PHASE TRANSITIONS IN THE FLUCTUATIONS OF DRIVEN SYSTEMS

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Tizón, Espigares, Garrido & PH
Phys. Rev. Lett. 119, 090602 (2017)

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DYNAMIC PHASE TRANSITIONS

- Phase transitions (1st- and 2nd-order) are ubiquitous in nature.
- Ideas extended to **fluctuations**, where **dynamic phase transitions (DPTs)** have been identified in the **trajectory statistics** of **classical and quantum systems**
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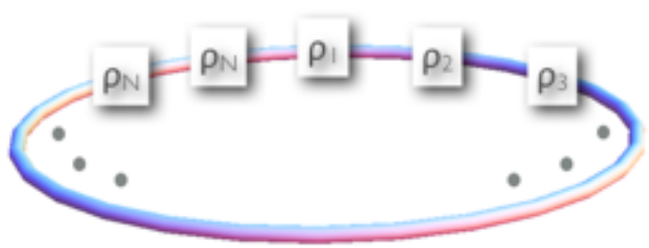
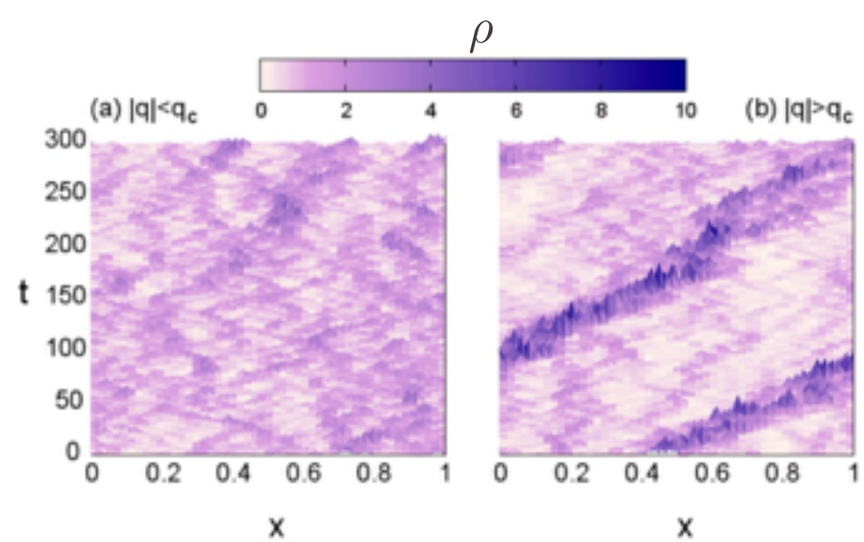
- DPTs appear **when conditioning a system to have a fixed value of some time-integrated observable**, such as, e.g., the **current** or the **activity**
- **Dynamical phases** correspond to **different types of trajectories** : some may display **emergent order, collective rearrangements, and symmetry-breaking**

Kipnis-Marchioro-Presutti (KMP) model



PH&Garrido, PRL **107**, 180601 (2011)



- The **large deviation functions (LDFs)** controlling the statistics of these fluctuations exhibit **nonanalyticities and Lee-Yang singularities at the DPT reminiscent of standard critical behavior**

WHY TO STUDY DPT'S ?

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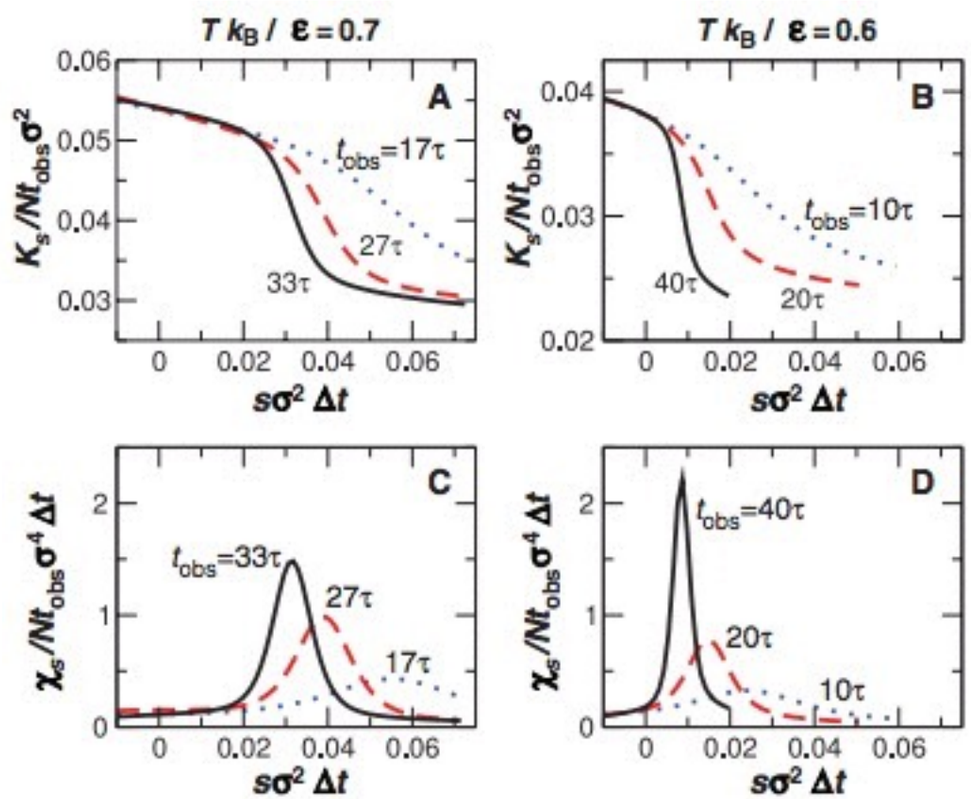
- **LDFs play a role akin to thermodynamic potentials for nonequilibrium systems**, so their nonanalyticities are interesting

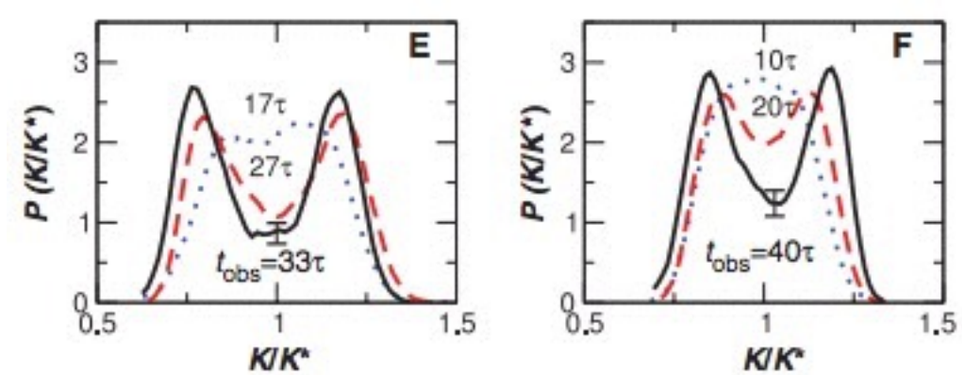
- **Rare events far more probable than anticipated** due to self-organized structures

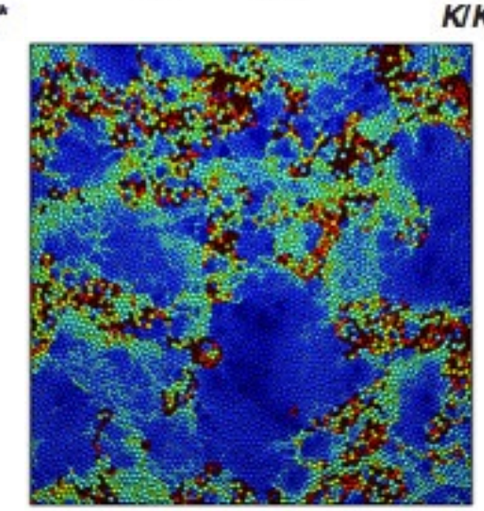
- **Control-theory (or active) interpretation of fluctuations** allows to see **DPTs as singular changes in optimal control field** (**experimentally observable**)

- May **help understanding some elusive phenomena**, e.g. **glass transition**



Hedges, Jack, Garrahan & Chandler, Science **323**, 1309 (2009)
(see also Pinchaipat et al, PRL **119**, 028004 (2017) for experiments)





Hedges (2009)

DYNAMIC PHASE TRANSITIONS IN D>1

- However, study of DPTs **mostly restricted to 1d models or fluctuations of scalar observables in d>1**

Bodineau at al, PRE **72**, 066110 (2005); Pérez-Espigares et al, PRE **87**, 032115 (2013); Vaikuntanathan et al PRE **89**, 062108 (2014); Jack et al PRL **114**, 060601 (2015); Shpielberg et al, PRL **116**, 240603 (2016); Zarfaty et al J. Stat. Mech. (2016) P033304; Baek al, PRL **118**, 030604 (2017); Garrahan et al, PRL **104**, 160601 (2010); Lesanovsky et al, PRL **110**, 150401 (2013).

Garrahan et al, PRL **98**, 195702 (2007); Hedges et al, Science **323**, 1309 (2009); Chandler et al, Annu. Rev. Phys. Chem. **61**, 191 (2010); Pitard et al, EPL **96**, 56002 (2011); Withelam et al, arXiv: 1709.03951; Speck et al, PRL **109**, 195703 (2012); Pinchaipat et al, PRL **119**, 028004 (2017).

- Challenge: bridge the gap to **fluctuations of vectorial observables in d>1** and how they are affected by the **(possible) system anisotropy**

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- Challenge: bridge the gap to **fluctuations of vectorial observables in d>1** and how they are affected by the **(possible) system anisotropy**

- Current statistics: important for nonequilibrium statistical physics.
  Fundamental observable: current LDF

- Aim: **DPTs in the vectorial current statistics of d>1 driven diffusive systems**

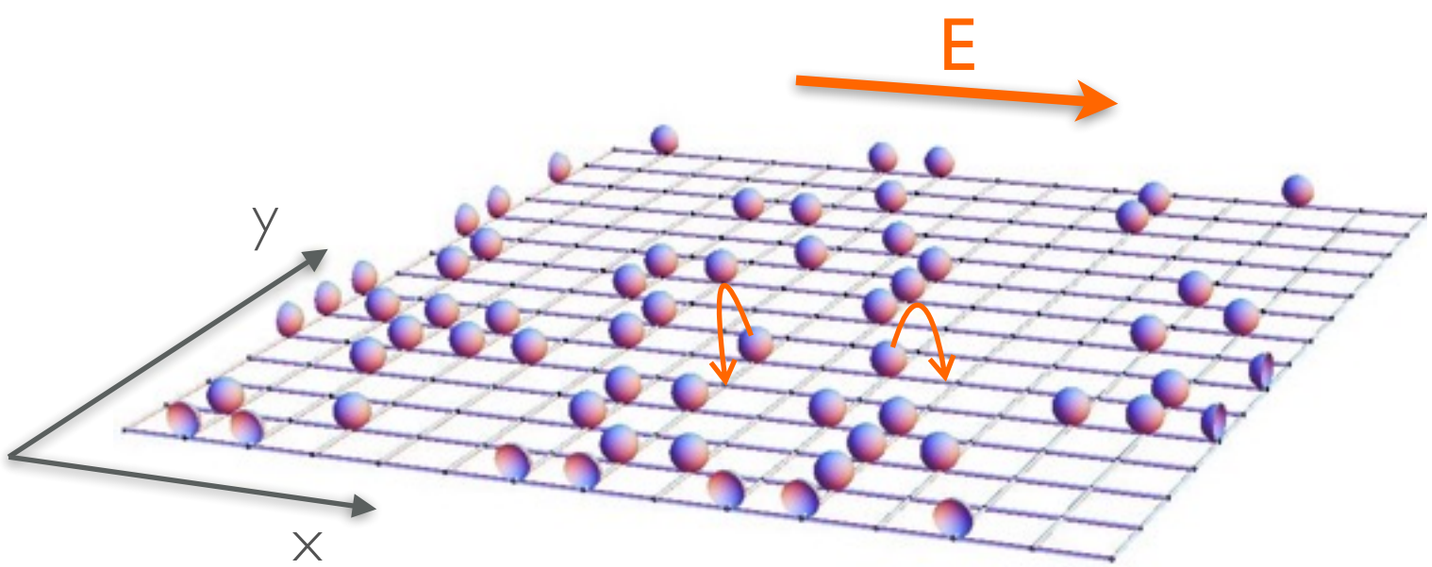
- **Tools**: Macroscopic Fluctuation Theory (MFT) and advanced cloning Monte Carlo simulations for rare events

2D WEAKLY ASYMMETRIC SIMPLE EXCLUSION PROCESS

- WASEP in 2d: **Diffusive particle transport under external field**
- Occupation numbers n_i=0,1 + particle jumps to empty neighbors with rates

$$r_{\pm}^{(\alpha)} \equiv \frac{1}{2}\exp(\pm E_\alpha/L)$$

- **Periodic boundary conditions**



$$\mathbf{n} \equiv \{n_{ij} = 0, 1; i, j \in [1, L]\}$$

$$N = L^2 \qquad M = \sum_{i,j=1}^{L} n_{ij}$$

$$\rho_0 = \frac{M}{N}$$

- For large E and moderate L, **the field per unit length E/L is strong** ⇒ **effective anisotropy ϵ**, enhancing diffusivity and mobility along E.

MACROSCOPIC FLUCTUATION THEORY (MFT)

Bertini, Gabrielli, De Sole, Jona-Lasinio & Landim, 2001-2016
Rev. Mod. Phys. **87**, 593 (2015)

- Evolution equation for **broad family of driven diffusive systems** (like WASEP):

$$\partial_t \rho(\mathbf{r},t) + \boldsymbol{\nabla} \cdot \Big(-\hat{D}(\rho) \cdot \boldsymbol{\nabla} \rho(\mathbf{r},t) + \hat{\sigma}(\rho) \cdot \mathbf{E} + \boldsymbol{\xi}(\mathbf{r},t) \Big) = 0 \qquad \mathbf{r} \in \Lambda \equiv [0,1]^d$$

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- **Gaussian white noise**: Accounts for **microscopic fluctuations at the macroscale**

$$\langle\boldsymbol{\xi}(\mathbf{r},t)\rangle = 0 \qquad \langle\xi_\alpha(\mathbf{r},t)\xi_\beta(\mathbf{r}',t')\rangle = L^{-d}\sigma_\alpha(\rho)\delta_{\alpha\beta}\delta(\mathbf{r}-\mathbf{r}')\delta(t-t') \qquad \alpha,\beta\in[1,d]$$

- **Anisotropy** in diffusivity and mobility matrices:

$$\hat{D}(\rho) = D(\rho)\hat{\mathcal{A}}$$
$$\hat{\sigma}(\rho) = \sigma(\rho)\hat{\mathcal{A}}$$

$$\hat{\mathcal{A}} = \begin{pmatrix} 1+\epsilon & 0 \\ 0 & 1-\epsilon \end{pmatrix}$$

WASEP

$$\boxed{\begin{aligned} D(\rho) &= 1/2 \\ \sigma(\rho) &= \rho(1-\rho) \end{aligned}}$$

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- Probability of a **trajectory** $\{\rho(\mathbf{r},t),\mathbf{j}(\mathbf{r},t)\}_0^\tau \longrightarrow \mathrm{P}\left(\{\rho,\mathbf{j}\}_0^\tau\right) \sim \mathrm{e}^{+L^d I_\tau[\rho,\mathbf{j}]}$

$$I_\tau[\rho,\mathbf{j}] = -\frac{1}{2}\int_0^\tau dt \int_\Lambda d\mathbf{r}[\mathbf{j}-\mathbf{Q}_\mathbf{E}(\rho)]^T\cdot\hat{\sigma}^{-1}(\rho)[\mathbf{j}-\mathbf{Q}_\mathbf{E}(\rho)]$$

$$\mathbf{Q}_\mathbf{E}(\rho) \equiv -\hat{D}(\rho)\cdot\boldsymbol{\nabla}\rho + \hat{\sigma}(\rho)\cdot\mathbf{E} \quad\Big|\quad \partial_t\rho(\mathbf{r},t) + \boldsymbol{\nabla}\cdot\mathbf{j}(\mathbf{r},t) = 0 \quad\Big|\quad \rho_0 = \int_\Lambda d\mathbf{r}\,\rho(\mathbf{r},t)$$

CURRENT STATISTICS IN MFT

- Statistics of trajectories conditioned on total current $\mathbf{J} = \frac{1}{\tau}\int_0^\tau dt \int_\Lambda d\mathbf{r}\, \mathbf{j}(\mathbf{r},t)$
- Dynamical partition function and dynamical free-energy (dFE):

$$Z_\tau(\boldsymbol{\lambda}) = \sum_{\mathbf{J}} P_\tau(\mathbf{J}) \mathrm{e}^{\tau \boldsymbol{\lambda}\cdot\mathbf{J}} \longrightarrow \mu(\boldsymbol{\lambda}) = \lim_{\tau\to\infty} \tau^{-1} \ln Z_\tau(\boldsymbol{\lambda})$$

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- MFT leads to variational problem for μ(λ). Optimal trajectory≠steady profile:

$$\rho_{\mathbf{J}}(\mathbf{r},t) \qquad \mathbf{j}_{\mathbf{J}}(\mathbf{r},t)$$

- Homogeneous steady state: $\rho_{\mathrm{st}}(\mathbf{r}) = \rho_0 \quad \mathbf{j}_{\mathrm{st}}(\mathbf{r}) = \langle \mathbf{J} \rangle = \sigma_0 \hat{\mathcal{A}} \mathbf{E}$

$$D_0 \equiv D(\rho_0)$$
$$\sigma_0 \equiv \sigma(\rho_0)$$

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- Small current fluctuations result from weakly-correlated local events which sum incoherently ⇒ homogeneous optimal fields

$$|\mathbf{J} - \langle \mathbf{J} \rangle| \ll 1 \;\Rightarrow\; \left| \begin{array}{l} \rho_{\mathbf{J}}(\mathbf{r},t) = \rho_0 \\ \mathbf{j}_{\mathbf{J}}(\mathbf{r},t) = \langle \mathbf{J} \rangle \end{array} \right.$$

- Leads to quadratic dFE and Gaussian current statistics

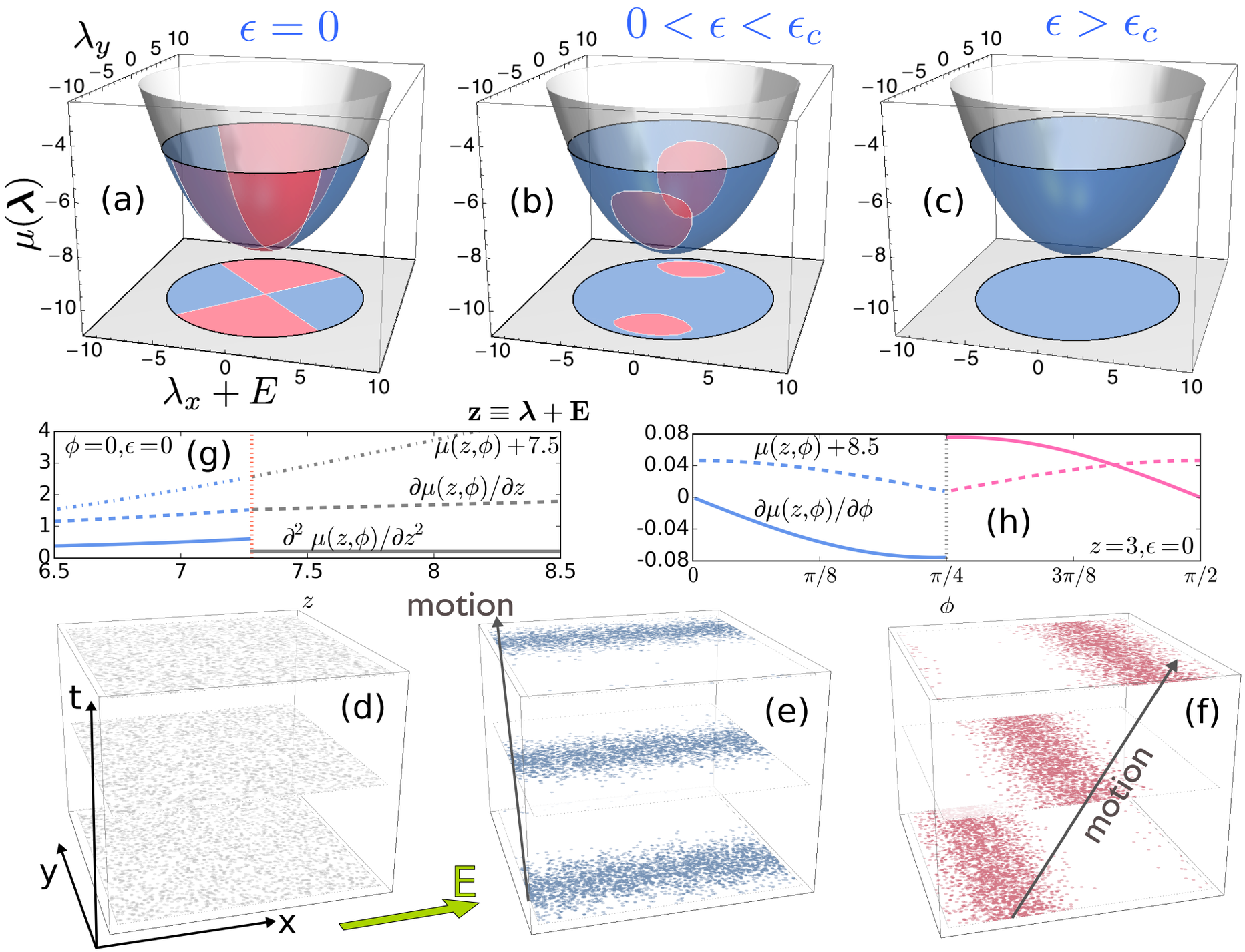
$$\mu_{\mathrm{G}}(\mathbf{z}) = (\mathbf{z}\cdot\hat{\sigma}_0\mathbf{z} - \mathbf{E}\cdot\hat{\sigma}_0\mathbf{E})/2 \qquad \mathbf{z} \equiv \boldsymbol{\lambda} + \mathbf{E}$$

- Stability of this homogeneous solution?

A RICH PHASE DIAGRAM (2D WASEP)

$\mathbf{J} \cdot \hat{\mathcal{A}}^{-1}\mathbf{J} \leq \sigma_0^2 \Xi_c$

$\mathbf{E} \cdot \hat{\mathcal{A}}\mathbf{E} \geq |\Sigma_c|$



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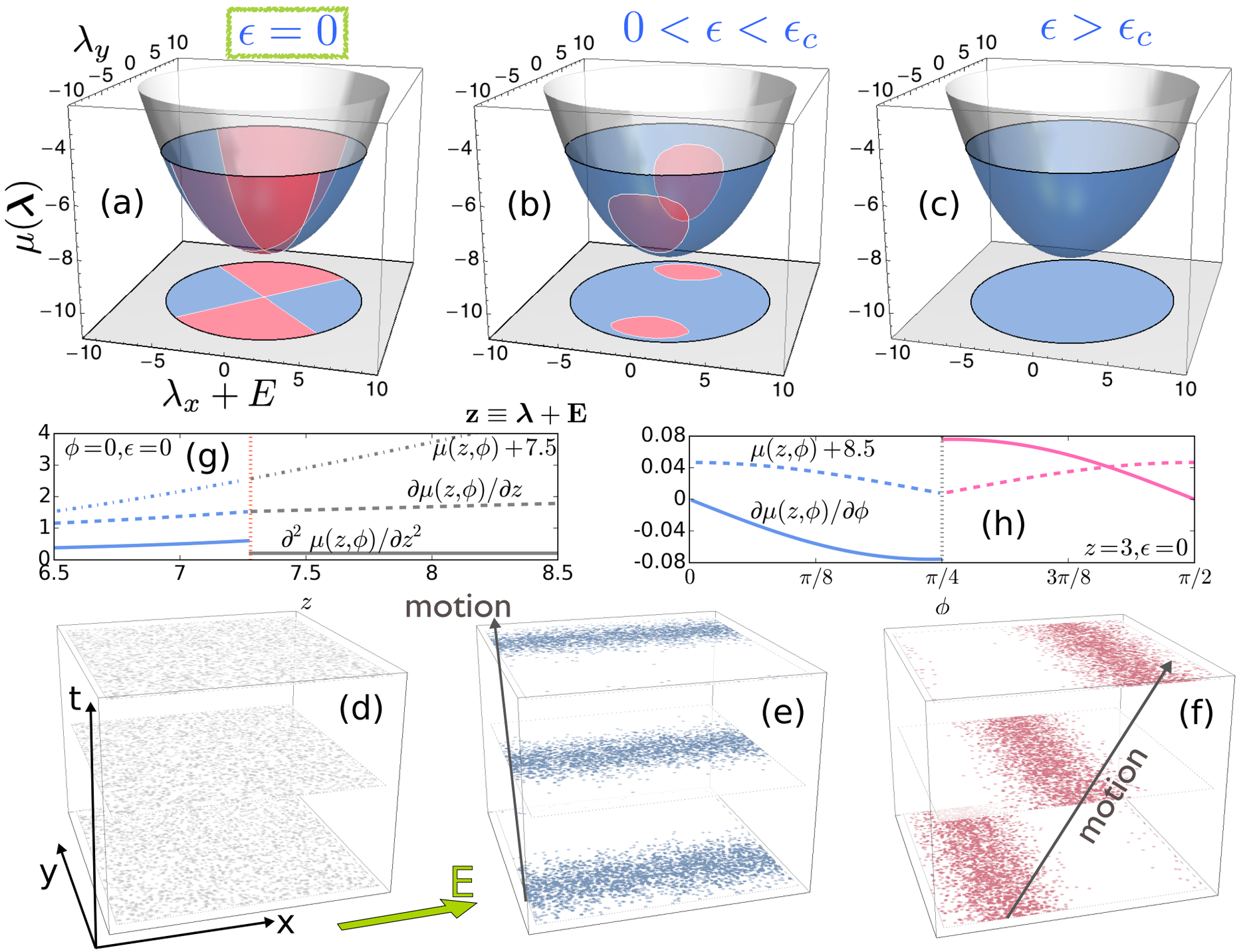

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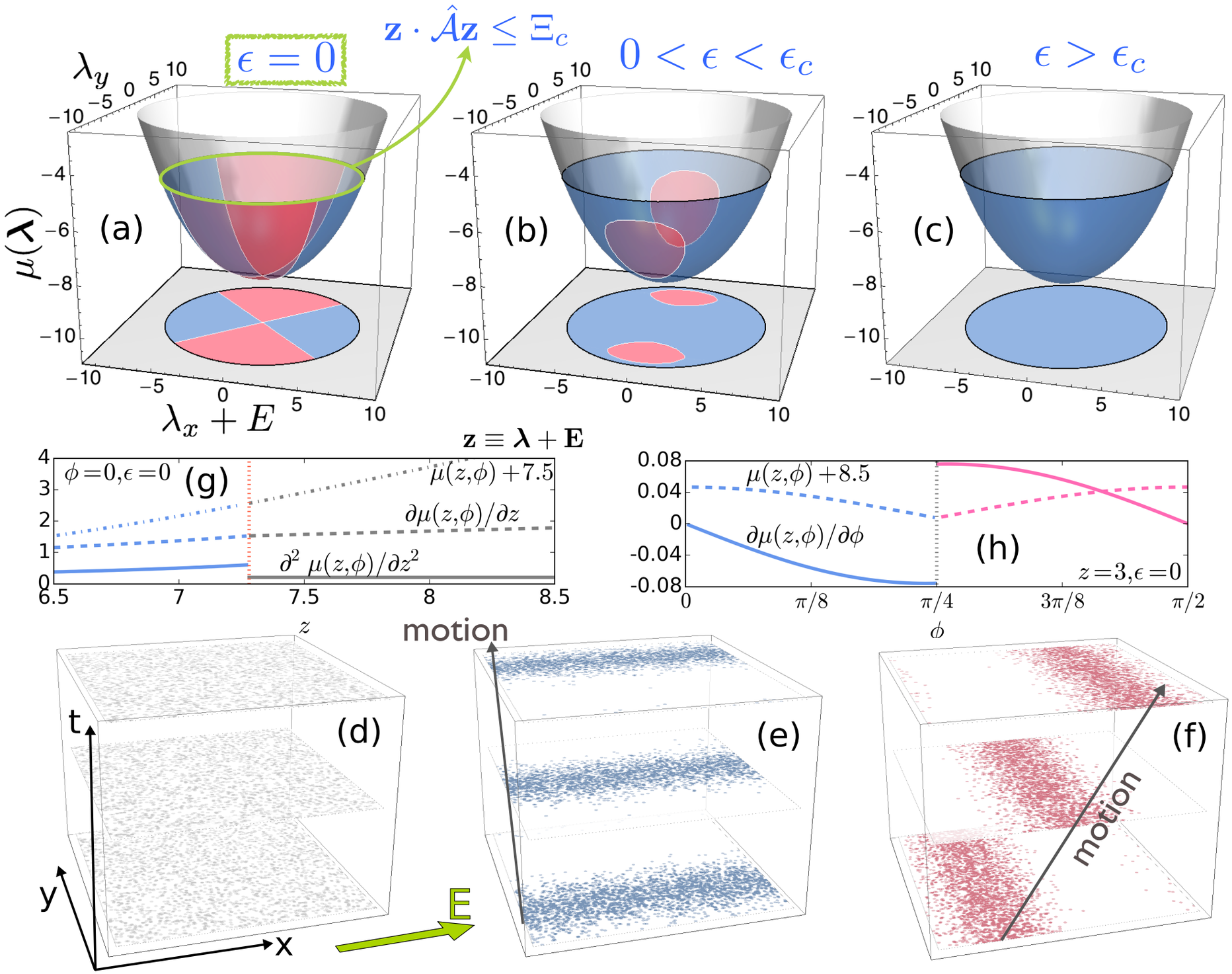

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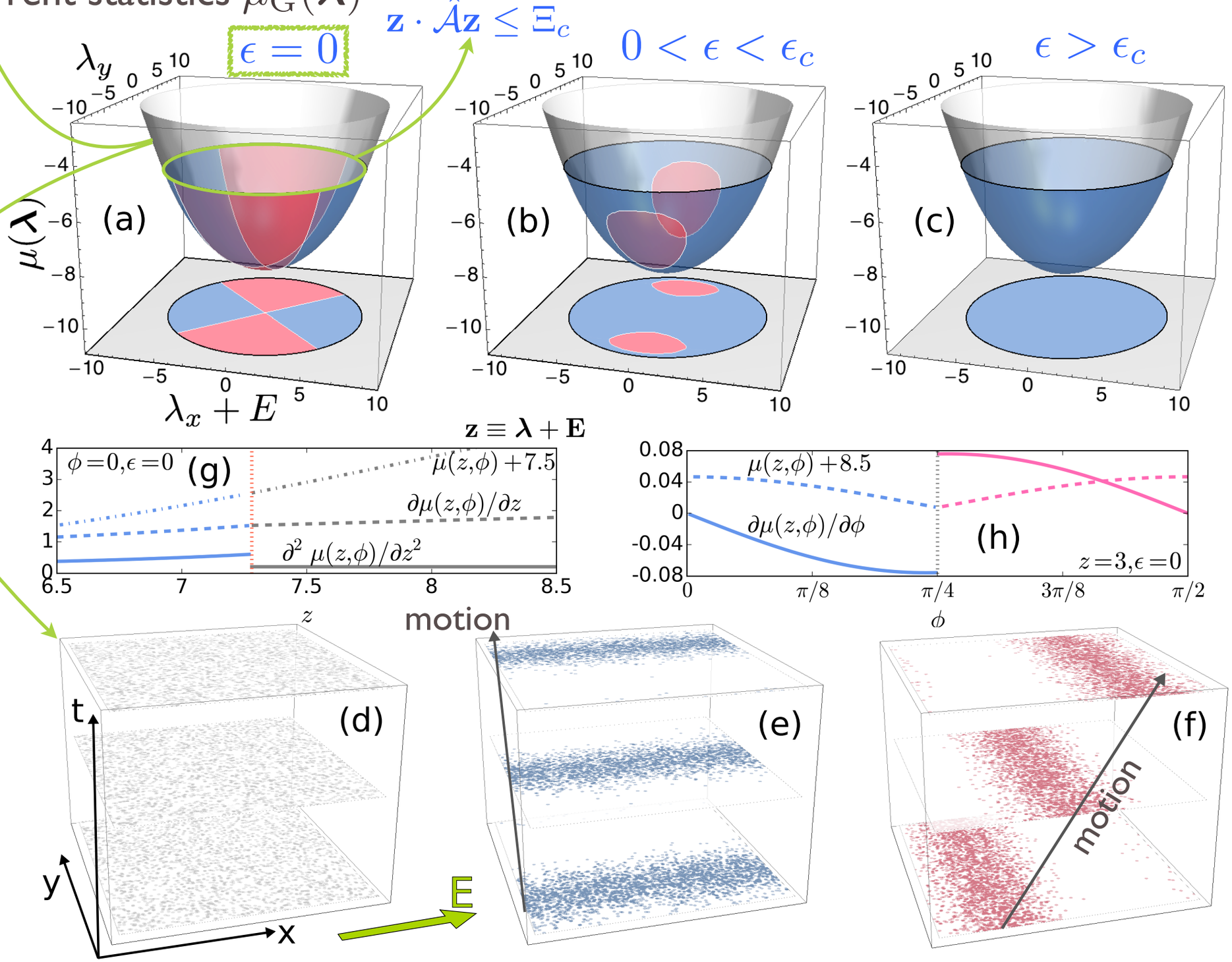
Gaussian current statistics $\mu_{\mathrm{G}}(\boldsymbol{\lambda})$



Homogeneous, structureless typical trajectories

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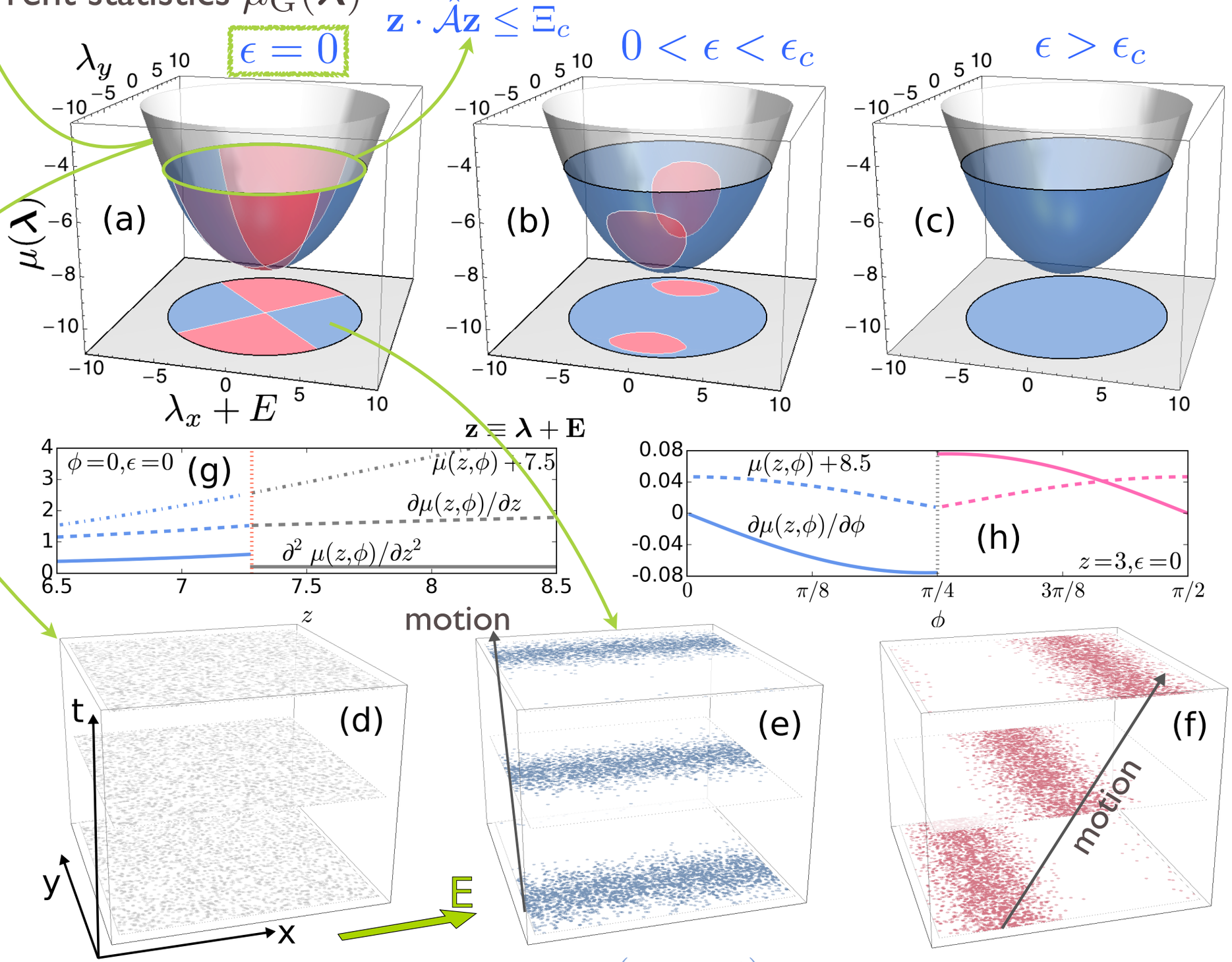

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$\omega_{\mathbf{J}}(y - vt)$

Traveling wave in y, jam in x

Broken spacetime symmetry

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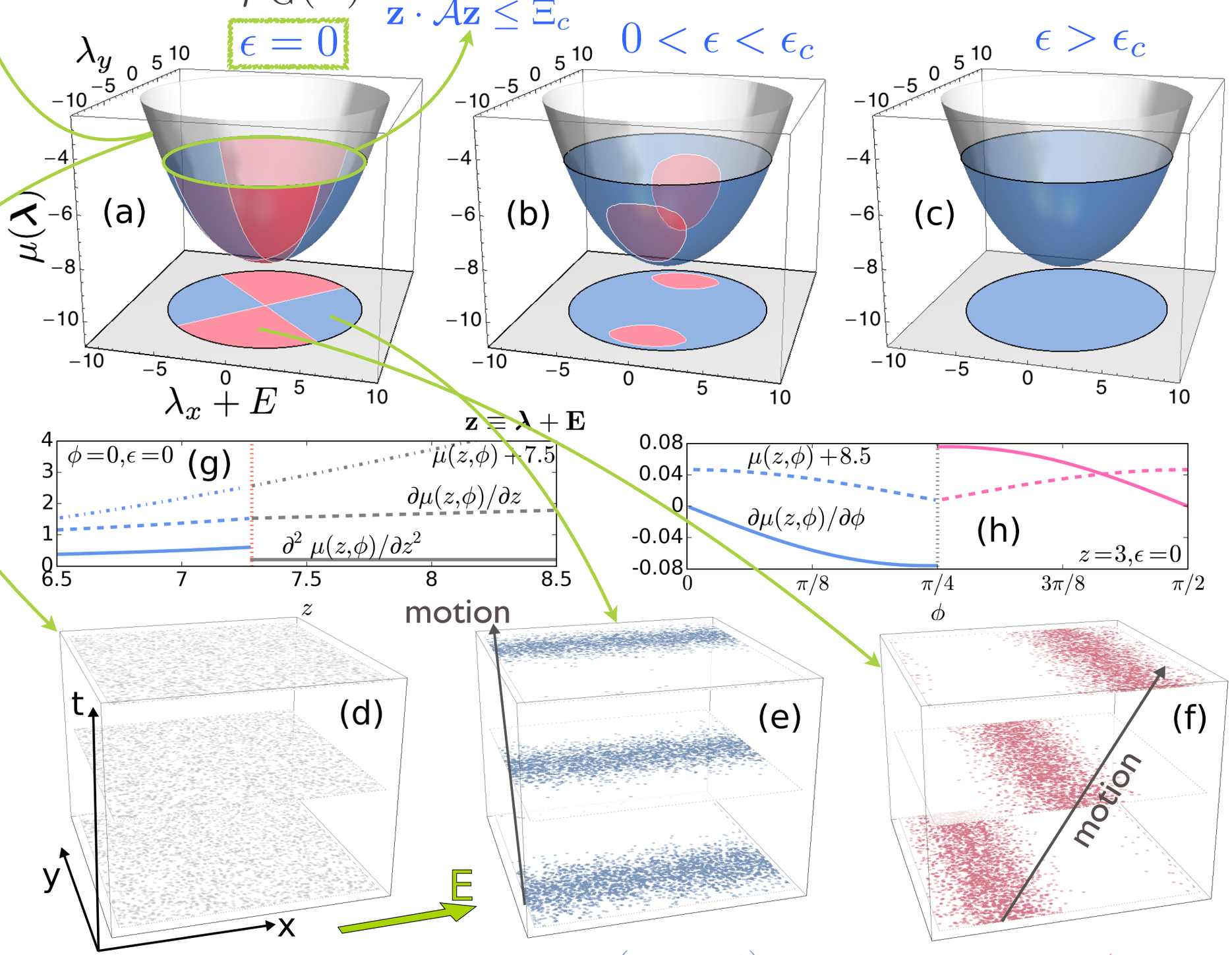
$\mathbf{z}\cdot\hat{\mathcal{A}}\mathbf{z} \le \Xi_c$



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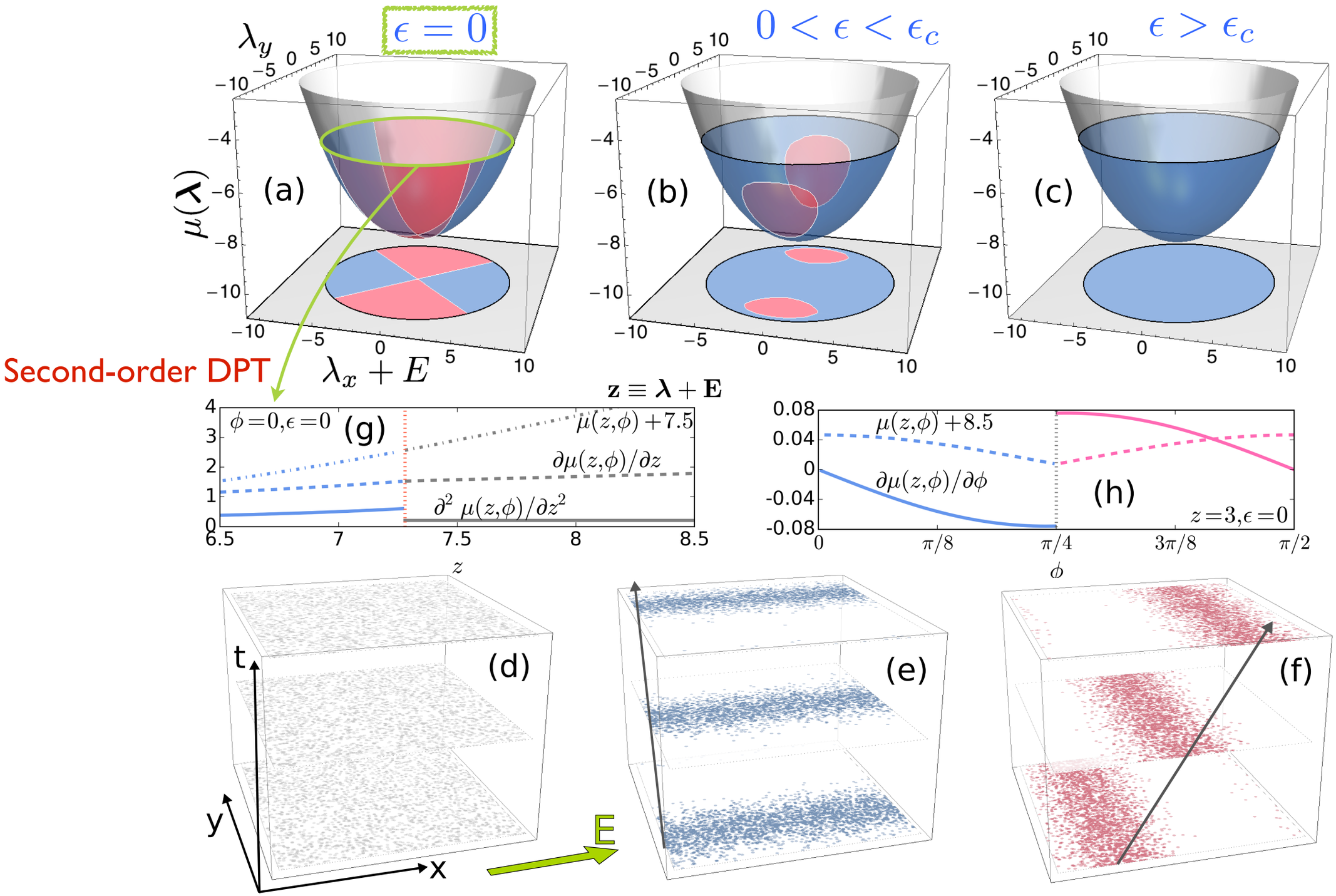

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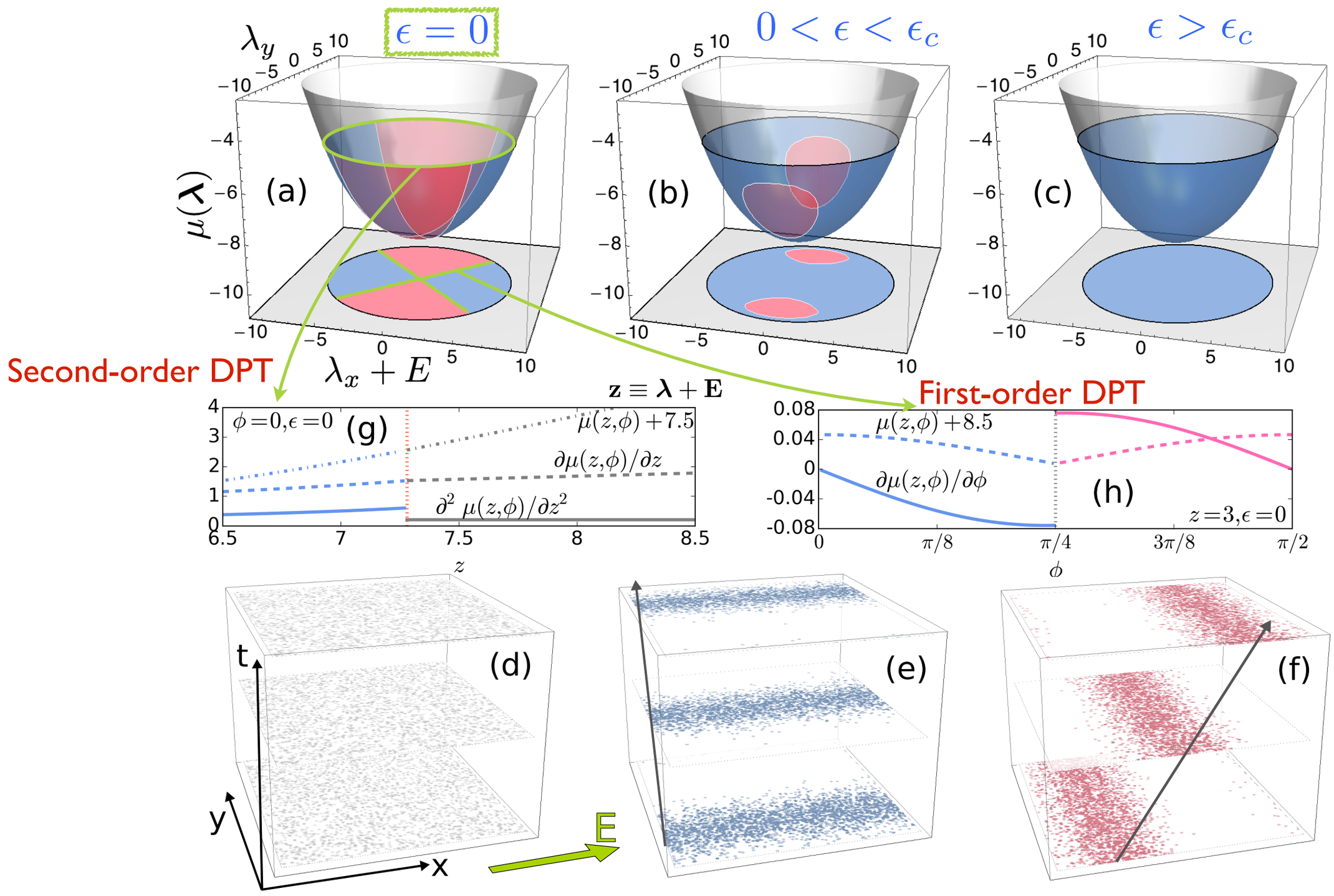

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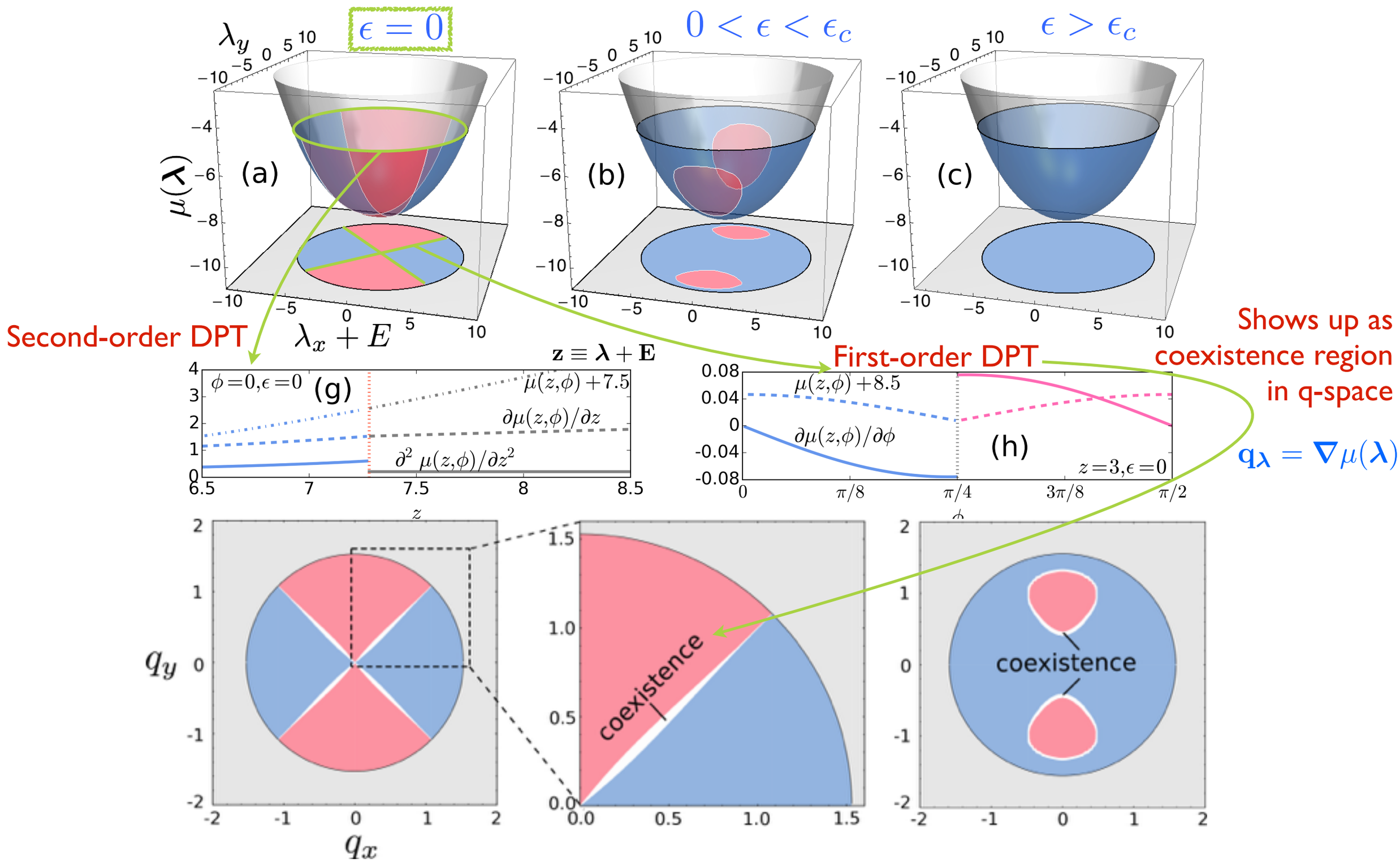

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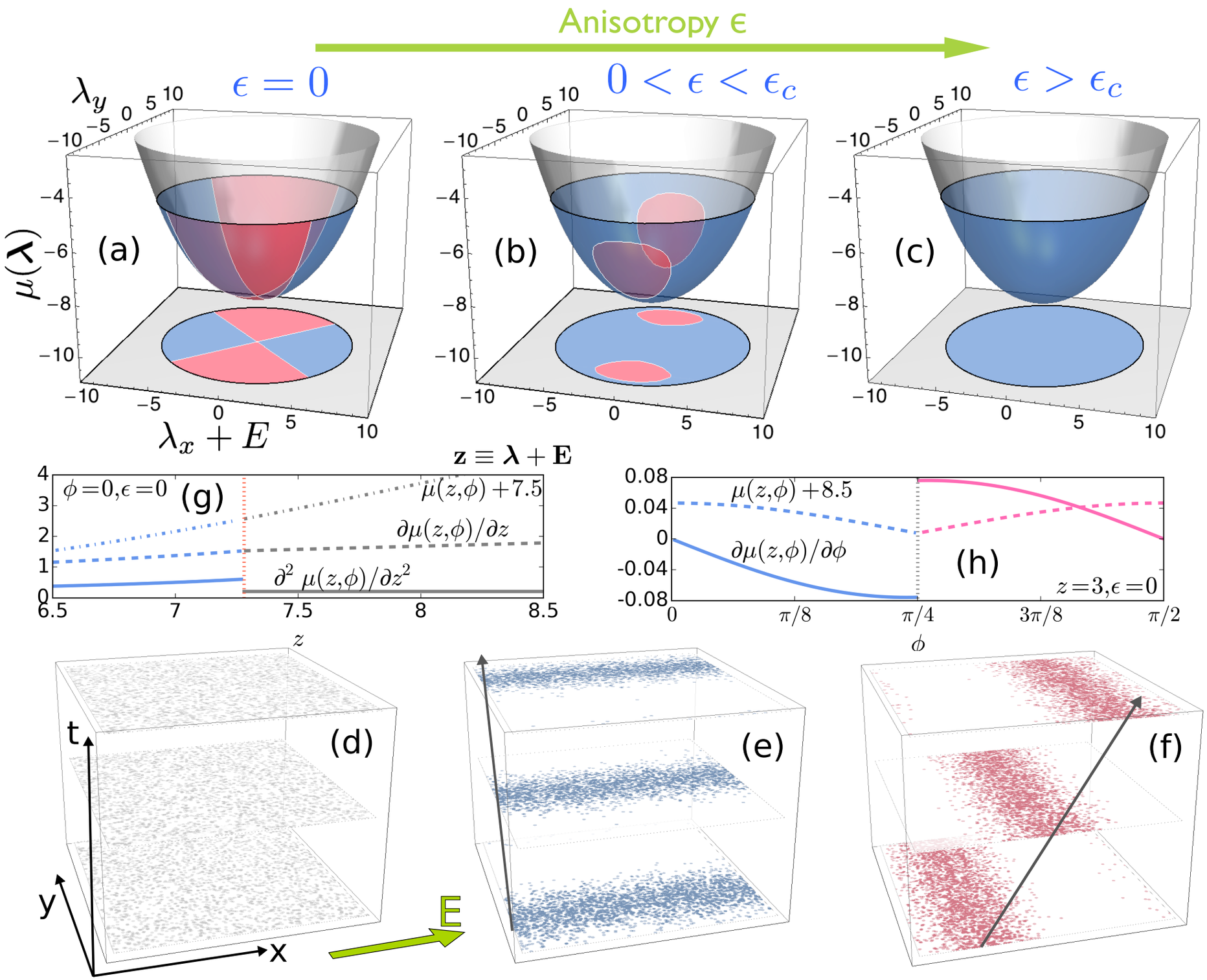

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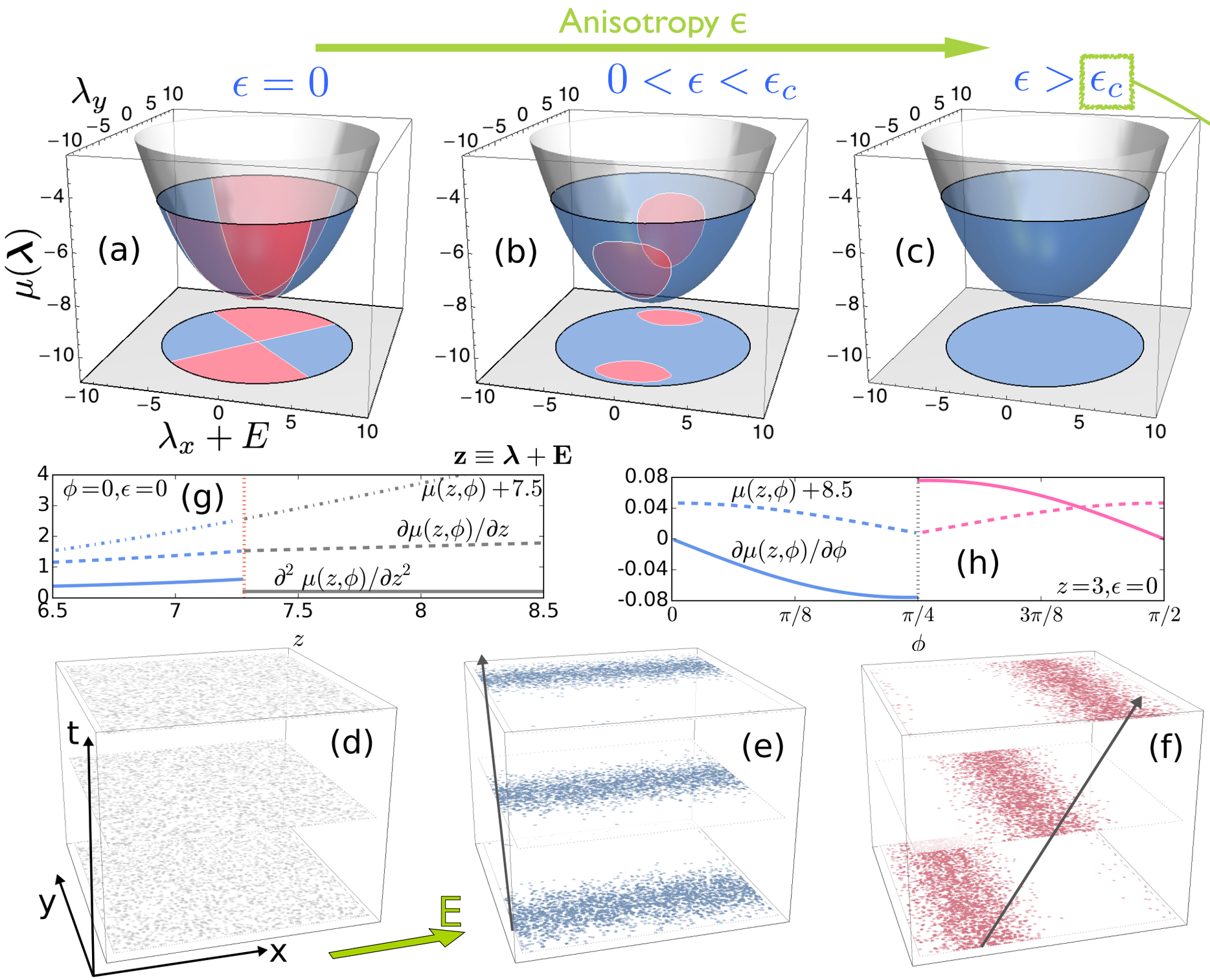
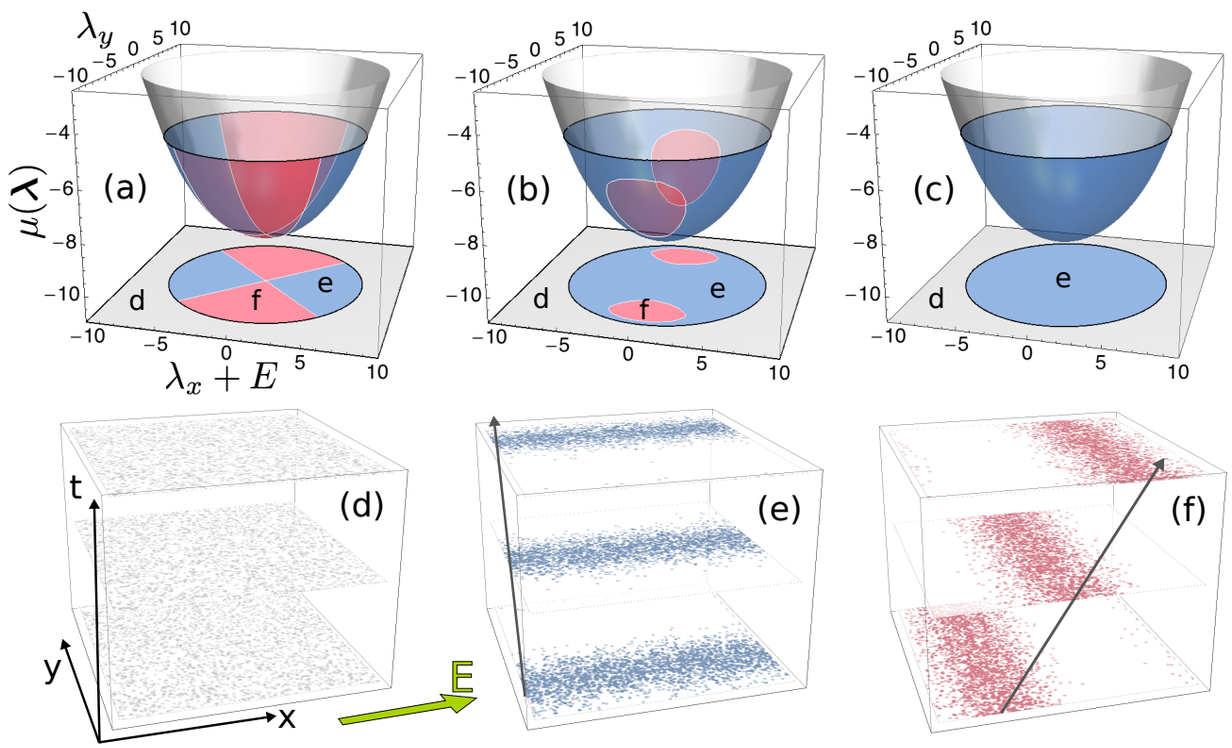


There exists a critical anisotropy $\epsilon_c \approx 0.035$ beyond which only one symmetry-broken phase appears

KEY INGREDIENTS BEHIND NEW PHYSICS ?

- First, by considering **vectorial currents**, it becomes apparent that **current rotations can trigger first-order transitions** between different dynamical phases.

- Second, by including **anisotropy** it becomes clear its **strong effect on the relative shape and position of the different jammed phases**



- Mathematically, the competition between different traveling-waves stems from a **structured vector field coupled to the current**

$$\mathbf{j}_{\mathbf{J}}(\mathbf{r},t) = \mathbf{J} - \mathbf{v}\left[\rho_0 - \omega_{\mathbf{J}}(\mathbf{r}-\mathbf{v}t)\right] + \boxed{\boldsymbol{\phi}_{\mathbf{J}}(\mathbf{r}-\mathbf{v}t)}$$

$$\int_{\Lambda} \boldsymbol{\phi}_{\mathbf{J}}(\mathbf{r})\, d\mathbf{r} = 0$$

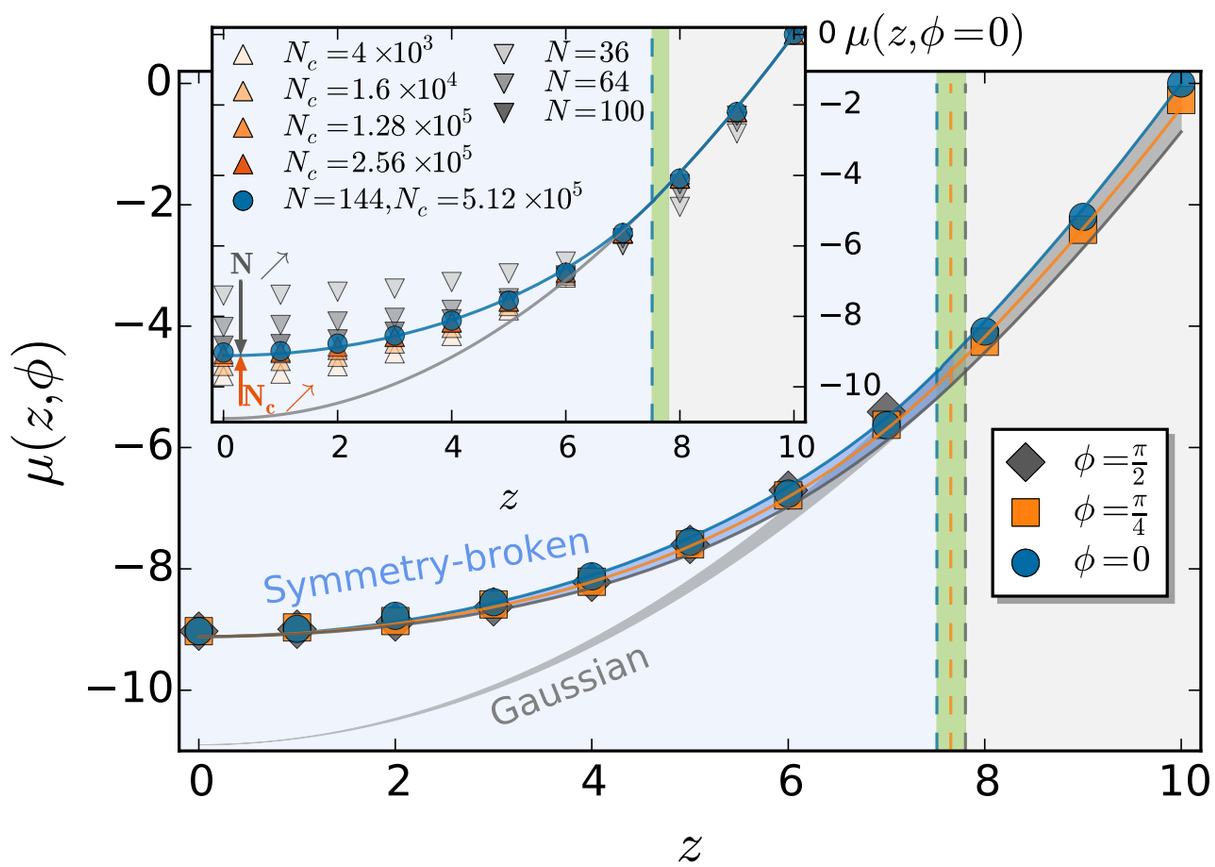
$$\boldsymbol{\nabla}\cdot\boldsymbol{\phi}_{\mathbf{J}}(\mathbf{r}) = 0$$

Pérez-Espigares, Garrido & PH, PRE **93**, 040103(R) (2016)
Villavicencio & Harris, PRE **93**, 032134 (2016)
Tizón, PH & Garrido, PRE **95**, 032119 (2017)

CURRENT STATISTICS FOR 2D WASEP

$\mathbf{z} \equiv \boldsymbol{\lambda} + \mathbf{E}$
$z = |\mathbf{z}|$
$\phi = \tan^{-1}(z_y/z_x)$

- **Simulations: Cloning Monte Carlo** method for rare events. N_c=5.12x10^5 !!
- Parameters: $\rho_0 = 0.3$ $\quad N \leq 144$ $\quad \mathbf{E} \cdot \hat{\mathcal{A}}\mathbf{E} \geq |\Sigma_c|$



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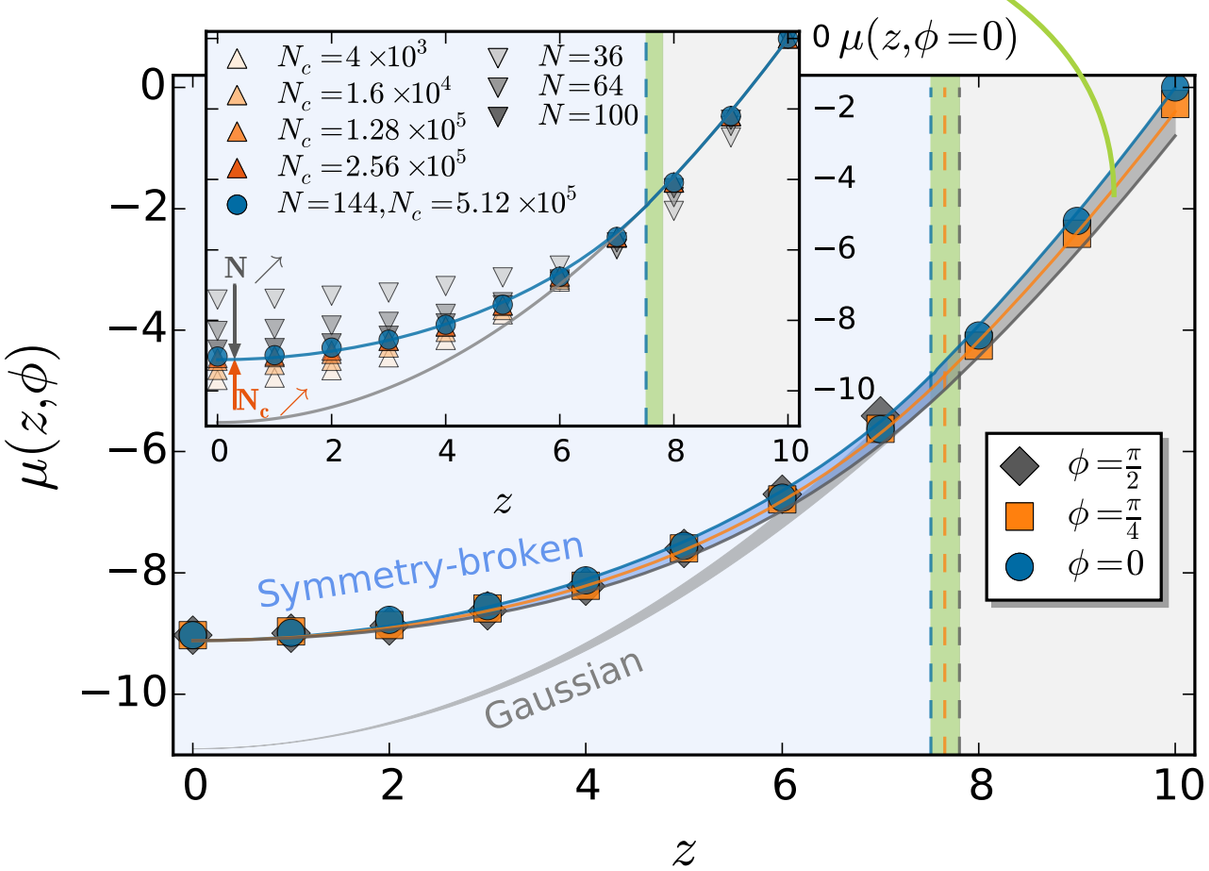
Gaussian current statistics $\mu_{\mathrm{G}}(\boldsymbol{\lambda})$
for mild current fluctuations



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$\mathbf{z} \equiv \boldsymbol{\lambda} + \mathbf{E}$
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$\phi = \tan^{-1}(z_y/z_x)$

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Gaussian current statistics $\mu_{\mathrm{G}}(\boldsymbol{\lambda})$
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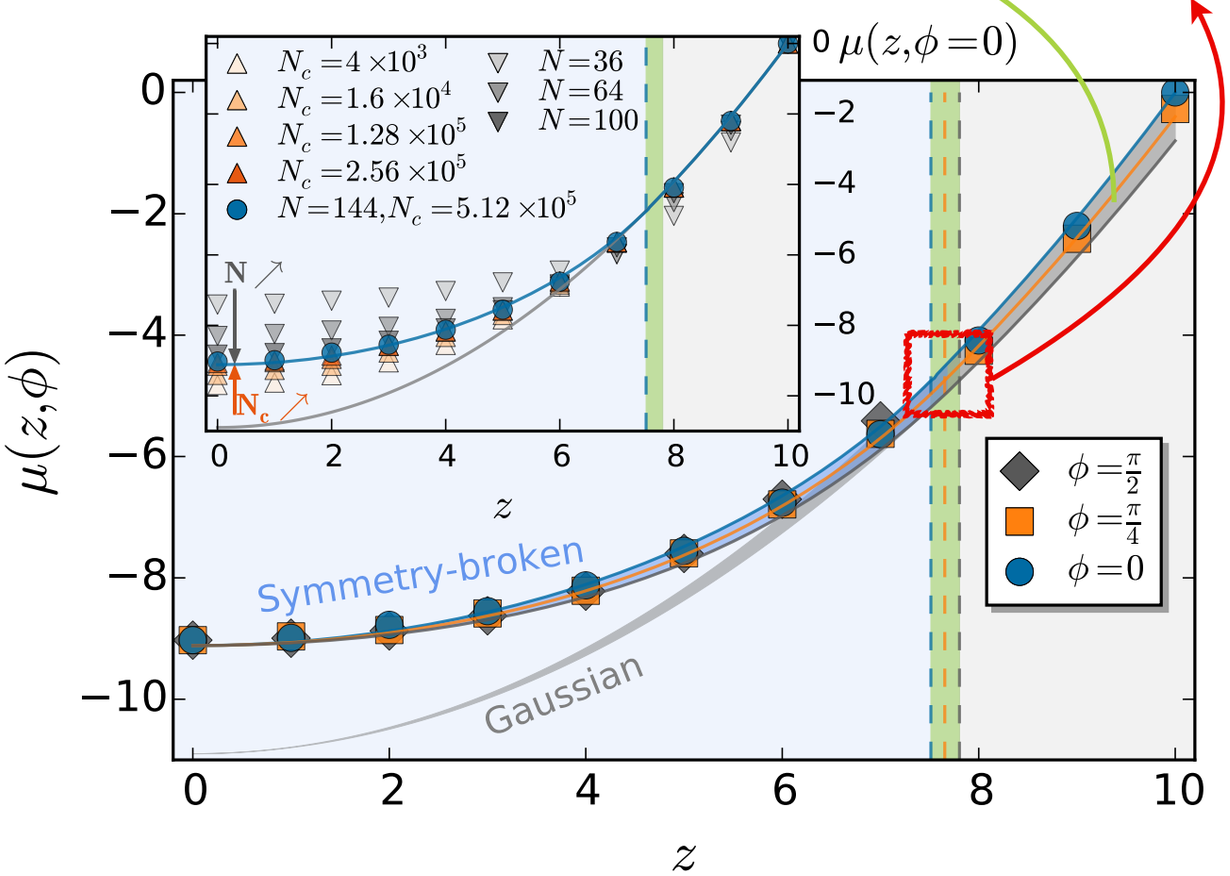

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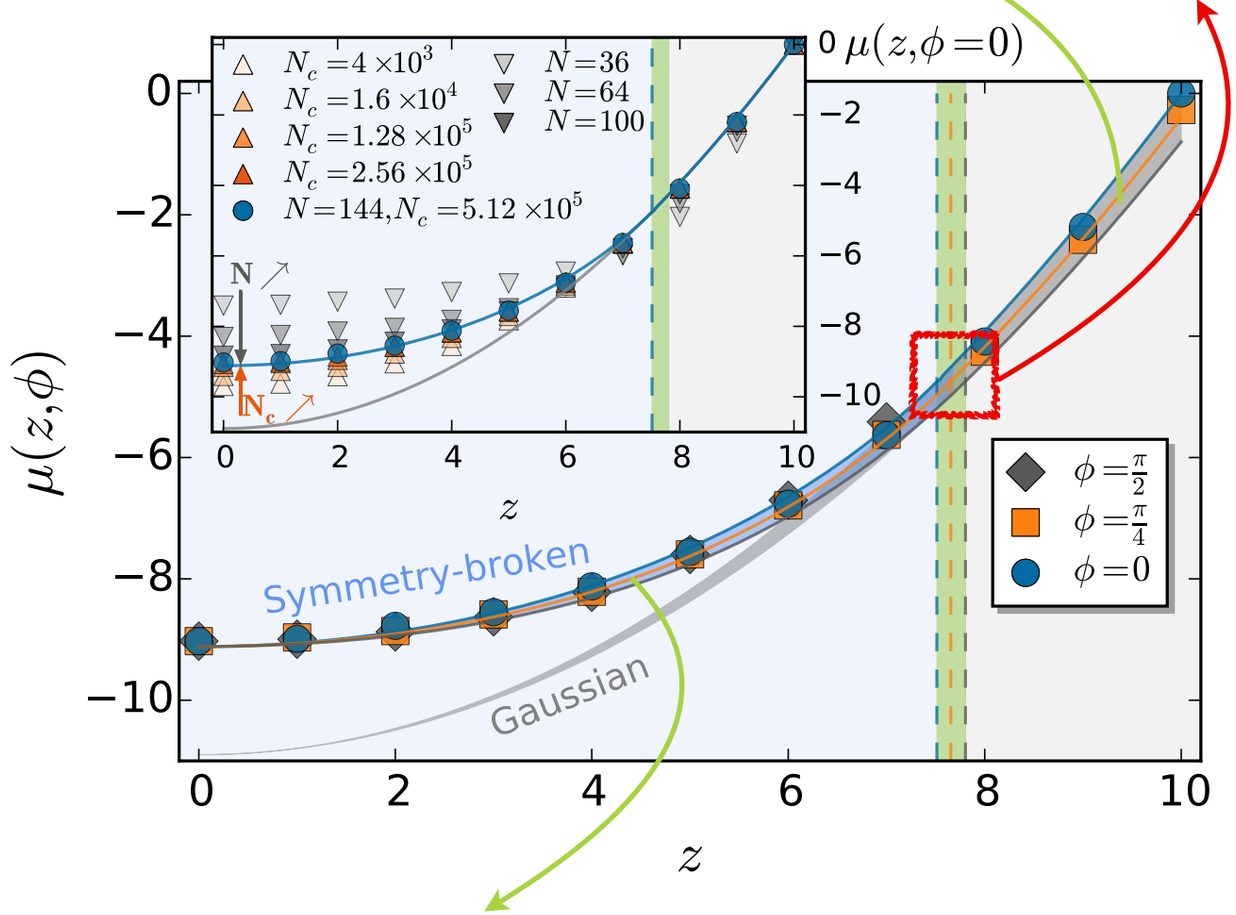

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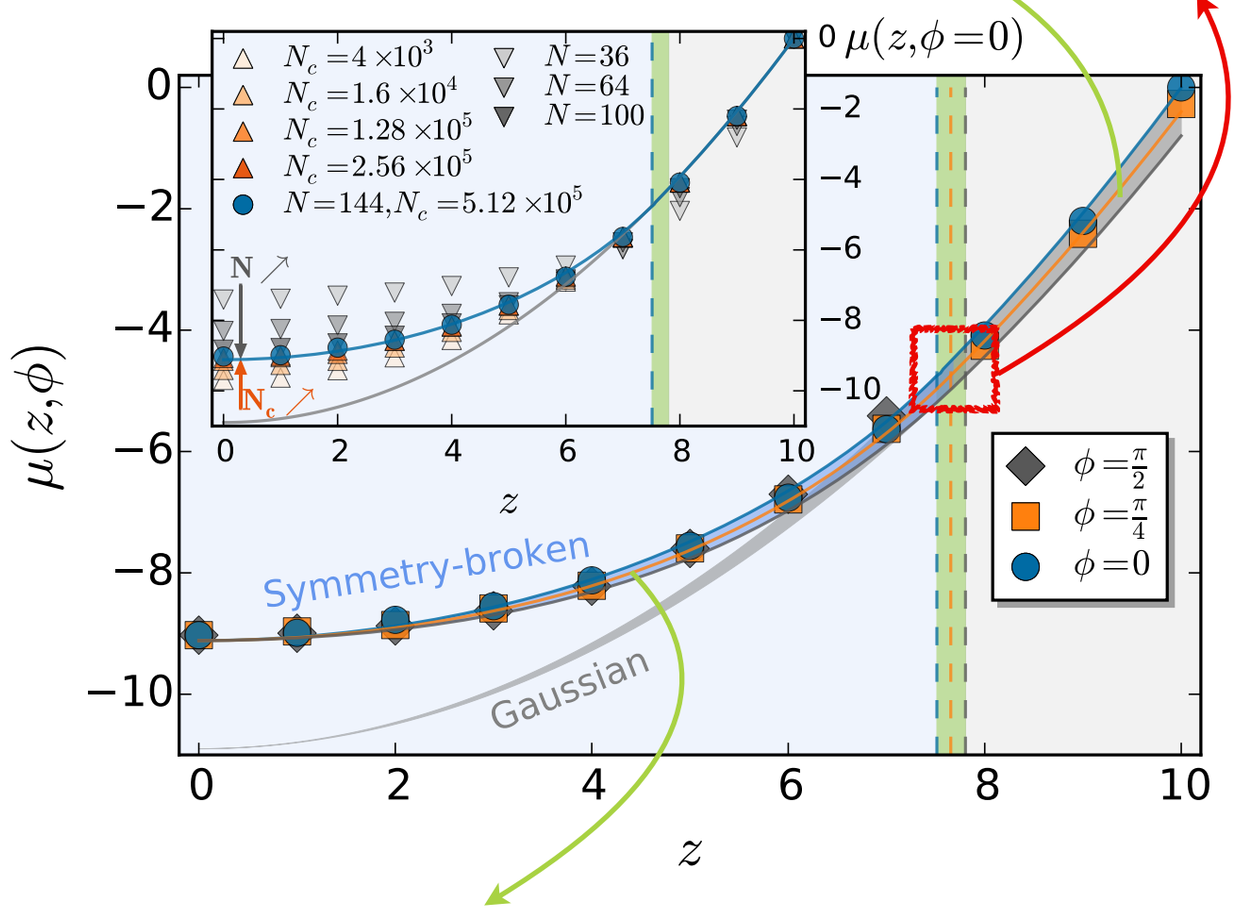

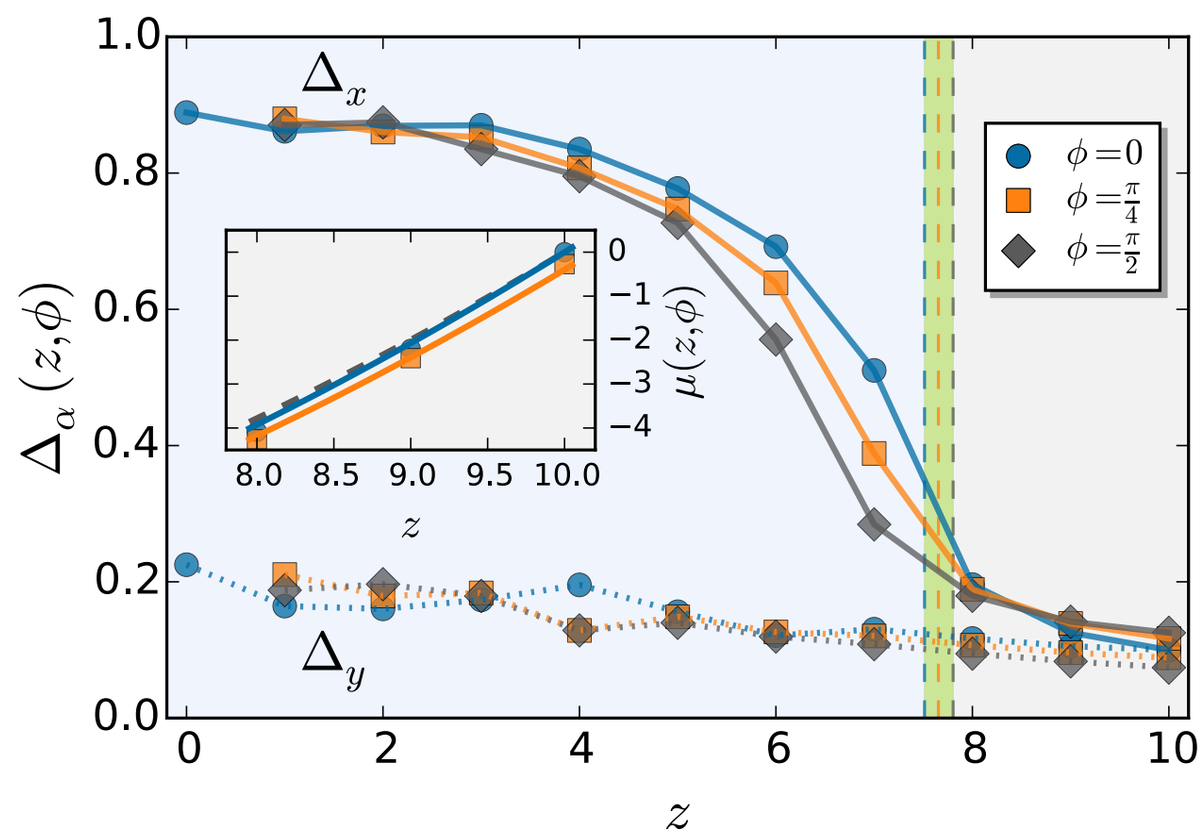

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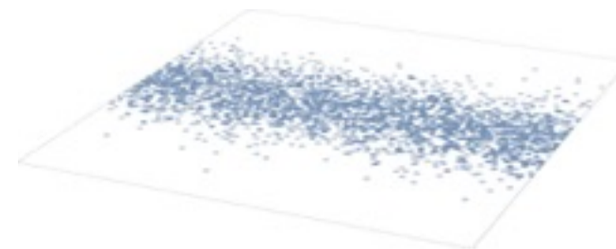

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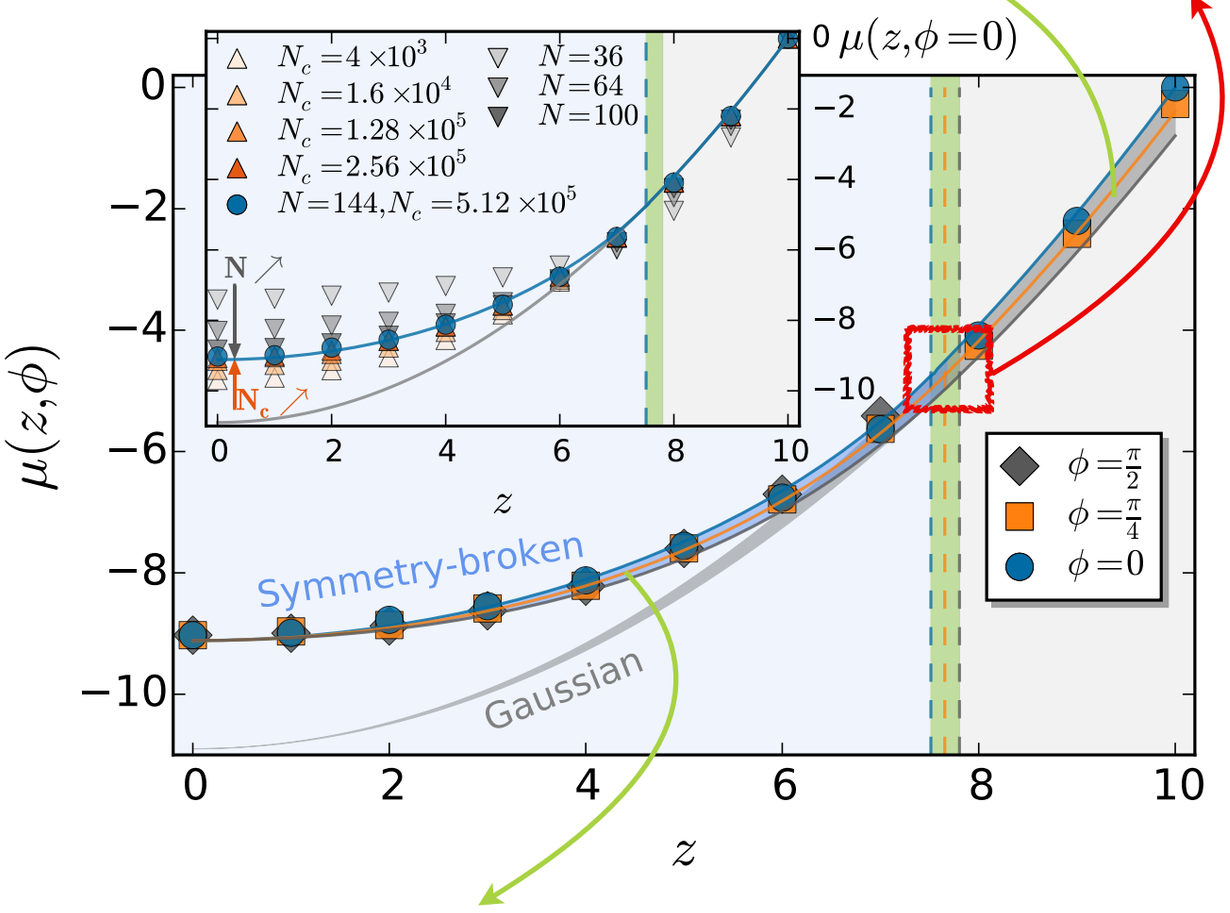
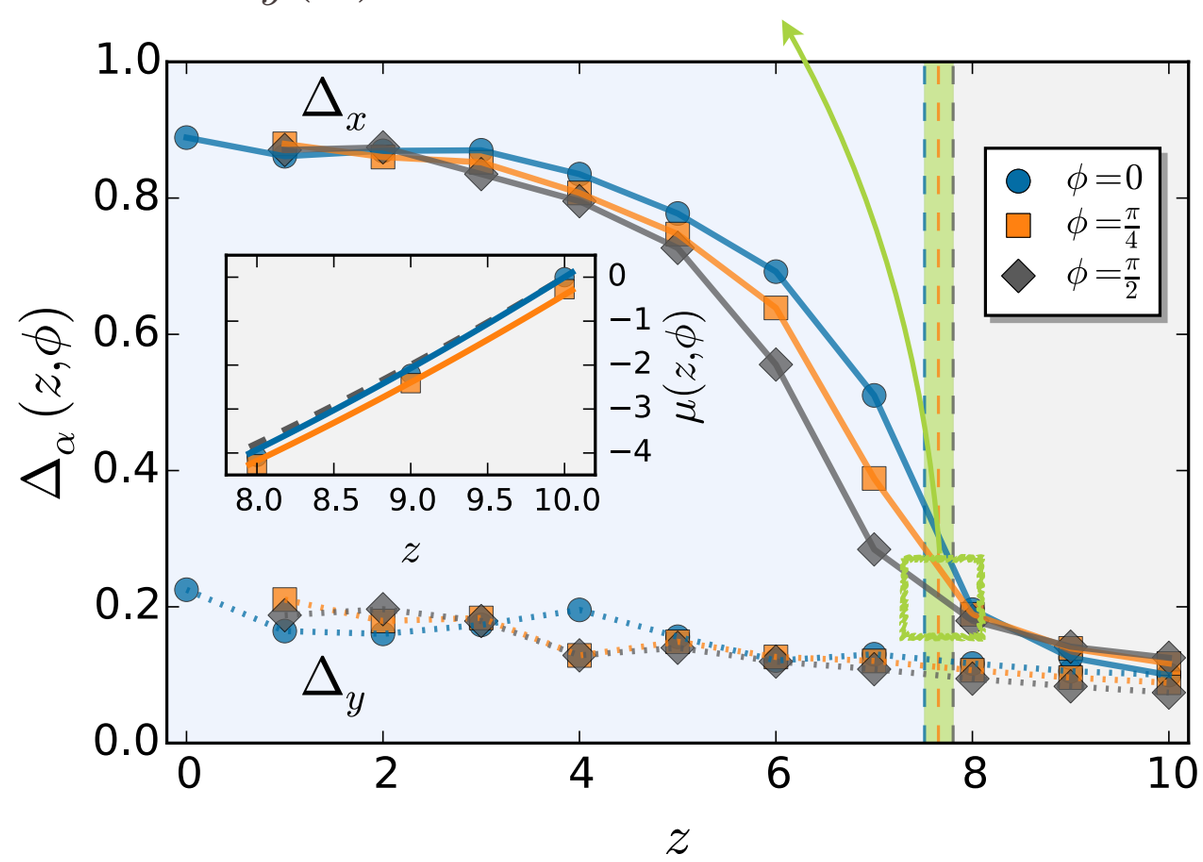
$\Delta_x(\boldsymbol{\lambda})$ increases steeply across DPT $\forall\phi$
$\Delta_y(\boldsymbol{\lambda})$ remains small and unaffected





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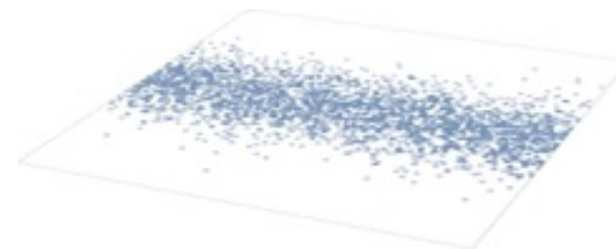

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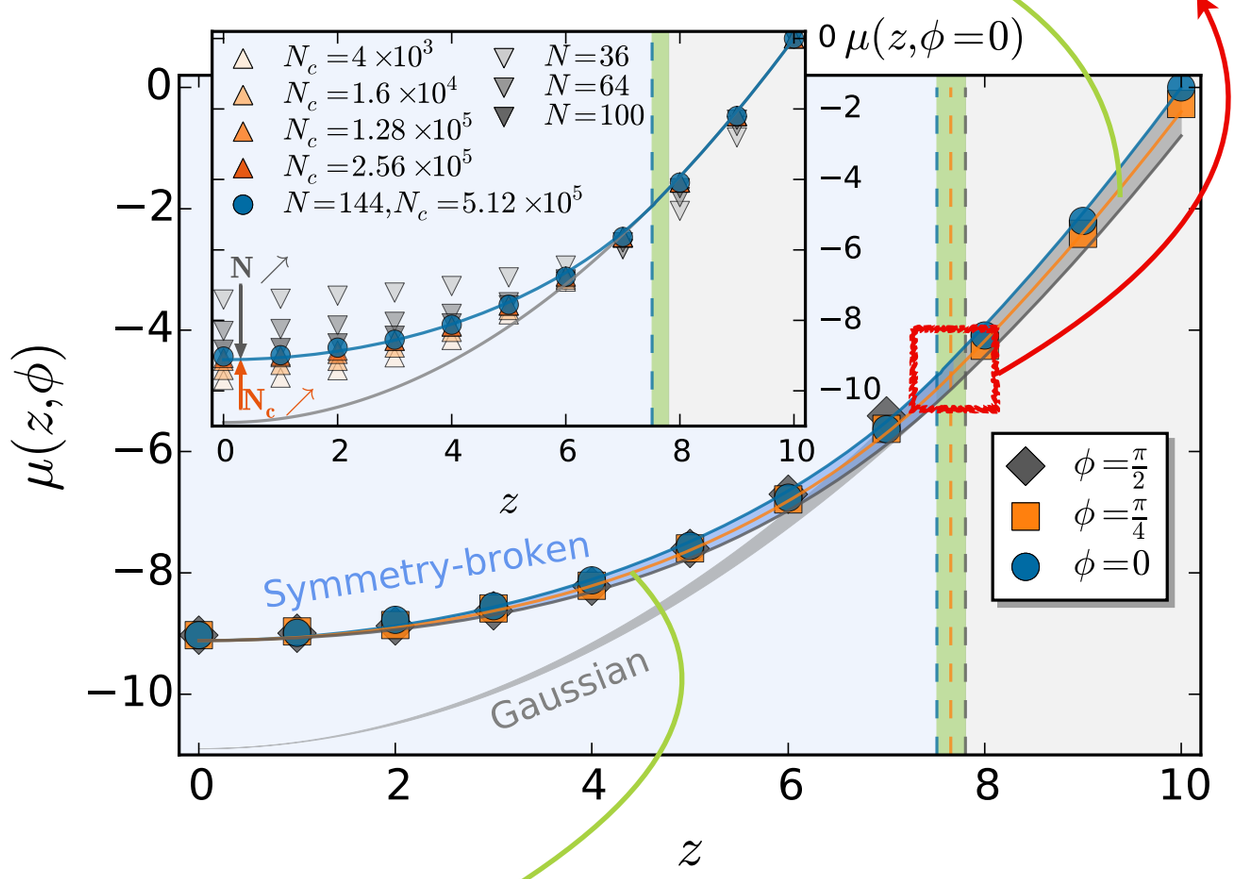

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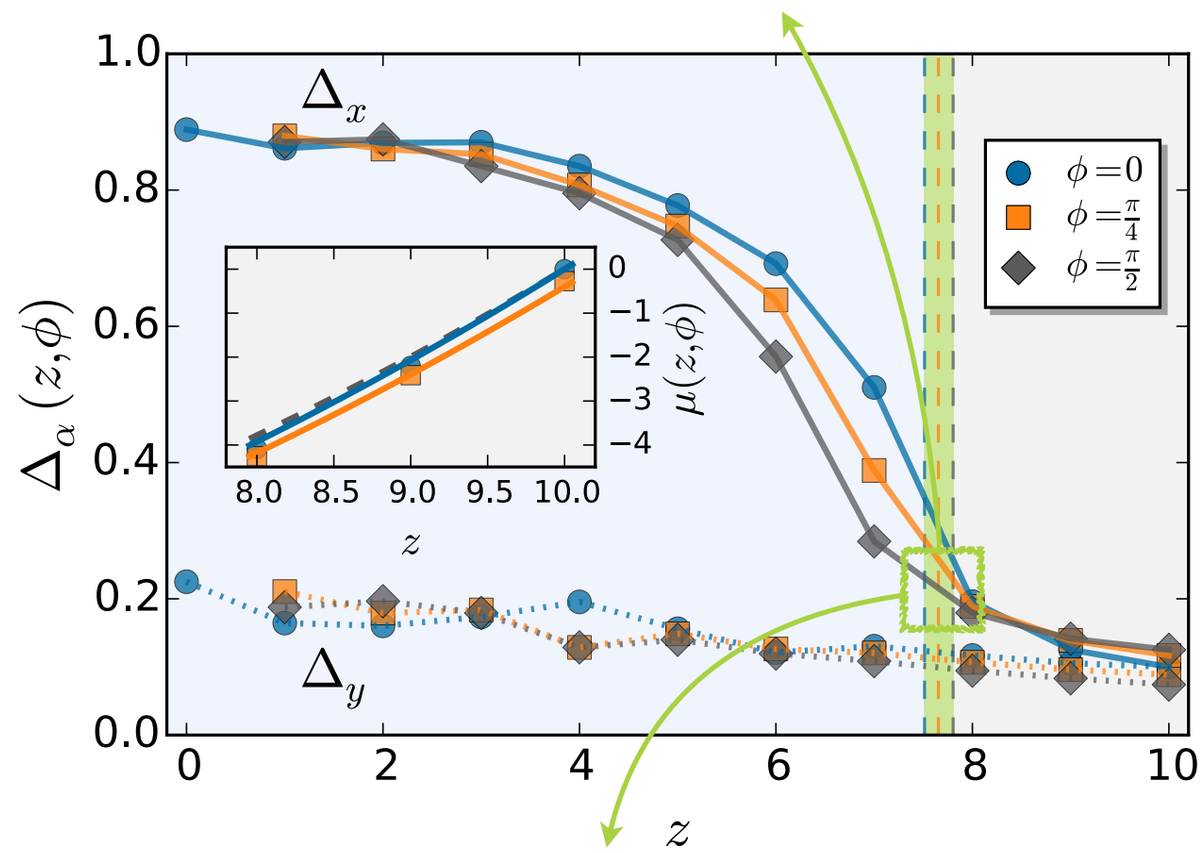


Non-Gaussian current fluctuations

Sharp but continuous change in $\Delta_x(\boldsymbol{\lambda})$
consistent with 2nd-order DPT

WORK IN PROGRESS
DPT'S IN OPEN SYSTEMS (1D)

DPT'S IN OPEN SYSTEMS (1D)

- Up to now, DPT's in current statistics associated to **emergence of time-dependent optimal profiles** (e.g. traveling waves). Only possible in **periodic systems**

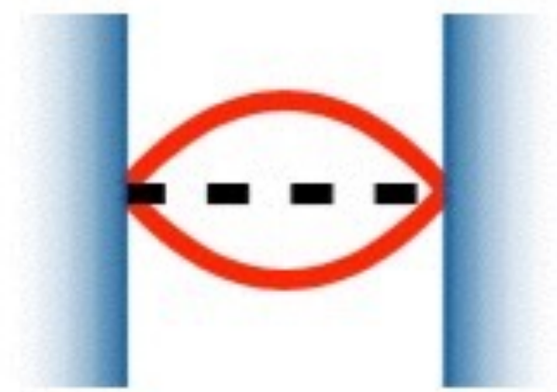
- Baek, Kafri & Lecomte: **DPT's for current fluctuations in open systems**

  Baek, Kafri & Lecomte, PRL **118**, 030604 (2017); arXiv:1710.07139

- **Need an underlying symmetry**, e.g. particle-hole symmetry around, say, **ρ=1/2**

$$D(\frac{1}{2} - \delta\rho) = D(\frac{1}{2} + \delta\rho) \qquad \sigma(\frac{1}{2} - \delta\rho) = \sigma(\frac{1}{2} + \delta\rho)$$

- Beautiful **Landau theory within MFT** leads to predictions of **1st and 2nd order DPT's in open 1d systems** and symmetry-breaking phenomena



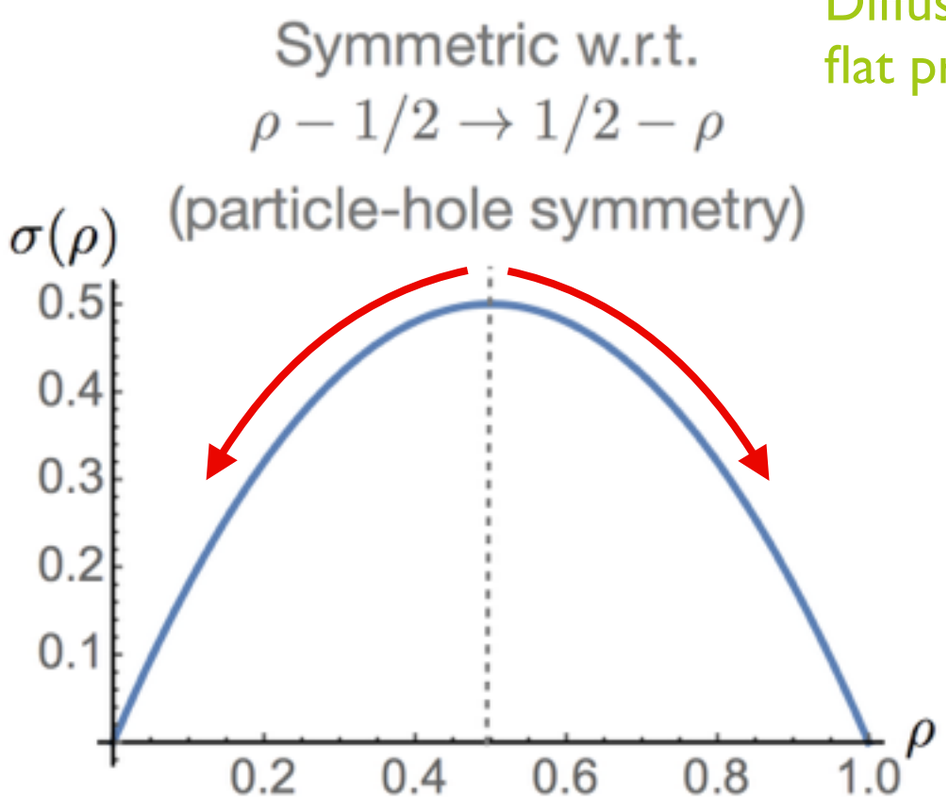
DPT'S IN OPEN SYSTEMS (1D)

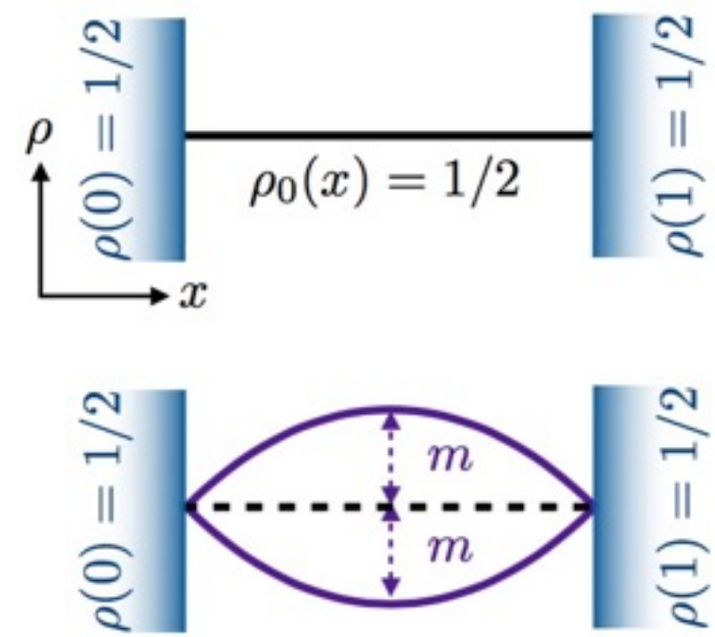
- Physical picture for 1d WASEP (from Y. Kafri talk in Bangalore)

$$\partial_t \rho = -\partial_x [\underline{-D(\rho)\partial_x \rho} + \underline{\sigma(\rho)E + \sqrt{\sigma(\rho)}\eta}]$$

Diffusion favors flat profile

Small J is easier if σ(ρ) small

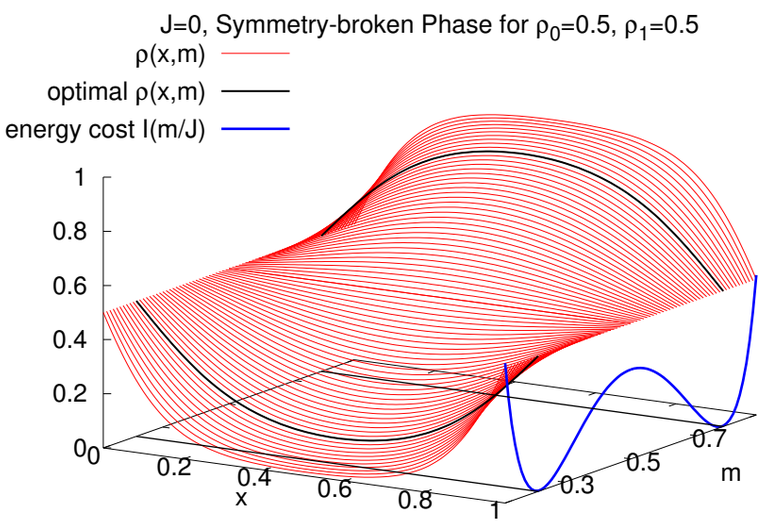




- Limitations:
  - perturbative predictions (*a la* Landau)
  - only in equilibrium (no gradient -though perturbatively extended to linear regime)
  - not yet observed

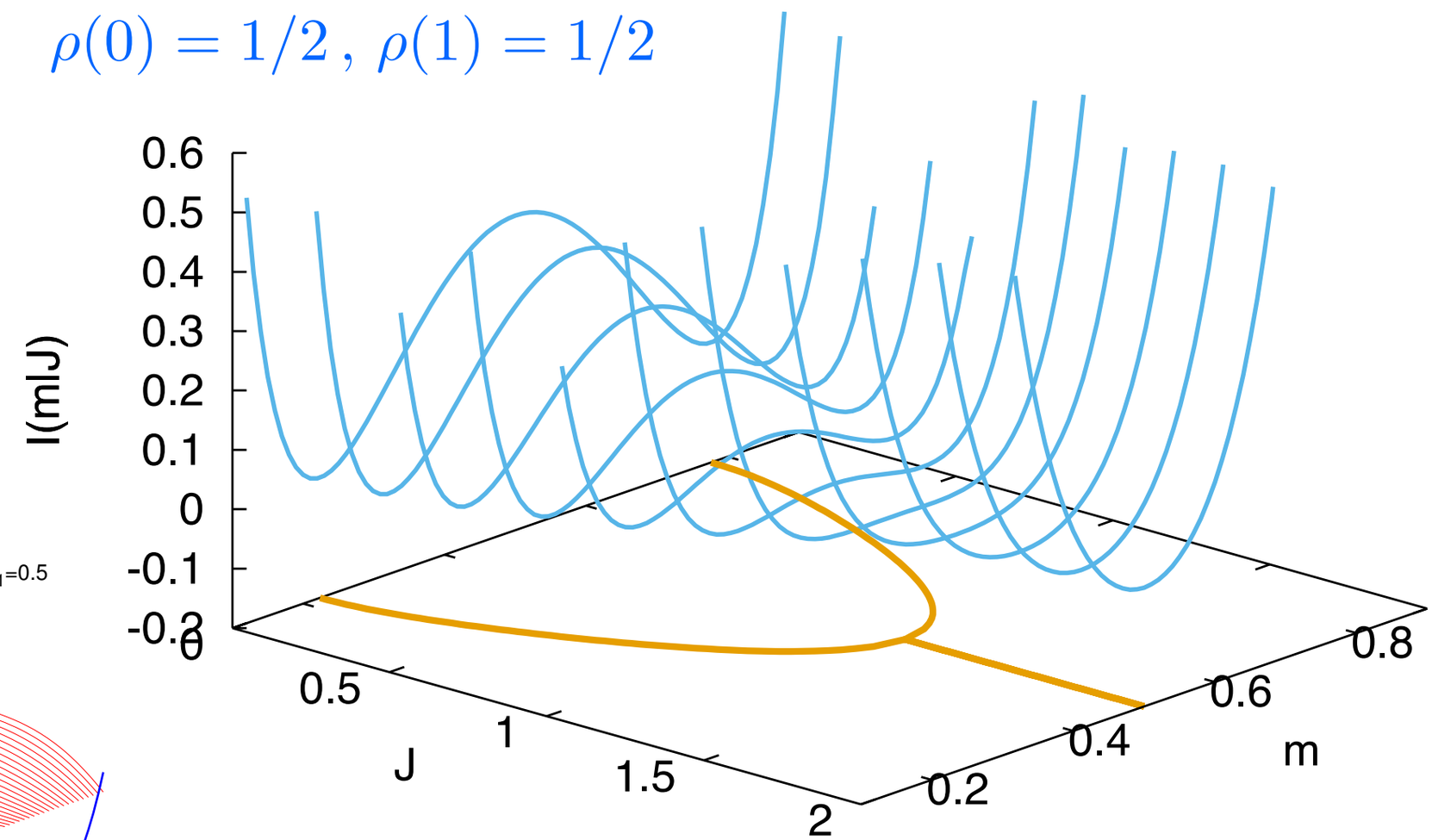
ONGOING WORK: DPT'S IN OPEN SYSTEMS (1D)

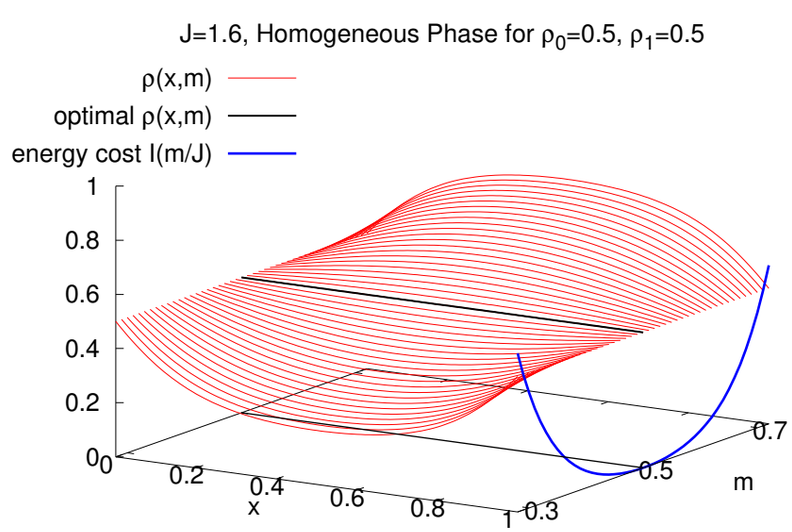
- **Joint fluctuations of the current and order parameter** (total mass) in **1d open WASEP**
- … or fluctuations of total mass in trajectory ensembles with constrained current
- **Exact MFT result (non-perturbative) + arbitrary gradients + numerical observation**



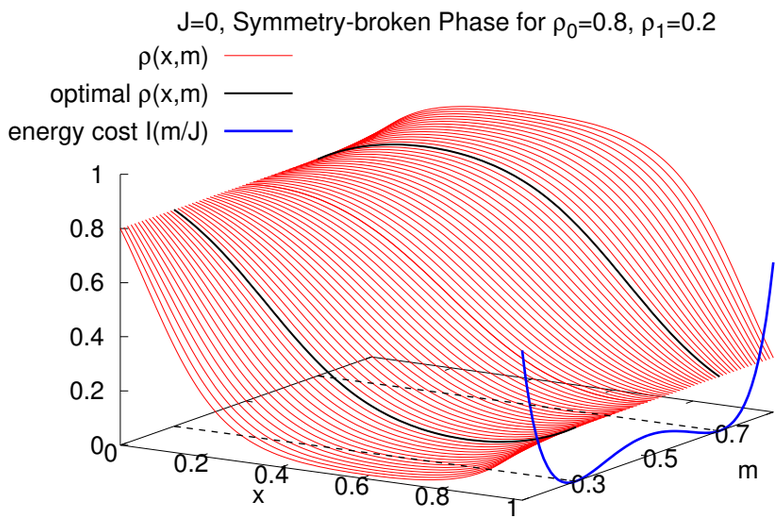
Equilibrium

$$\rho(0) = 1/2\,,\ \rho(1) = 1/2$$



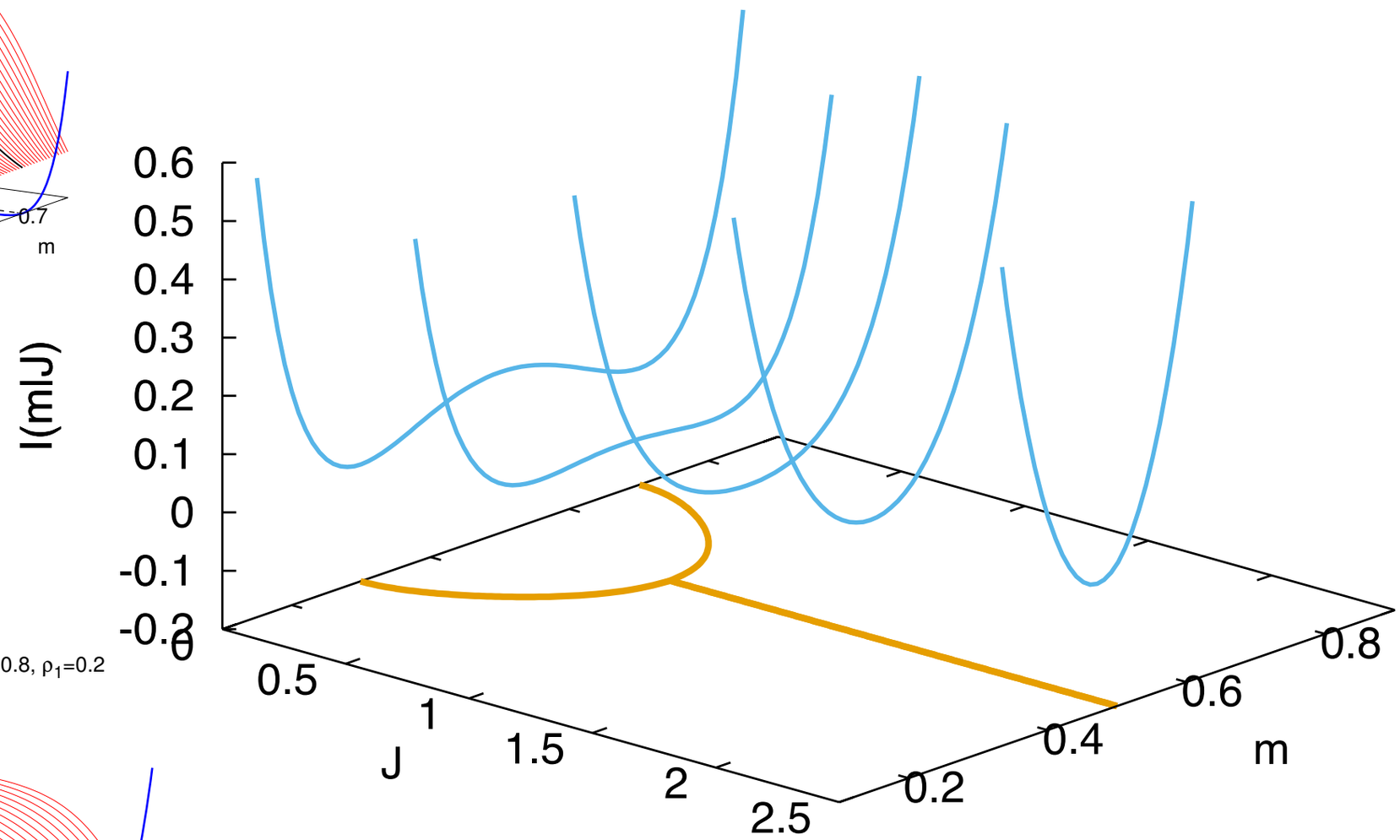


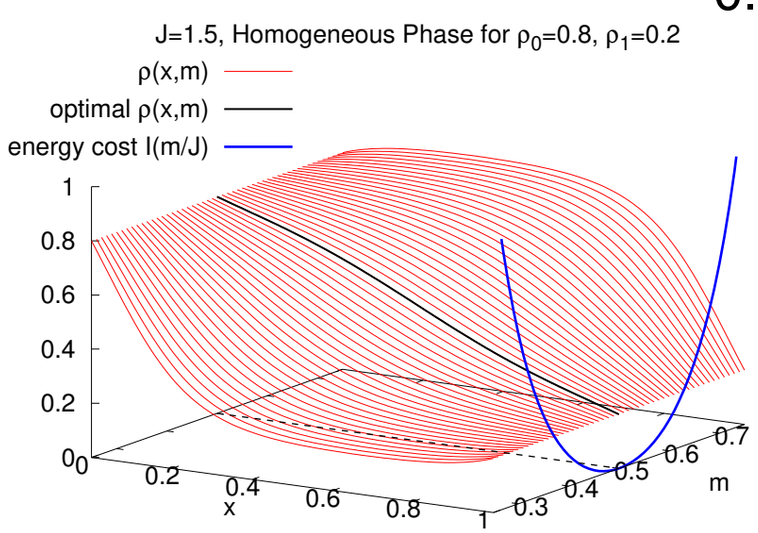
ONGOING WORK: DPT'S IN OPEN SYSTEMS (1D)



Symmetric gradient

$$\rho(0) = 0.8\,,\ \rho(1) = 0.2$$





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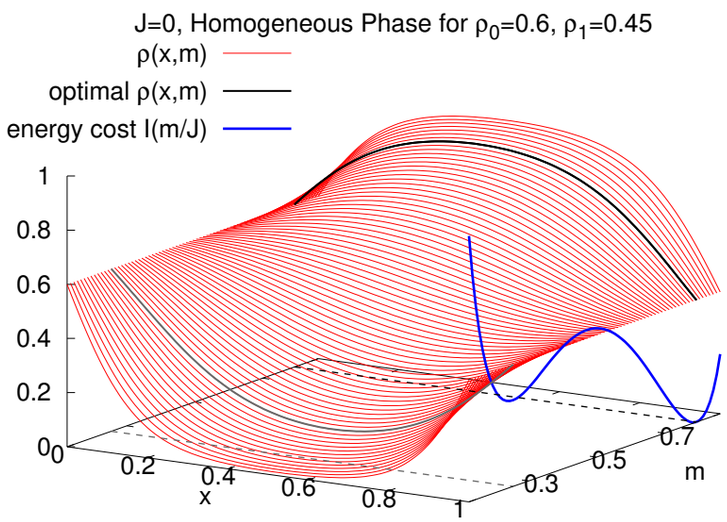
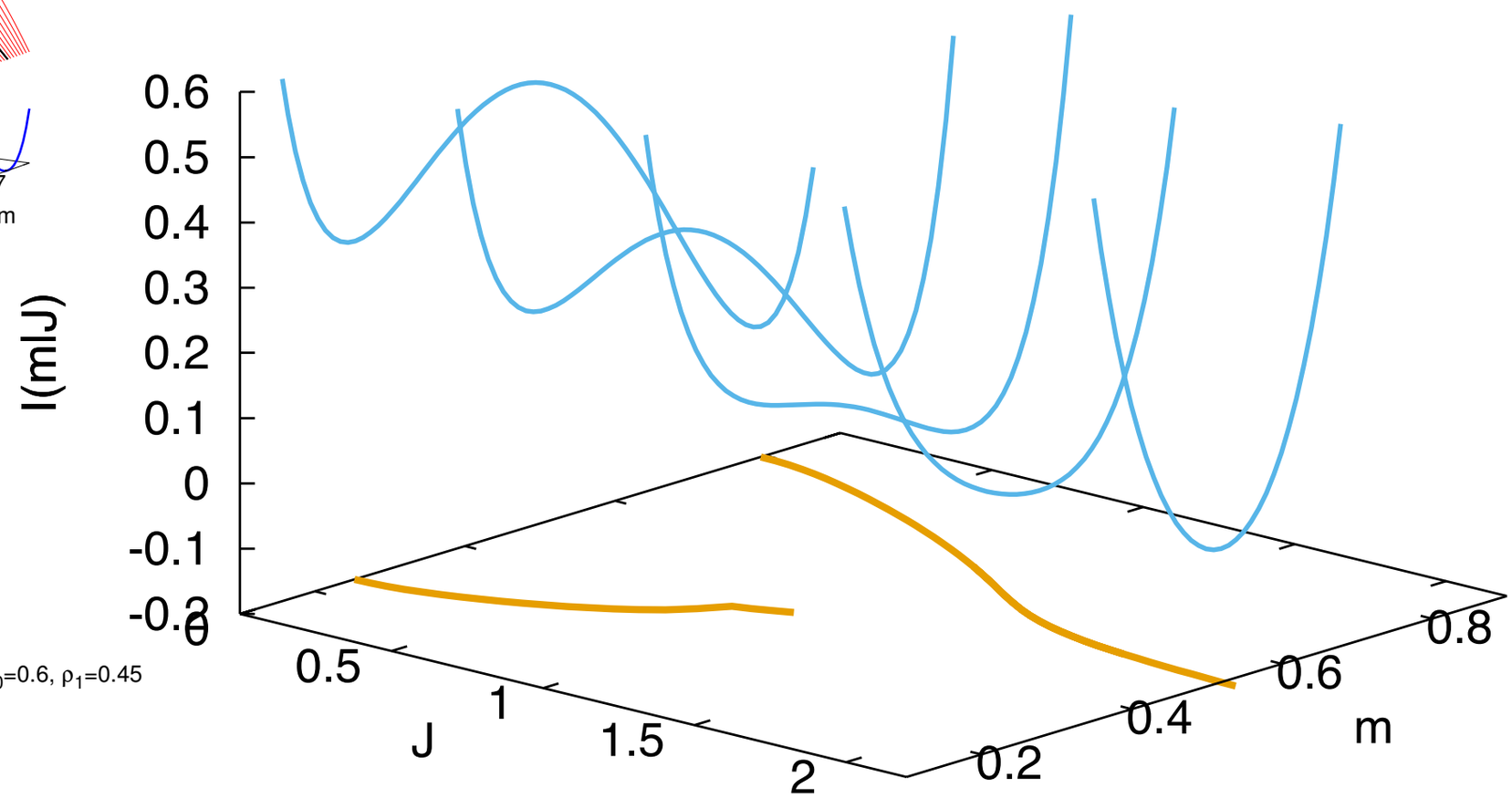
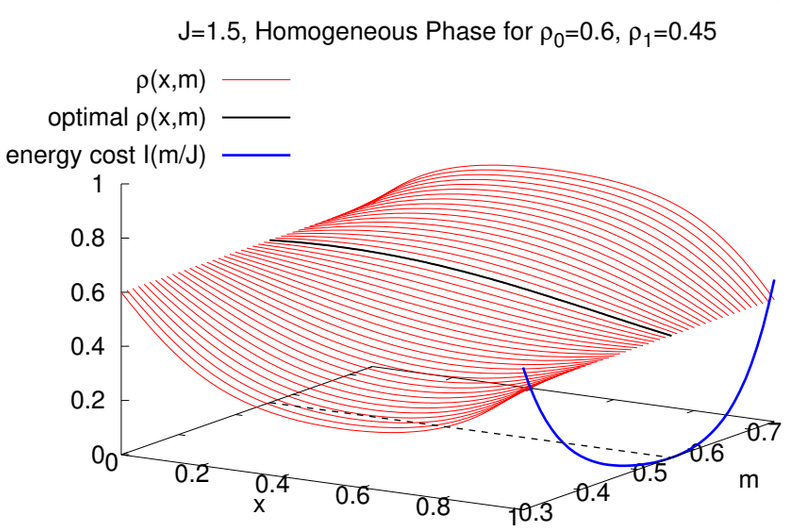


Asymmetric gradient

$$\rho(0) = 0.6\,,\ \rho(1) = 0.45$$





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- The **complex interplay among the external field, anisotropy and currents in 2d** leads to a **rich phase diagram**
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- Key role of divergence-free but **structured current fields:** **weak additivity principle**

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- Order in the form of **coherent jammed states** emerges to hinder transport for low-current fluctuations
- Also **DPT's in the current statistics of open 1d systems** … **work in progress**

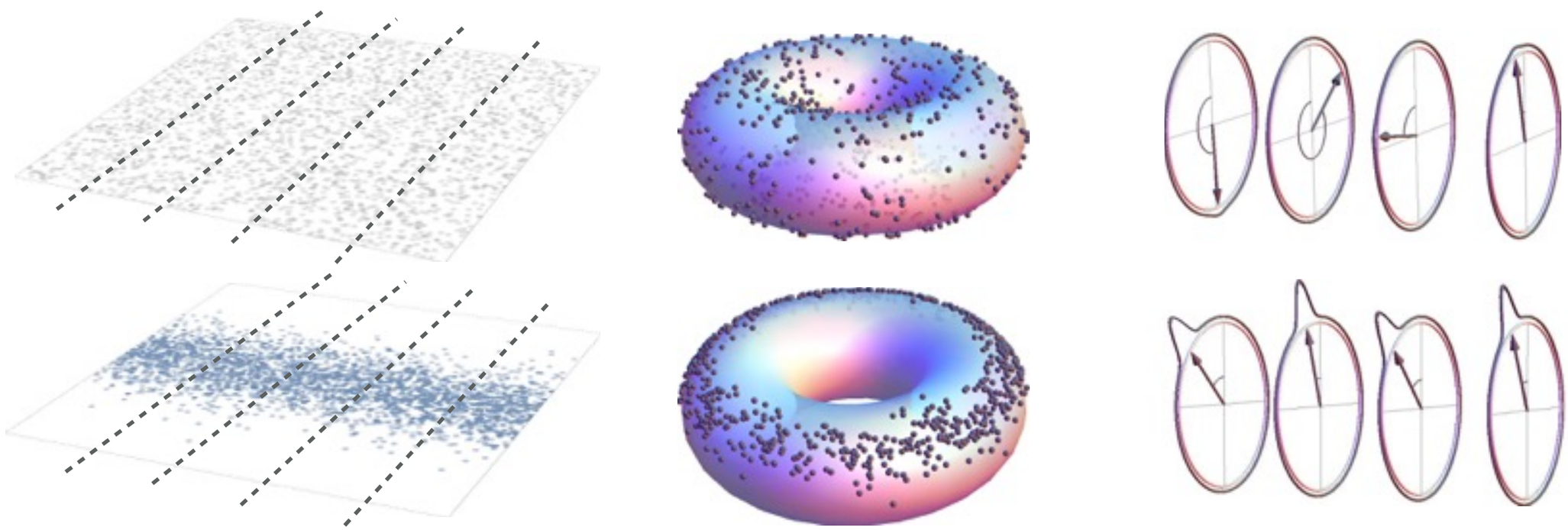
Thank you

Backup slides

ORDER PARAMETER: TOMOGRAPHIC COHERENCES

- MFT: **1d density waves** in symmetry-broken phase ⇒ jam particle flow

- **Tomographic analysis**: slice system along **α**-direction ⇒ **j-slice is a ring in 2d (due to pbc)** ⇒ compute **angular position of center of mass, θ^j_{cm}**



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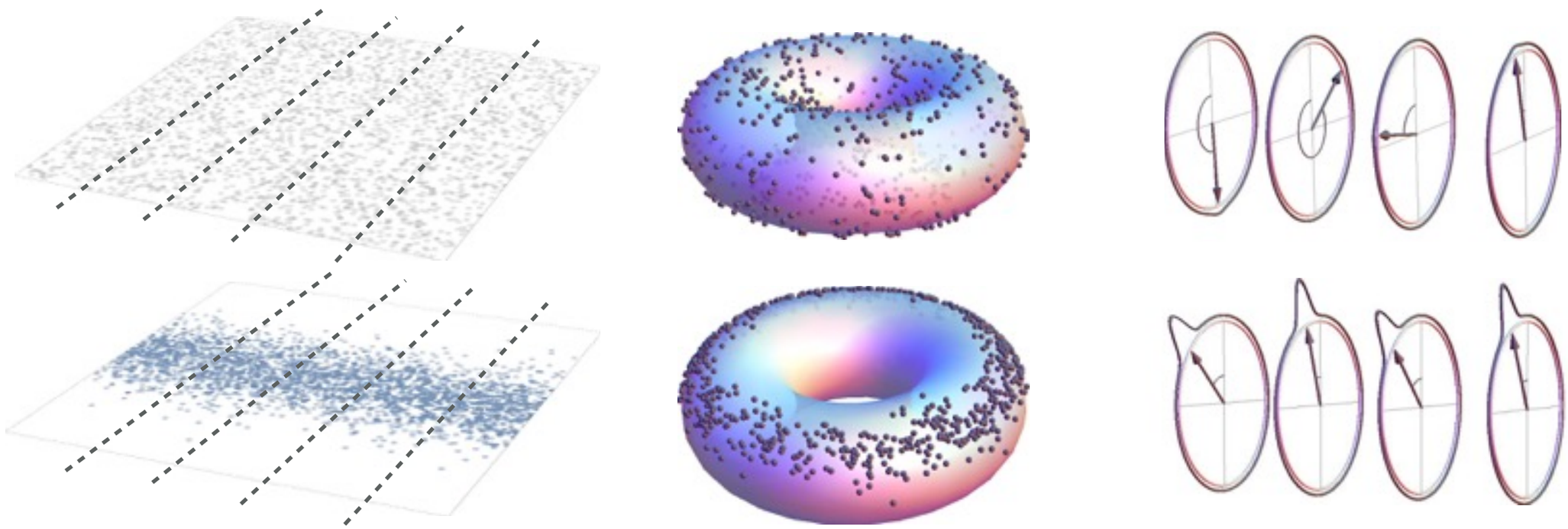

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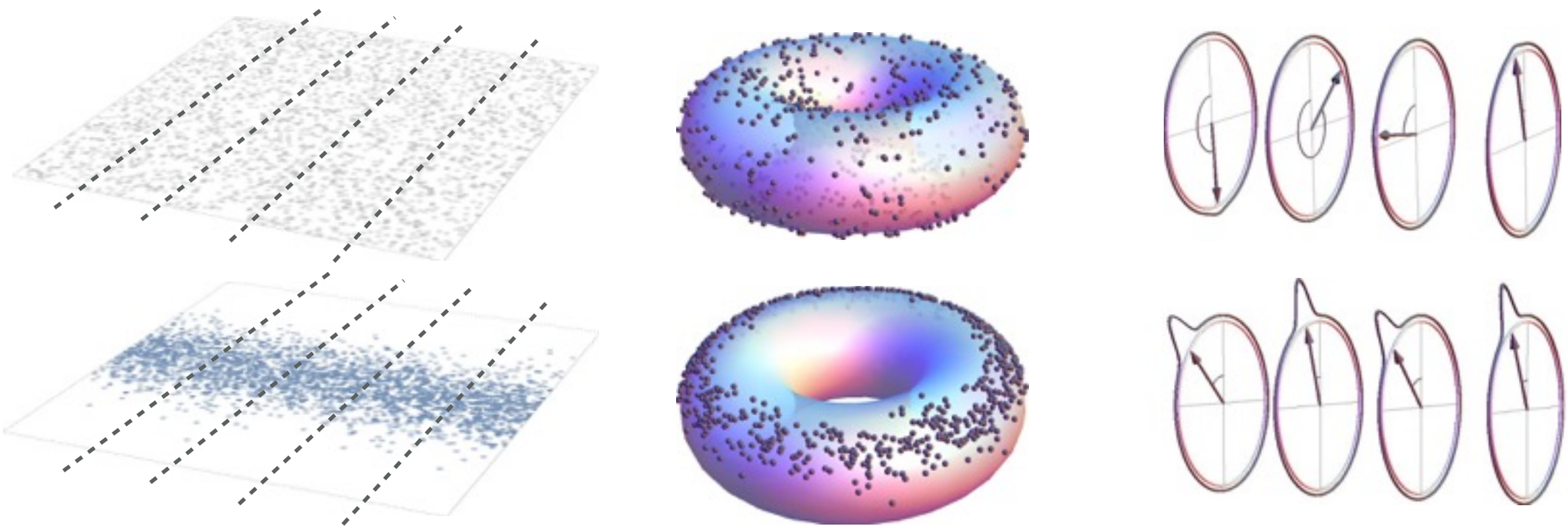

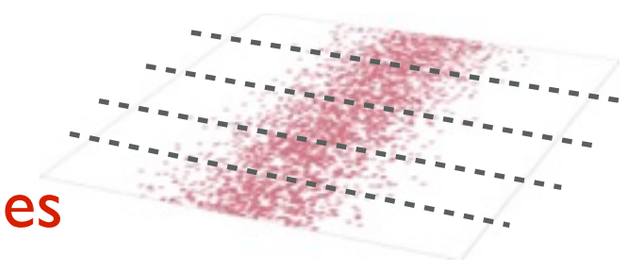

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