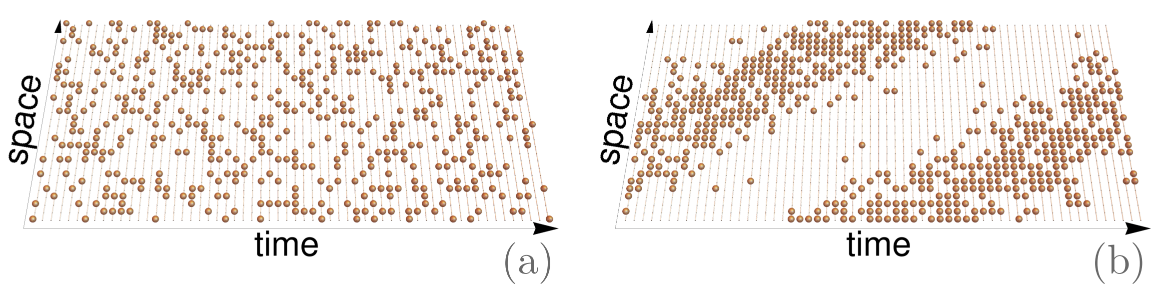

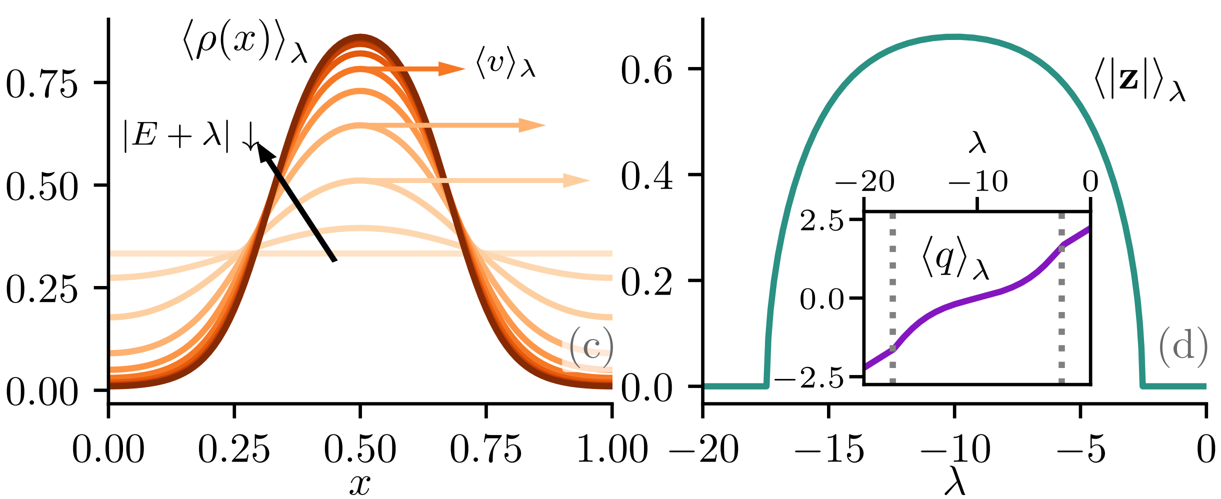

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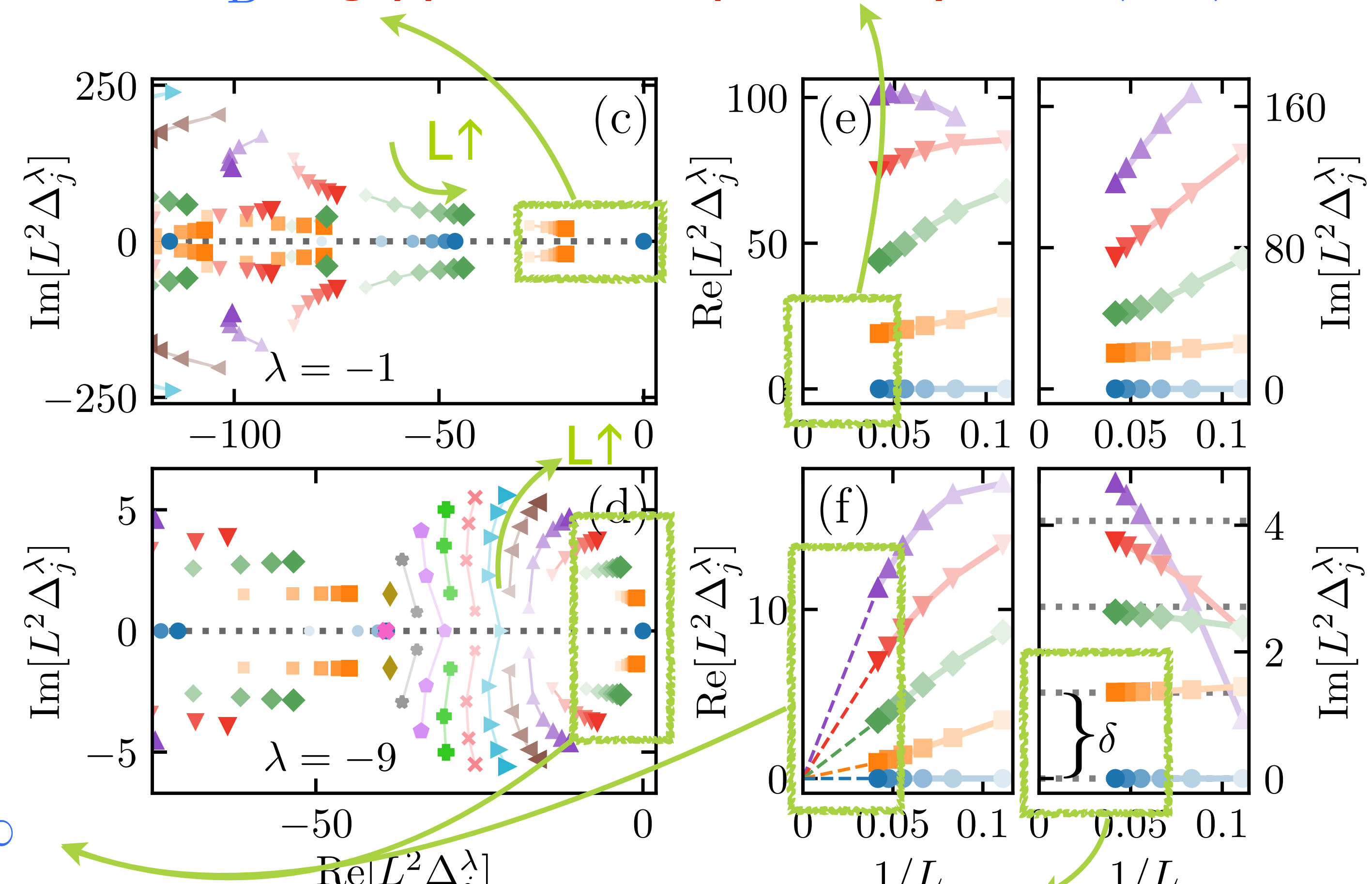

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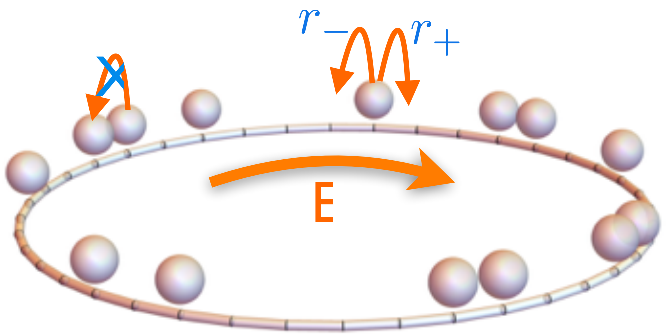
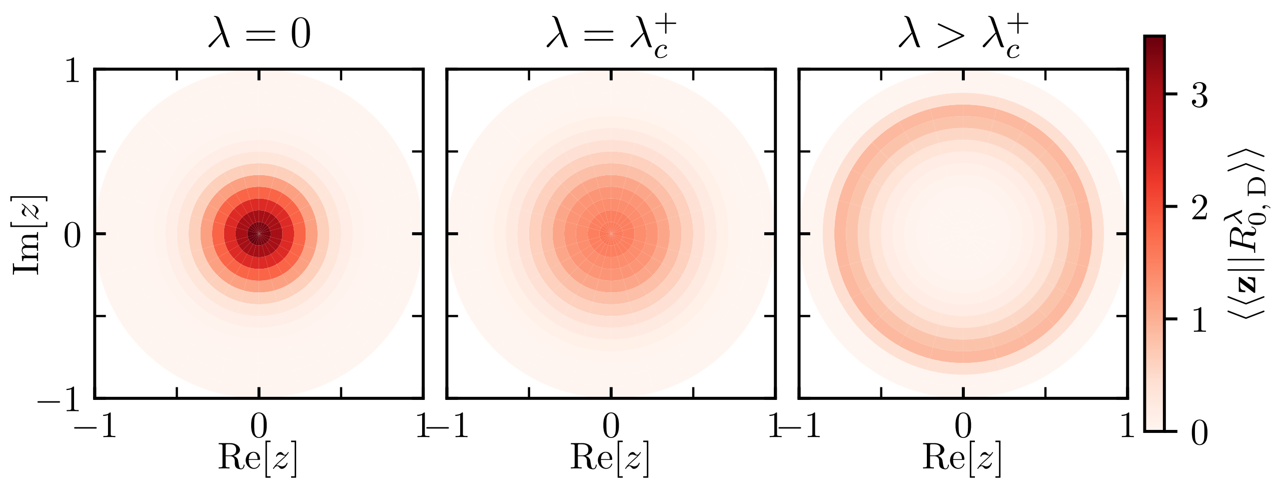
$$|P(t)\rangle = |P(t+T)\rangle$$
$$T = 2\pi/\delta$$

Band structure in **imaginary part** of gap-closing eigenvals.
$v = L/T = L\delta/2\pi$

A TIME CRYSTAL DPT



- **Packing order parameter**: $\mathbf{z} = \frac{1}{N}\sum_{k=1}^{L} n_k\, \mathrm{e}^{i2\pi k/L} = |\mathbf{z}|\mathrm{e}^{i\varphi}$



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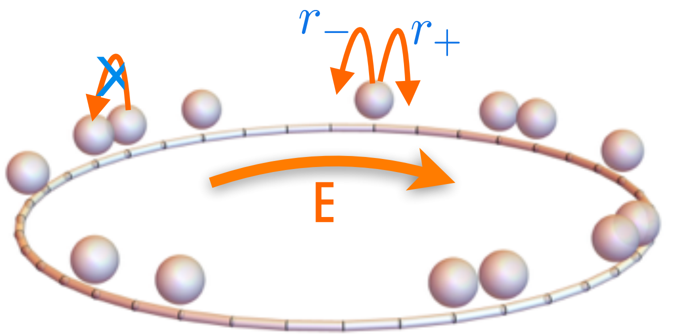

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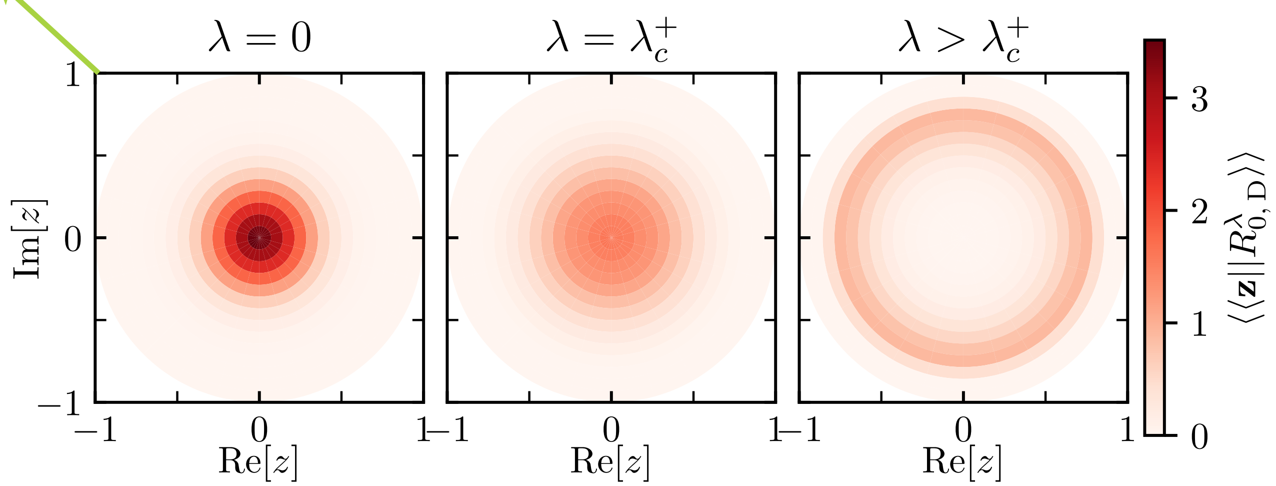
$\langle\langle \mathbf{z} || R^{\lambda}_{0,\mathrm{D}} \rangle\rangle$ peaked at $|\mathbf{z}| \approx 0$ before the DPT



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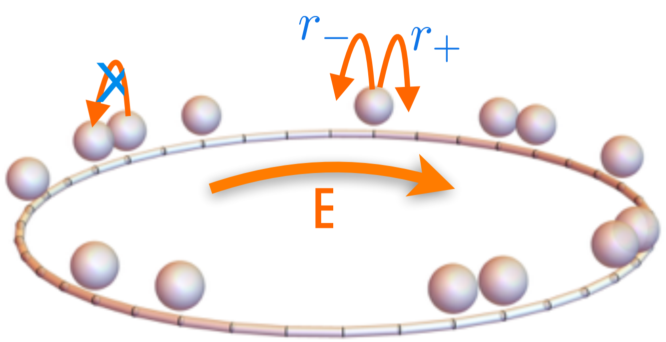

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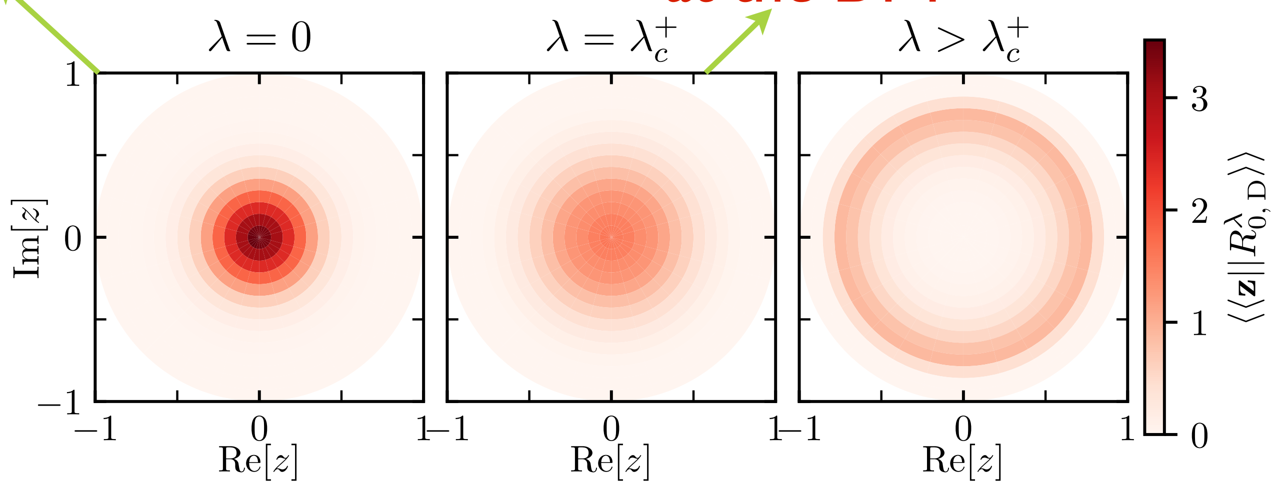
Flat $\langle\langle \mathbf{z} || R_{0,\mathrm{D}}^{\lambda} \rangle\rangle$ at the DPT



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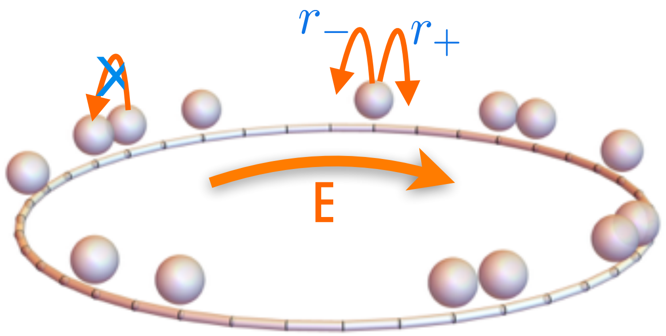

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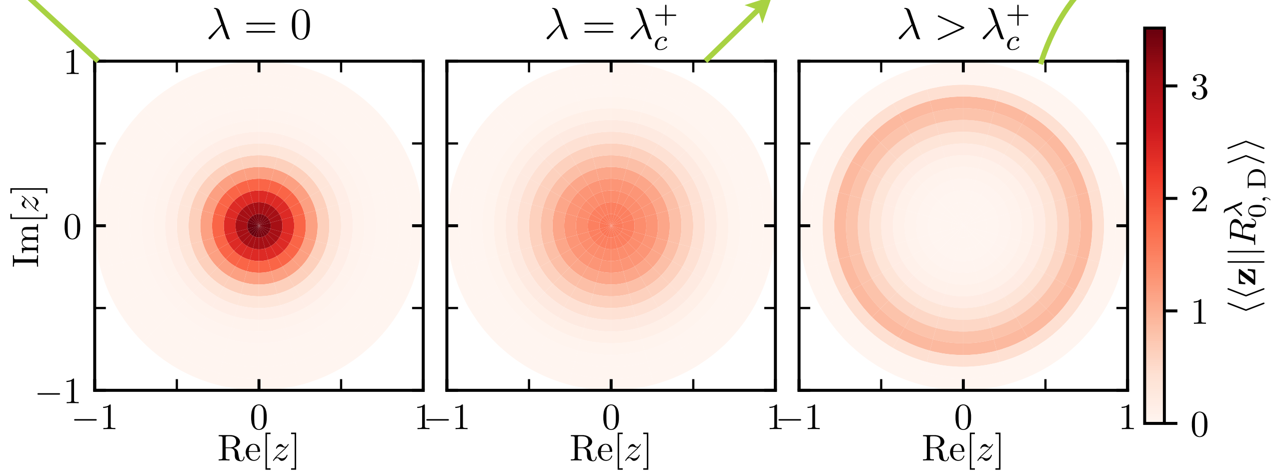
$\langle\langle \mathbf{z}||R^{\lambda}_{0,\mathrm{D}}\rangle\rangle$ peaked at $|\mathbf{z}| \approx 0$ before the DPT

Flat $\langle\langle \mathbf{z}||R^{\lambda}_{0,\mathrm{D}}\rangle\rangle$ at the DPT

Inverted mexican hat $\langle\langle \mathbf{z}||R^{\lambda}_{0,\mathrm{D}}\rangle\rangle$ with ridge at $|\mathbf{z}| \approx 0.7$ after the DPT



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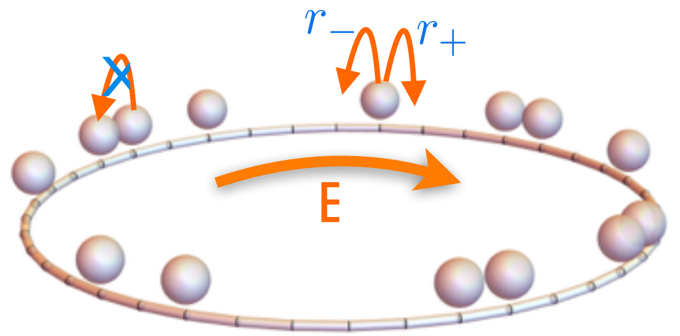

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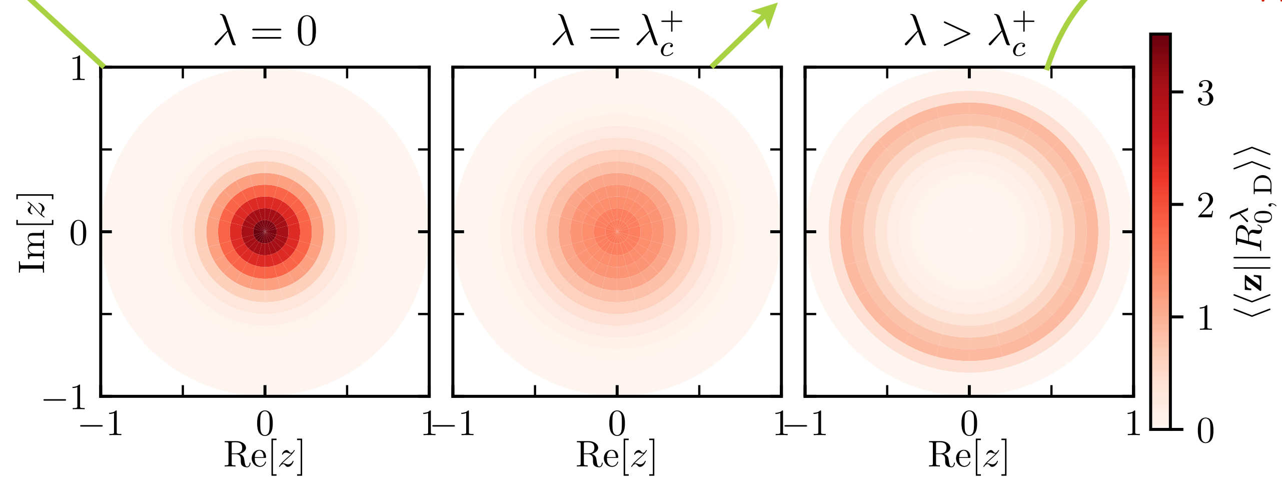
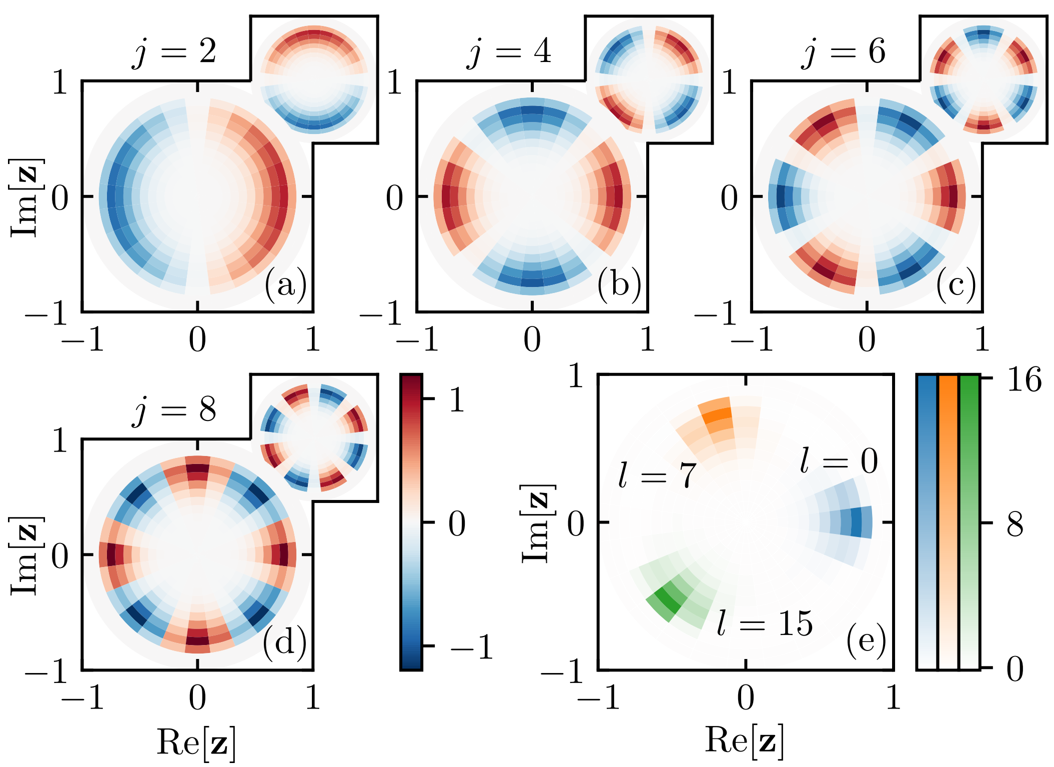


Degenerate eigenvectors $\langle\langle \mathbf{z} | R^{\lambda}_{j,\mathrm{D}} \rangle\rangle$ with j/2-fold angular symmetry



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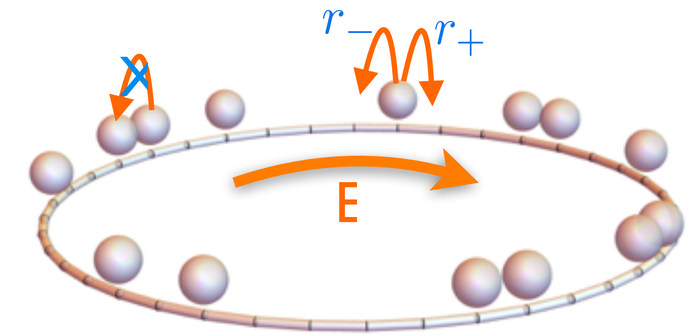

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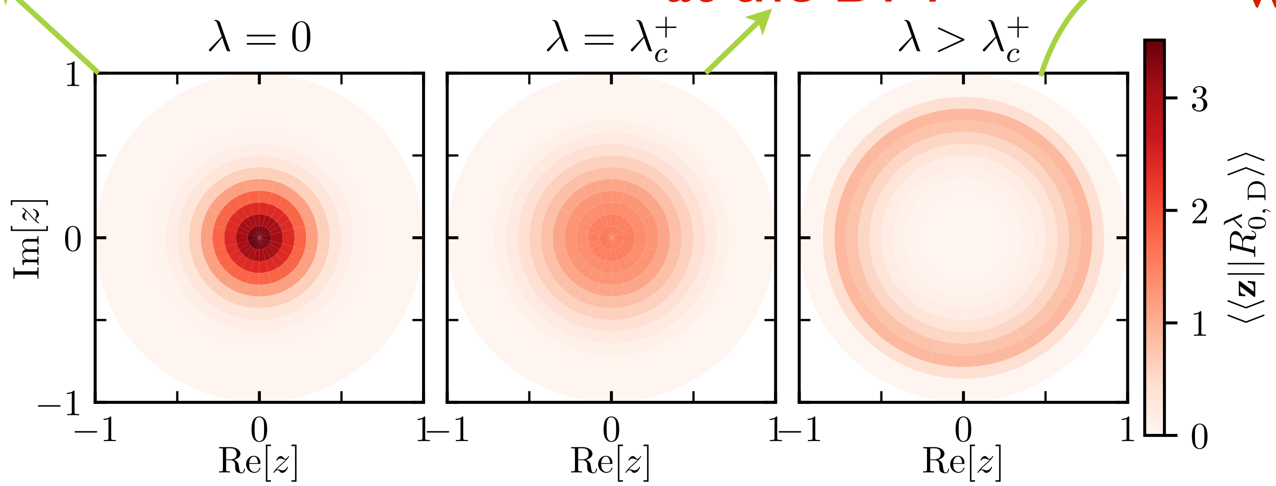

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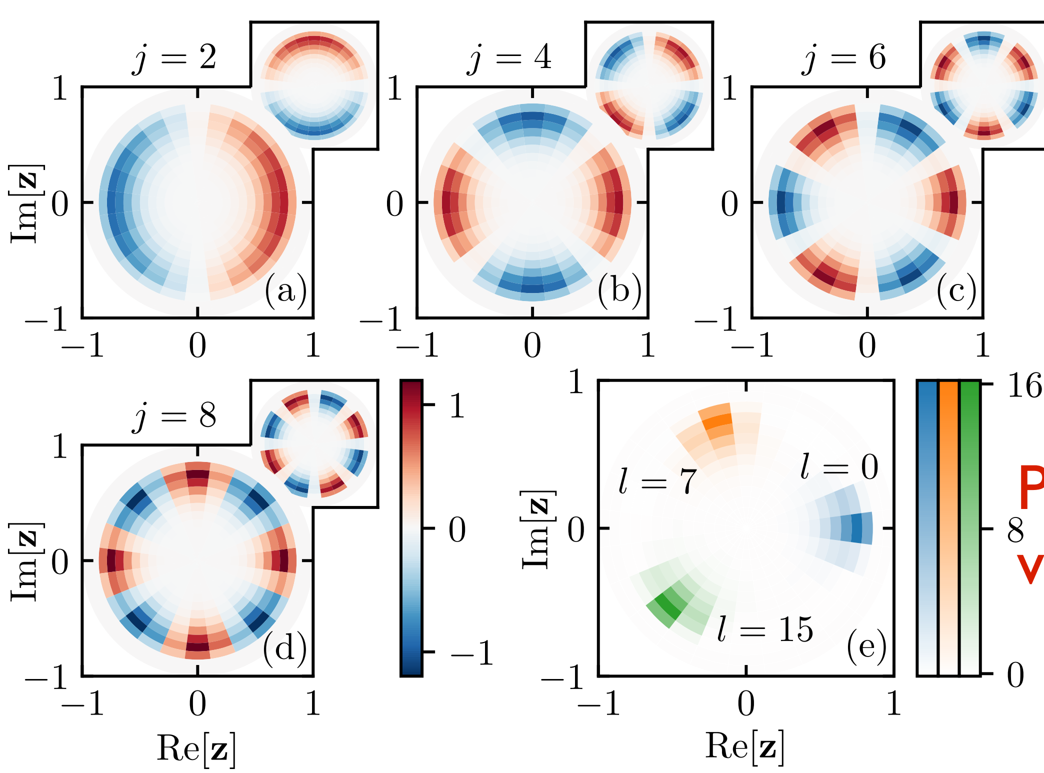
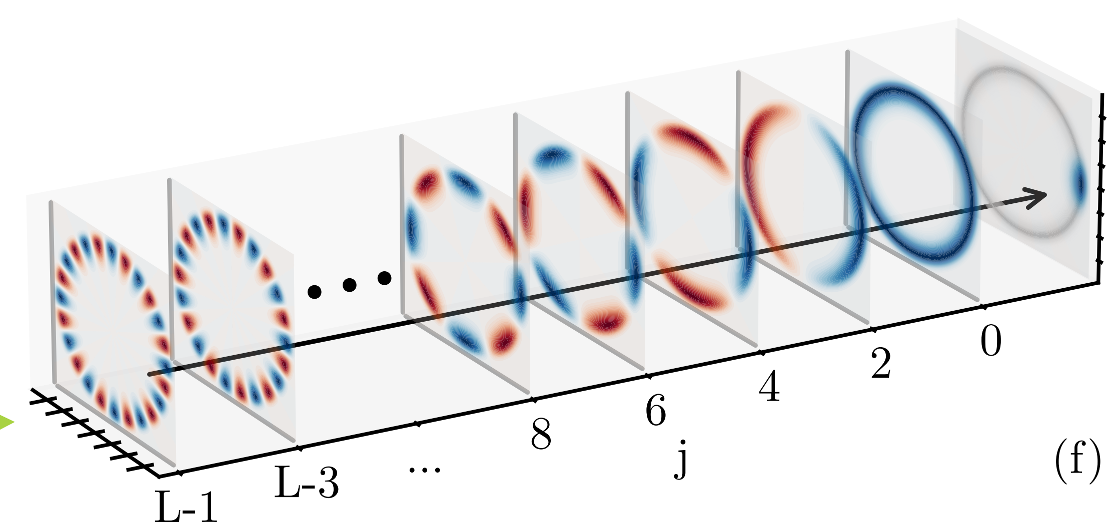
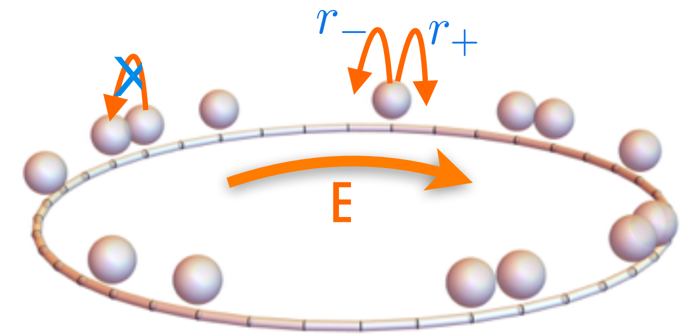


Phase prob. vectors $|\Pi_l^{\lambda}\rangle$



$$|\Pi_l^{\lambda}\rangle = |R_{0,\mathrm{D}}^{\lambda}\rangle + 2\sum_{\substack{j=2 \\ j\ \mathrm{even}}}^{L-1} \mathrm{Re}\left[e^{+i\frac{\pi l}{L}j}|R_{j,\mathrm{D}}^{\lambda}\rangle\right]$$

A TIME CRYSTAL DPT



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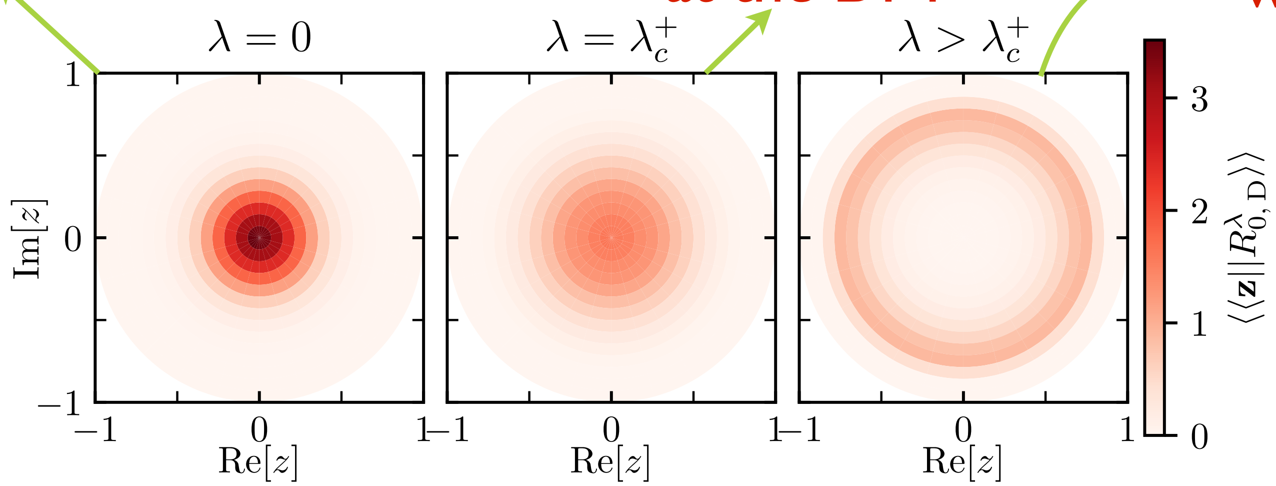
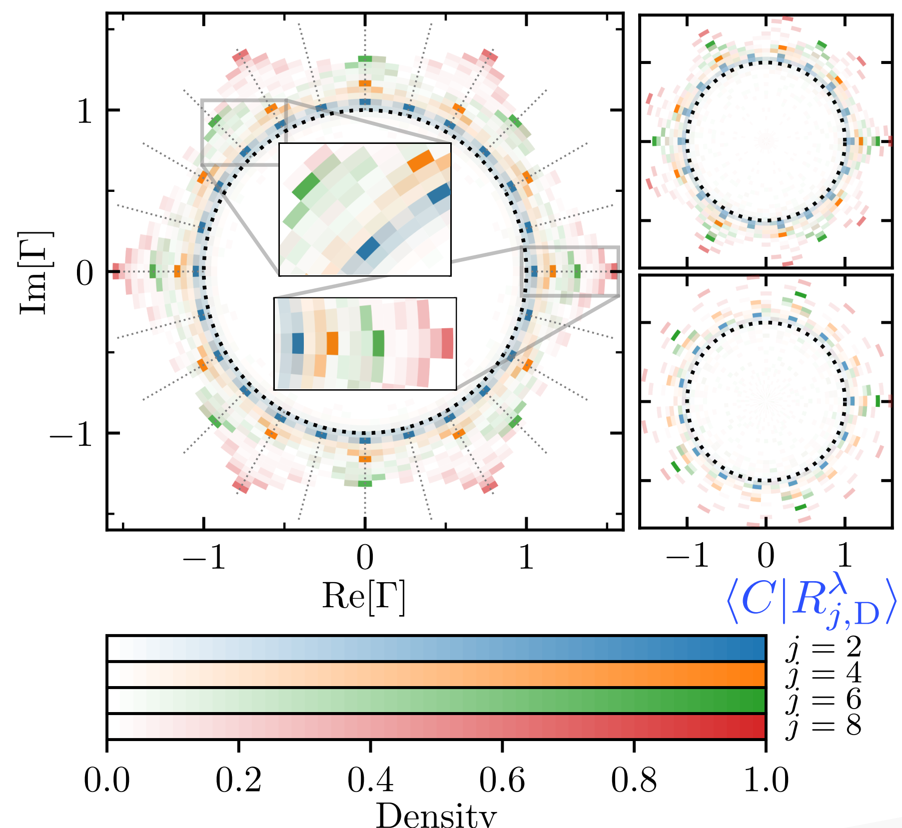
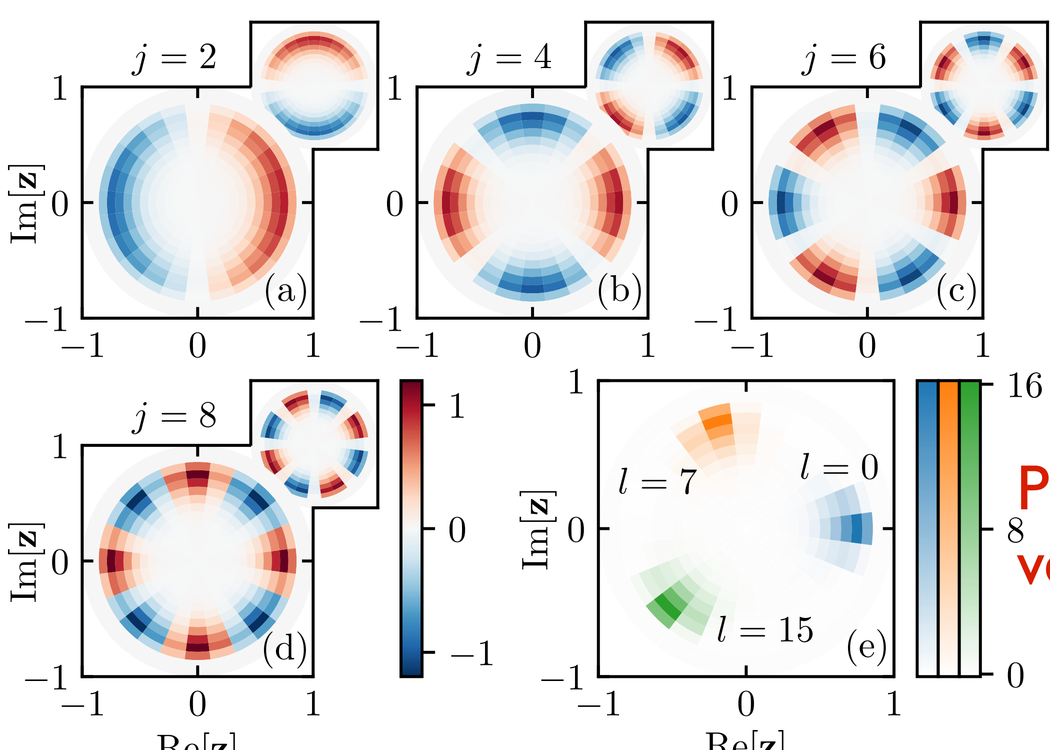




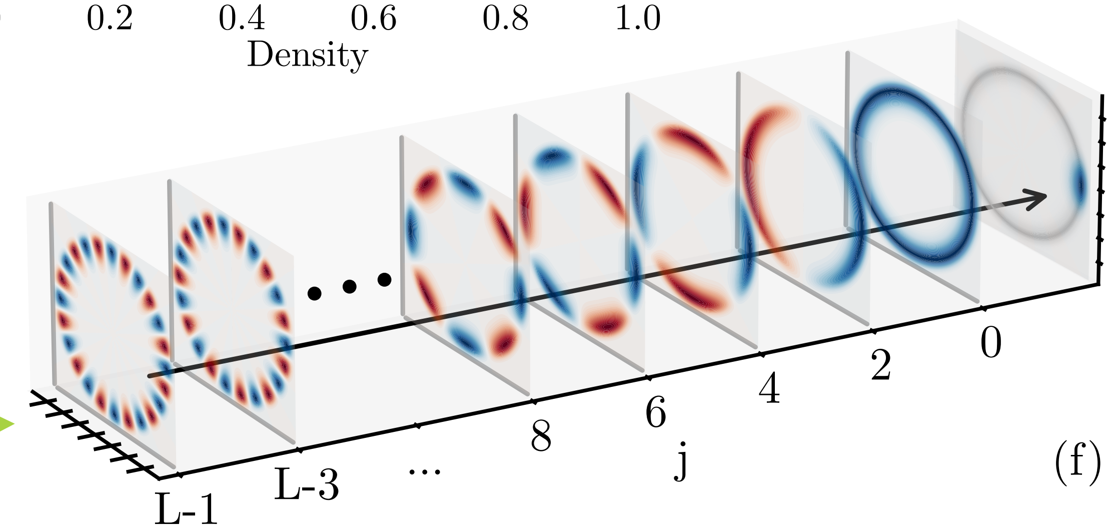
$\Gamma_j(C) = \frac{\langle C|R^{\lambda}_{j,\mathrm{D}}\rangle}{\langle C|R^{\lambda}_{0,\mathrm{D}}\rangle}$

$\langle C|R^{\lambda}_{j,\mathrm{D}}\rangle \approx \mathrm{e}^{-i\frac{\pi j}{L}\ell_C}\langle C|R^{\lambda}_{0,\mathrm{D}}\rangle$

Degenerate eigenvectors $\langle\langle \mathbf{z}|| R^{\lambda}_{j,\mathrm{D}}\rangle\rangle$ with j/2-fold angular symmetry



Phase prob. vectors $|\Pi^{\lambda}_{l}\rangle$



$$|\Pi^{\lambda}_{l}\rangle = |R^{\lambda}_{0,\mathrm{D}}\rangle + 2\sum_{\substack{j=2\\ j\ \mathrm{even}}}^{L-1} \mathrm{Re}\left[\mathrm{e}^{+i\frac{\pi l}{L}j}|R^{\lambda}_{j,\mathrm{D}}\rangle\right]$$

SUMMARY

- We have unveiled the **spectral signatures of symmetry-breaking DPTs**
- The **spectral hallmark** of a symmetry-breaking DPT is the emergence of a **degeneracy in the stationary subspace of Doob eigenvectors**
- **Once the DPT kicks in, different steady states coexist**, characterized by **physical phase probability vectors** connected via symmetry
- **The system breaks the symmetry** by singling out a particular dynamical phase out of the multiple possible phases present in the first Doob eigenvector
- **Symmetry imposes a stringent spectral structure on DPTs**: the components of subleading degenerate eigenvectors are related to those of the leading eigenvector
- Projecting our results to a **reduced order-parameter space** allows for a **quantitative confirmation of our predictions in several paradigmatic examples of DPTs**

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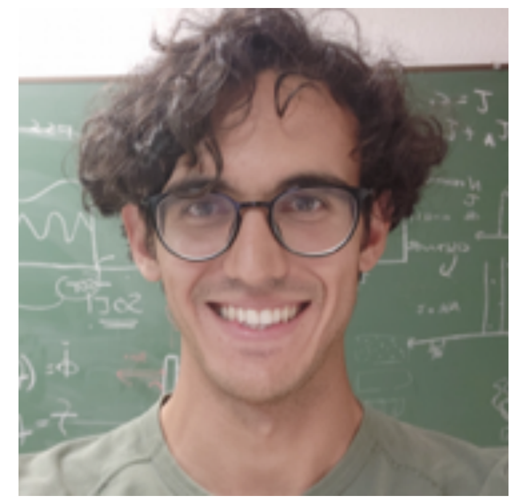
Carlos Pérez-Espigares
(Granada)



Rubén Hurtado
(Granada)

Thanks for your attention

ugr Universidad de Granada