Reentrant behavior of the spinodal curve in a nonequilibrium ferromagnet

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The metastable behavior of a kinetic Ising-type ferromagnetic model system in which a generic type of microscopic disorder induces nonequilibrium steady states is studied by computer simulation and a mean-field approach. We pay attention, in particular, to the spinodal curve or intrinsic coercive field that separates the metastable region from the unstable one. We find that, under strong nonequilibrium conditions, this exhibits reentrant behavior as a function of temperature. That is, metastability does not happen in this regime for both low and high temperatures, but instead emerges for intermediate temperature, as a consequence of the nonlinear interplay between thermal and nonequilibrium fluctuations. We argue that this behavior, which is in contrast with equilibrium phenomenology and could occur in actual impure specimens, might be related to the presence of an effective multiplicative noise in the system.

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I. INTRODUCTION

The concept of metastability [1,2] is crucial to many branches of science. Metastable states occur in liquids and glasses [3], quark/gluon plasmas [4], globular proteins [5], cosmological phase transitions [6], the "standard model" of particle physics [7], climate models [8], black holes and protoneutronic stars [9], for instance. Understanding metastability from a microscopic point of view is therefore most interesting. It is also a difficult task, given that the concept is a kinetic feature which is not described by the Gibbs ensemble theory [1]. Consequently, a lot of activity still focuses on very simple cases, particularly, kinetic Ising-type models. Some recent studies along this line concern the details of nucleation during the relaxation processes [10], some exact results in the limit of zero temperature [11], the checking of theoretical predictions by means of computer simulation [12], and analysis of the effects of open borders [13], quenched impurities [14], and demagnetizing fields [15].

These studies deal with systems in which the metastable state decays towards the equilibrium stable state. In this case, some understanding can be achieved via nucleation theories in which Gibbs thermodynamic (equilibrium) free energy plays a central role. However, more general and intriguing is the case in which relaxation is towards a nonequilibrium steady state [16–19]. Nonequilibrium conditions appear ubiquitously in nature, and they characterize the evolution of most real systems [17]. Under such conditions, no free energy can be defined in general [17], and no coherent theoretical framework exists that accounts for the observed far-from-equilibrium behavior. Some important questions regarding metastability concern the existence and properties of a nonequilibrium macroscopic potential capturing the essential physics of the metastable-stable transition under nonequilibrium conditions, and the limit of metastability when such conditions hold.

In this paper we therefore deal with aspects of metastability in a kinetic Ising-type model with nonequilibrium steady states. Our interest is on the effects of the nonequilibrium condition on the properties of the metastable state as one varies the system parameters. In particular, we study the magnetic-field strength for the onset of instability. This is the intrinsic coercive field [20] which locates in magnets the spinodal curve which is familiar from studies of density-conserved systems [21]. The system behavior around this curve is the consequence of a complex interplay between thermal and nonequilibrium fluctuations. This results in a spinodal curve that depicts novel behavior as compared to the equilibrium case. An interesting observation is that metastability occurs in the strong nonequilibrium regime at intermediate temperatures but not in the low temperature limit, pointing out that, in this regime, noise enhances metastability.

Recent studies on critical behavior of some nonequilibrium models have predicted similar reentrance phenomena. That is, in a large class of model systems, one observes that, under nonequilibrium fluctuations, a disordered phase which characterizes the system at both low and high temperatures becomes ordered at intermediate temperatures [17,22–27]. Further research is still needed, however, before one may conclude on the relevance of such model behavior on the reentrance phenomena reported, for instance, in nonequilibrium phase transitions driven by competition between quantum and thermal fluctuations in superconductors and vortex matter [28–30], and concerning different liquid, glassy and amorphous phases in water and silica [31–33]. Despite the similarities between our results and these studies, they are different in essence: the latter concern reentrance in phase diagrams associated with nonequilibrium phase transitions, while our work concern reentrance of the nonequilibrium spinodal curve, which characterizes the limit of metastability.

The paper is organized as follows: Secs. II and III describe, respectively, the model and a dynamic mean-field approximation. Section IV contains our main results on the static properties of nonequilibrium metastability; in particular, we evaluate the intrinsic coercive field. In Sec. V we measure this spinodal field in computer simulations, and numerical results are compared in this section with our mean-field theory. Section VI is devoted to conclusions.

II. THE MODEL

Consider the two-dimensional Ising model on the square lattice of side L, $\Lambda=\{1,\ldots,L\}^2\subset\mathbb{Z}^2$, with periodic (toroidal) boundary conditions. There is a spin variable at each lattice site with two possible states, $s_i=\pm1$, $i\in\Lambda$, and spin-spin interactions and influence of an external magnetic field h as described by the function

$$\mathcal{H}(\mathbf{s}) = -\sum_{\langle i,j\rangle} s_i s_j - h\sum_{i\in\Lambda} s_i, \tag{1}$$

where $\mathbf{s}\equiv\{s_i\}$ and the first sum runs over all nearest-neighbor pairs. Futhermore, the spin system evolves with time via stochastic single-spin-flip dynamics as determined by the master equation:

$$\frac{\mathrm{d}P(\mathbf{s};t)}{\mathrm{d}t} = \sum_{i\in\Lambda}[\omega(\mathbf{s}^i\to\mathbf{s})P(\mathbf{s}^i;t) - \omega(\mathbf{s}\to\mathbf{s}^i)P(\mathbf{s};t)]. \tag{2}$$

Here, $P(\mathbf{s};t)$ is the probability of configuration $\mathbf{s}$ at time t, $\mathbf{s}^i$ stands for $\mathbf{s}$ after performing a flip at i, i.e., $s_i\to -s_i$, and $\omega(\mathbf{s}\to\mathbf{s}^i)$ stands for the corresponding transition rate. In order to ensure nonequilibrium conditions, we introduce a weighted competition between two different temperatures (one "infinite" and the other finite). This has been shown to be the simplest way of inducing nonequilibrium behavior in lattice models [17]. The rate is then chosen to be

$$\omega(\mathbf{s}\to\mathbf{s}^i) = p + (1-p)\frac{\mathrm{e}^{-\beta\Delta\mathcal{H}(s_i,n_i)}}{1+\mathrm{e}^{-\beta\Delta\mathcal{H}(s_i,n_i)}}, \tag{3}$$

where $\beta=1/T$ stands for the lattice (inverse) temperature—so that we are fixing the Boltzmann's constant to unity—and $\Delta\mathcal{H}(s_i,n_i)\equiv\mathcal{H}(\mathbf{s}^i)-\mathcal{H}(\mathbf{s})=2s_i[2(n_i-2)+h]$, where $n_i\in[0,4]$ is the number of up nearest-neighbors of s_i.

The rate (3) describes spin-flips under the action of two competing heat baths. For $p=0$, the system goes asymptotically towards the unique Gibbs, equilibrium state for temperature T and energy $\mathcal{H}$. This has a critical point at $h=0$ and $T=T_c(p=0)=T_{\text{ons}}\approx 2.2691$, the Onsager, equilibrium critical temperature. For $0<p<1$, the conflict in (3) impedes canonical equilibrium, and the system evolves asymptotically towards a nonequilibrium steady state which may essentially differ from a Gibbs state [17,34]. In this case, no equilibrium thermodynamic global temperature can be defined. Now parameter T can be thought as a source of thermal fluctuations, which compete with the nonthermal (nonequilibrium) noise induced by p. The system now exhibits a critical point, at $h=0$ and $T=T_c(p)<T_c(0)$, which is apparently of the Ising universality class [35,36], but only as far as p is small enough. In fact, the nonequilibrium disorder which is implied by (3) washes out the critical point for any $p>p_c\approx 0.17$ [17]. One may think of the dynamic random perturbation parameterized by p as an extra source of (nonequilibrium) disorder and randomness which is likely to occur also in many actual systems in nature [16,17]. A main question here is how the metastable states in the system depend on the competition between this nonthermal noise and the standard thermal fluctuations parameterized by T.

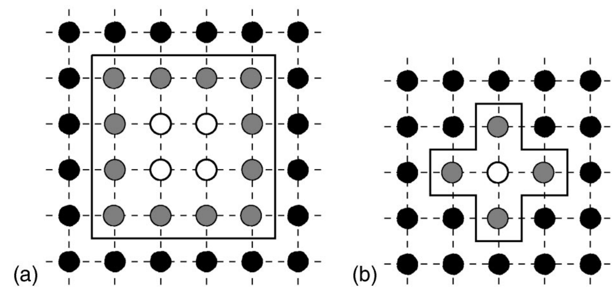

FIG. 1. Two examples of spin domains, each following from a different lattice partition $\mathbb{P}(\Lambda)$; see the main text. The spins that do not belong to the domain are in black, surface spins are gray, and the spins forming the domain interior are empty.

III. A MEAN-FIELD APPROXIMATION

We first study a mean-field solution of (2) in the pair approximation [17,37]. This approach is a dynamic generalization of Kikuchi's method known as cluster variation method [38]. Consider a partition $\mathbb{P}$ of the lattice such that resulting domains, $q_j\in\mathbb{P}(\Lambda)$, satisfy $q_j\cap q_{j'}=\varnothing$ if $j\neq j'$, and $\cup_j q_j=\Lambda$. We define the surface $\mathcal{S}_j$ of q_j as the set of all its spins that have at least one nearest neighbor outside the domain [39]; the rest is the interior, namely, $\mathcal{I}_j\equiv q_j-\mathcal{S}_j$ and it follows that $q_j=\mathcal{I}_j\cup\mathcal{S}_j$. These definitions are illustrated in Fig. 1. Next, consider a local observable $A(\mathbf{s}_{q_j};j)$ which exclusively depends on spins belonging to q_j. One readily has from (2) for the average $\langle A(j)\rangle_t\equiv\Sigma_{\mathbf{s}}A(\mathbf{s}_{q_j};j)P(\mathbf{s};t)$ that

$$\frac{\mathrm{d}\langle A(j)\rangle_t}{\mathrm{d}t} = \sum_{\mathbf{s}_{q_j}}\sum_{i\in\mathcal{I}_j}\Delta A(\mathbf{s}_{q_j};j;i)\,\omega(\mathbf{s}_{q_j}\to\mathbf{s}^i_{q_j})Q(\mathbf{s}_{q_j};t) + \sum_{\mathbf{s}}\sum_{i\in\mathcal{S}_j}\Delta A(\mathbf{s}_{q_j};j;i)\,\omega(\mathbf{s}\to\mathbf{s}^i)P(\mathbf{s};t), \tag{4}$$

where $\mathbf{s}_{q_j}$ is the configuration of the domain spins, $\Delta A(\mathbf{s}_{q_j};j;i)=A(\mathbf{s}^i_{q_j};j)-A(\mathbf{s}_{q_j};j)$, and $Q(\mathbf{s}_{q_j};t)\equiv\Sigma_{\mathbf{s}-\mathbf{s}_{q_j}}P(\mathbf{s};t)$ is the probability of having the configuration $\mathbf{s}_{q_j}$ at time t. The notation $\omega(\mathbf{s}_{q_j}\to\mathbf{s}^i_{q_j})$ in Eq. (4) stresses the fact that flipping a spin in the interior only depends on the spins belonging to the domain.

Let us assume that the system is spatially homogeneous, namely, that $\langle A(j)\rangle\equiv\langle A\rangle$, $q_j\equiv q$, $\mathcal{I}_j\equiv\mathcal{I}$, and $\mathcal{S}_j\equiv\mathcal{S}$ for any j. Equivalently, the partition $\mathbb{P}(\Lambda)$ is regular, so that all domains are topologically identical. One notices that the two r.h.s. terms in Eq. (4) concern the domain interior and surface, respectively; the latter couples the domain dynamics to its surroundings. Our second approximation consists in neglecting this surface term, i.e., any correlation during time evolution which extends outside the domain. Under these two approximations, homogeneity and kinetic isolation, Eq. (4) reduces to

$$\frac{\mathrm{d}\langle A\rangle_t}{\mathrm{d}t} = \sum_{\mathbf{s}_q}\sum_{i\in\mathcal{I}} \Delta A(\mathbf{s}_q;i)\omega(\mathbf{s}_q \to \mathbf{s}_q^i)Q(\mathbf{s}_q;t). \tag{5}$$

Next, one needs to estimate $Q(\mathbf{s}_q;t)$ in terms of n-body correlation functions. Assuming that only $\langle s\rangle$ and $\langle s_i s_j\rangle$, with i and j nearest-neighbor sites inside the domain, matter, $Q(\mathbf{s}_q;t)$ may be written as a function of the spin density $\rho(s)$ and the density $\rho(s,s')$ of nearest-neighbors pairs only. Furthermore, as only nearest-neighbors correlations are taken into account, we consider a domain with only one spin in its interior, which has four surface spins; see Fig. 1(b). With this choice, our mean-field theory turns out to be a nonequilibrium analog of the equilibrium Bethe-Peierls approximation. It follows that the probability of finding the central spin in state s surrounded by n up nearest-neighbor spins is

$$Q(\mathbf{s}_q;t) \equiv Q(s,n) = \binom{4}{n}\frac{\rho(+,s)^n\rho(-,s)^{4-n}}{\rho(s)^3}. \tag{6}$$

Therefore, using the relations $\rho(+,-)=\rho(-,+)=\rho(+)-\rho(+,+)$ and $\rho(-,-)=1+\rho(+,+)-2\rho(+)$, and writing $x\equiv\rho(+)$ and $z\equiv\rho(+,+)$, Eq. (5) reads

$$\frac{\mathrm{d}\langle A\rangle_t}{\mathrm{d}t} = \sum_{n=0}^{4}\binom{4}{n}\left[\Delta A(+,n)\frac{z^n(x-z)^{4-n}}{x^3}\omega(+,n) - \Delta A(-,n)\frac{(x-z)^n(1+z-2x)^{4-n}}{(1-x)^3}\omega(-,n)\right], \tag{7}$$

where $\omega(s,n)\equiv\omega(\mathbf{s}_q\to\mathbf{s}_q^i)$. This is for local isotropic observables for which the dependence on $\mathbf{s}_q$ is through the pair (s,n) only, $A(\mathbf{s}_q;t)\equiv A(s,n)$.

One may apply (7) to the observables $A_1(s,n)=\frac{1}{2}(1+s)$ and $A_2(s,n)=\frac{1}{8}n(1+s)$ whose averages are x and z, respectively. Then, $\Delta A_1(s,n)=-s$, $\Delta A_2(s,n)=-\frac{1}{4}sn$, and

$$\frac{\mathrm{d}x}{\mathrm{d}t} = F_1(x,z) \equiv \sum_{n=0}^{4} G(x,z;n), \tag{8}$$

$$\frac{\mathrm{d}z}{\mathrm{d}t} = F_2(x,z) \equiv \sum_{n=0}^{4} \frac{n}{4} G(x,z;n), \tag{9}$$

where

$$G(x,z;n) \equiv \binom{4}{n}\left[\frac{(x-z)^n(1+z-2x)^{4-n}}{(1-x)^3}\omega(-,n) - \frac{z^n(x-z)^{4-n}}{x^3}\omega(+,n)\right].$$

Together with (3) and (7), these equations provide $x(t)$ and $z(t)$ as well as any other local isotropic magnitude.

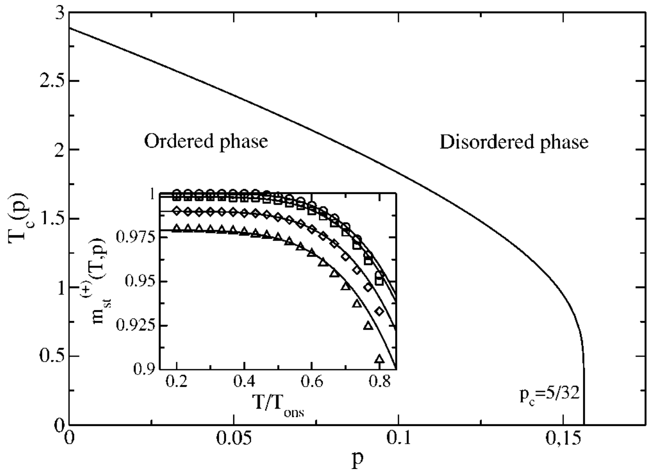

FIG. 2. Variation with p of the critical temperature for the nonequilibrium ferromagnetic system in the first-order mean-field approximation. The solid line in the inset stands for the locally-stable steady magnetization as a function of temperature (in units of the equilibrium critical value) for $h=0$ and, from top to bottom, $p=0$, 0.001, 0.005, and 0.01. The symbols in the inset are Monte Carlo results for a 53×53 lattice.

IV. STATIC PROPERTIES

Our main interest here is on the steady solutions

$$F_1(x_{\mathrm{st}},z_{\mathrm{st}}) = 0, \quad F_2(x_{\mathrm{st}},z_{\mathrm{st}}) = 0, \tag{10}$$

and on their stability [40]. Both stable and metastable states are locally stable under small perturbations, which requires the (necessary and sufficient) condition [41]

$$\left(\frac{\partial F_1}{\partial x}\right)_{\mathrm{st}} + \left(\frac{\partial F_2}{\partial z}\right)_{\mathrm{st}} < 0,$$

$$\left(\frac{\partial F_1}{\partial x}\right)_{\mathrm{st}}\left(\frac{\partial F_2}{\partial z}\right)_{\mathrm{st}} - \left(\frac{\partial F_1}{\partial z}\right)_{\mathrm{st}}\left(\frac{\partial F_2}{\partial x}\right)_{\mathrm{st}} > 0. \tag{11}$$

The condition

$$\left[\frac{\partial F_1(x,z)}{\partial x}\right]_{\mathrm{st}} = 0, \tag{12}$$

on the other hand, characterizes incipient or marginal stability, i.e., the presence of a critical point $(x_{\mathrm{st}}^c, z_{\mathrm{st}}^c)$ for $h=0$, $x_{\mathrm{st}}^c=\frac{1}{2}$ and $z_{\mathrm{st}}^c=\frac{1}{3}$, and it follows that

$$T_c(p) = -4\left[\ln\left(\frac{3}{4}\sqrt{\frac{1-4p}{1-p}} - \frac{1}{2}\right)\right]^{-1}. \tag{13}$$

This is the critical temperature for the nonequilibrium model in the present first-order mean-field approximation [17]; see Fig. 2. The existence is noticeable of a critical value of p such that $T_c(p_c)=0$, which gives $p_c=5/32=0.15625$ (to be compared with the exact value $p_c\simeq0.17$).

The stationary state $(x_{\mathrm{st}},z_{\mathrm{st}})$ may be obtained numerically from the nonlinear differential equations (10) subject to the local stability condition (11). For $h=0$; the up-down symmetry leads to pairs of locally-stable steady solutions, namely $(x_{\mathrm{st}},z_{\mathrm{st}})$ and $(1-x_{\mathrm{st}},1+z_{\mathrm{st}}-2x_{\mathrm{st}})$. The result is illustrated in

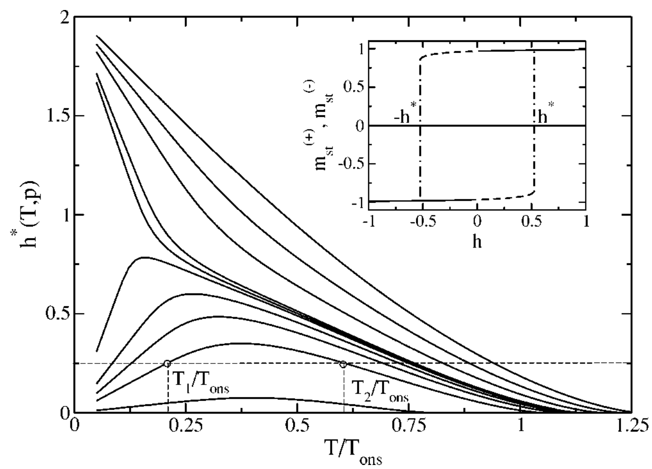

FIG. 3. $h^{*}(T,p)$, as a function of T for, from top to bottom, $p=0$, 0.01, 0.02, 0.03, 0.031, 0.032, 0.035, 0.04, 0.05, and 0.1. The qualitative change of behavior in the low temperature region occurs for $p\in(0.031,0.032)$. Inset: The two locally-stable steady magnetization branches as a function of h for $T=0.7T_c(0)$ and $p=0.005$. The solid (dashed) line represents stable (metastable) states. The dotted-dashed line signals the discontinuous transition, at $h=h^{*}(T,p)$, where metastable states disappear.

the inset of Fig. 2; this also shows a comparison with Monte Carlo results which confirms the expected agreement at low and intermediate temperatures for any p. The fact that increasing p at fixed T decreases the magnetization implies that the nonequilibrium perturbation tends to increase disorder. For small enough fields, the situation closely resembles the case $h=0$; the up-down symmetry is now broken, however, and locally-stable steady states with magnetization oriented opposite to the applied field are metastable.

The locally-stable steady magnetization exhibits two branches as a function of h; see inset in Fig. 3. This hysteresis loop reveals that metastability does not occur for any $|h|>h^{*}(T,p)\geqslant 0$, where $h^{*}(T,p)$ is the intrinsic coercive field [42]. Let $z=z(x)$ the solution of Eq. (9), and write Eq. (8) as

$$\frac{\mathrm{d}x}{\mathrm{d}t}=-\frac{\delta V(x)}{\delta x}. \tag{14}$$

This defines $V(x)$, a (nonequilibrium) bimodal potential that controls the system time evolution. An increase in the field tends to attenuate the local minimum in $V(x)$ associated with the metastable state. This minimum exists only for $|h|<h^{*}(T,p)$; the set of Eqs. (10) has only one solution, with magnetization sign equal to that of the applied field, for $|h|>h^{*}(T,p)$.

In order to evaluate $h^{*}(T,p)$, one may study how the metastable state responds to small perturbations of the applied field. If $(x_{\mathrm{st}}^{h_0},z_{\mathrm{st}}^{h_0})$ is a locally-stable stationary state for T, p, and h_0, with magnetization opposed to h, and we perturb $h=h_0+\delta h$, the new locally-stable stationary solution is modified: $x_{\mathrm{st}}^{h}=x_{\mathrm{st}}^{h_0}+\epsilon_x$ and $z_{\mathrm{st}}^{h}=z_{\mathrm{st}}^{h_0}+\epsilon_z$. One obtains at first order that

$$\epsilon_x=\left[\frac{\dfrac{\partial F_2}{\partial h}\dfrac{\partial F_1}{\partial z}-\dfrac{\partial F_1}{\partial h}\dfrac{\partial F_2}{\partial z}}{\dfrac{\partial F_1}{\partial x}\dfrac{\partial F_2}{\partial z}-\dfrac{\partial F_2}{\partial x}\dfrac{\partial F_1}{\partial z}}\right]_{x_{\mathrm{st}}^{h_0},z_{\mathrm{st}}^{h_0},h_0,T,p}\delta h. \tag{15}$$

This (linear) response diverges for

$$\left[\frac{\partial F_1}{\partial x}\frac{\partial F_2}{\partial z}-\frac{\partial F_2}{\partial x}\frac{\partial F_1}{\partial z}\right]_{x_{\mathrm{st}}^{h_0},z_{\mathrm{st}}^{h_0},h_0,T,p}=0, \tag{16}$$

which corresponds to a discontinuity in the metastable magnetization as a function of h. For fixed T and p, the field for which (16) holds is $h^{*}(T,p)$. Figure 3 shows the mean-field result for $h^{*}(T,p)$. In particular, for $p=0$ (the equilibrium case) we recover the standard equilibrium mean-field spinodal curve: converging to 2 as $T\to 0$, linearly decreasing with temperature for small T, and vanishing as $(T_c-T)^{3/2}$ at the mean-field equilibrium critical point. A main result derived from Fig. 3 is the existence of two different low temperature limits for $h^{*}(T,p)$. For small enough values of p, namely, $p\in[0,0.031]$, which includes the equilibrium case, $p=0$, the field $h^{*}(T,p)$ monotonously grows and extrapolates to 2 as $T\to 0$. For larger p, namely, $p\in[0.032,\frac{5}{32}]$, however, $h^{*}(T,p)\to 0$ as $T\to 0$, exhibiting a maximum at intermediate T. The value $p=\pi_c\approx 0.0315$ separates the two types of asymptotic behavior.

When we cool the system in the regime $p<\pi_c$, the field $h^{*}(T,p)$ increases, so in this case we require a stronger field to destroy the metastable state. This may be understood on simple grounds. The tendency of spins to line up in the direction of the field competes with the tendency to maintain order implied by their mutual interactions. A metastable state lasts for a long time because the latter prevails over the action of the field. Both T and p induce disorder; therefore, lowering T increases order, so that a stronger field is needed to destroy the metastable state, which is in fact observed for $p<\pi_c$. On the other hand, as p is increased, the disorder increases, and $h^{*}(T,p)$ needs to decrease for a fixed T, according to our observations.

The picture for $p>\pi_c$ is more intriguing. Consider the case $|h|=0.25$ and $p=0.05>\pi_c$. As illustrated in Fig. 3, one may define two temperatures, $T_1<T_2$, such that metastable states only occur for $T\in(T_1,T_2)$. The fact that $h^{*}(T,p)$ extrapolates to zero in the low temperature limit for $p=0.05>\pi_c$ indicates that such amount of nonequilibrium noise is able to destroy on its own any metastability. Following the above reasoning, increasing T adds disorder, so that no metastability should, in principle, show up in this case. However, there is a regime of intermediate temperatures, $T\in(T_1,T_2)$, for which metastability occurs. This noise-enhanced metastability is a consequence of the complex interplay between the standard thermal fluctuations and nonequilibrium noise: although both noises add independently disorder, their combination determines the existence of regions in the parameter space (T,p) in which no metastable states occur at low T but only at intermediate temperatures. This reentrance phenomenon is reminiscent of the one observed in the annealed Ising model [22] and other closely

TABLE I. Spin classes for the two-dimensional Ising model with periodic boundary conditions. The last column shows the energy cost of flipping the central spin.

Class	Central spin	Number of up neighbors	$\Delta\mathcal{H}$
1	+1	4	$8J+2h$
2	+1	3	$4J+2h$
3	+1	2	$2h$
4	+1	1	$-4J+2h$
5	+1	0	$-8J+2h$
6	−1	4	$-8J-2h$
7	−1	3	$-4J-2h$
8	−1	2	$-2h$
9	−1	1	$4J-2h$
10	−1	0	$8J-2h$

related systems [17,23–27] where multiplicative noise seems to play an essential role [25].

V. MONTE CARLO SIMULATIONS: GROWTH AND SHRINKAGE OF THE STABLE PHASE

In this section, we check further our theoretical predictions against computer simulation data. With this aim, we need a simple criterion to conclude that the model system exhibits metastable states. Let us first characterize all the possible local configurations in terms of the spin state, $s=\pm 1$, and the number of its up nearest neighbors, $n\in[0,4]$. For periodic boundary conditions, there are 10 different spin classes, as shown in Table I. The cost $\Delta\mathcal{H}(s,n)$ of flipping any spin within a class is the same. That is, the rate (3) only depends on s and n, which define the class. If $n_k(\mathbf{s})$ is the number of spins in class k when the system is is configuration $\mathbf{s}$, and noticing that classes $k\in[1,5]$ are characterized by a central up spin, we may write the number of up spins which flip per unit time in the state $\mathbf{s}$ as

$$G(\mathbf{s}) = \sum_{k=1}^{5} n_k(\mathbf{s})\omega_k. \qquad (17)$$

As far as $h<0$, this is the growth rate of the stable phase in state $\mathbf{s}$. The shrinkage rate of the stable phase follows similarly as [43]

$$S(\mathbf{s}) = \sum_{k=6}^{10} n_k(\mathbf{s})\omega_k. \qquad (18)$$

Given a phase-space point $\mathbf{s}$, the rates $G(\mathbf{s})$ and $S(\mathbf{s})$ yield the local tendency of the system to evolve toward the stable or metastable phases, respectively.

For $h<0$, a state with all spins up, $\mathbf{s}_1\equiv\{s_i=+1, i=1,\ldots,N\equiv L^2\}$, will relax after some time toward the stable steady state, which corresponds in this case to a configuration with negative magnetization, $m<0$. For a given experiment j of a total of N_{exp} runs, this relaxation will proceed

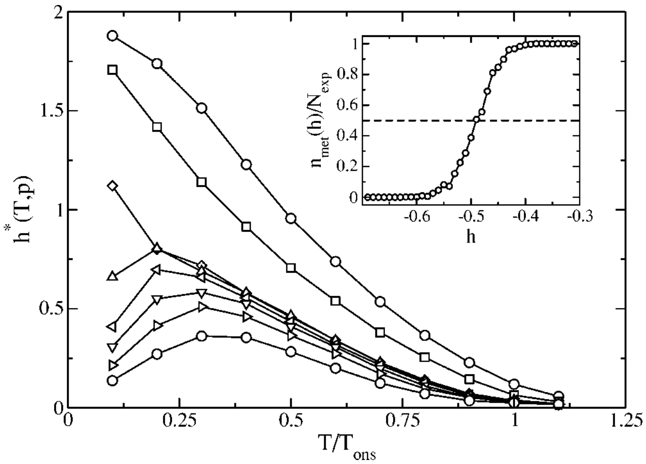

FIG. 4. Monte Carlo results for $h^*(T,p)$ as a function of T for $L=53$ and, from top to bottom, $p=0$, 0.01, 0.03, 0.0305, 0.0320, 0.0350, 0.04, and 0.05. Notice the change of asymptotic behavior in the low temperature limit for $p\in(0.03,0.0305)$. Inset: The probability of the metastable state, as defined in the main text, as a function of $h<0$ for $L=53$, $T=0.7T_c(0)$, and $p=0$. Data here correspond to an average over 500 independent demagnetization experiments for each value of h. Error bars are smaller than the symbol sizes.

through certain path in phase space, which we note as $\sigma_j\equiv\{\mathbf{s_1},\mathbf{s_2^{(j)}},\ldots,\mathbf{s_{\Gamma(j)}^{(j)}}\}$. Here $\mathbf{s_l^{(j)}}$ is the lth configuration, starting from $\mathbf{s_1}$, of a total number of $\Gamma(j)$ configurations which make up the path in experiment j. At any stage $\mathbf{s_l}^{(j)}$ of this path, the difference $G(\mathbf{s_l^{(j)}})-S(\mathbf{s_l^{(j)}})$ defines the net tendency of the system to evolve toward the final steady stable state. A metastable state is characterized by the presence of free energy barriers hampering the relaxation toward the truly stable state. In this case, relaxation is an activated process controlled by large fluctuations. On the other hand, an unstable state decays without any hindrance. Therefore, we may divide relaxation paths in two different types. On one hand, metastablelike paths, in which at least one configuration $\mathbf{s_k^{(j)}}\in\sigma_j$ exists, excluding the final stable one, such that $G(\mathbf{s_k^{(j)}})-S(\mathbf{s_k^{(j)}})<0$, and on the other hand, unstablelike paths, such that $G(\mathbf{s_k^{(j)}})-S(\mathbf{s_k^{(j)}})>0\ \forall\mathbf{s_k^{(j)}}\in\sigma_j$, excluding again the final stable state.

For fixed T, p and $h<0$, given the stochasticity of the dynamics, one needs to be concerned with the probability of occurrence of metastability, defined as $n_{\text{met}}(T,p,h)/N_{\text{exp}}$, where $n_{\text{met}}(T,p,h)$ is the number of experiments out of the total N_{exp} in which the relaxation path in phase space belongs to the class of metastablelike paths. This is shown in the inset of Fig. 4. The intrinsic coercive field, $h^*(T,p)$, is defined in this scheme as the field for which $n_{\text{met}}(T,p,h^*)/N_{\text{exp}}=0.5$; this is shown in Fig. 4 for a system with $L=53$.

A detailed comparison of these numerical results with the theory in Fig. 3 depicts semiquantitative agreement, namely, the agreement is excellent except—as one should have expected—near the critical temperature. In particular, the numerical critical value π_c for p is $\pi_c^{\text{MC}}\approx 0.03025$, rather close to the theoretical prediction $\pi_c^{\text{pair}}\approx 0.0315$. This nicely confirms that the addition of sufficient thermal noise in the presence of a large enough nonequilibrium perturbation, $p > \pi_c$, tends to restore metastability. We have also looked for finite-size corrections to the measured spinodal field by simulating larger systems, finding that these corrections are very small, and can be neglected for all practical purposes.

Finally, let us remark that the stable phase growth and shrinkage rates have been introduced in literature as projected on the slow observable characterizing the relaxation process, namely the system magnetization m [43]. In this case, the rate $G(m)$ $(S(m))$ yields the average number of up (down) spins which flip per unit time when magnetization is m. One may then define

$$G(m) = \sum_{\{\mathbf{s}|m\}} \mathcal{P}(\mathbf{s})G(s), \quad S(m) = \sum_{\{\mathbf{s}|m\}} \mathcal{P}(\mathbf{s})S(s), \qquad (19)$$

where $\{\mathbf{s}|m\}$ are all system configurations with fixed magnetization m, and $\mathcal{P}(\mathbf{s})$ is the probability of observing a configuration $\mathbf{s}$ during the relaxation from the initial state, $\mathbf{s_1}$, toward the stable one. Steady states are usually determined by the condition $G(m)=S(m)$. However, the lack of intersection between the curves $G(m)$ and $S(m)$ for $h<0$ in the $m>0$ interval does not contain information about the limit between metastable and unstable states. Instead, the magnetic field for which such lack of intersection first develops defines the so-called dynamic spinodal field, $h_{\mathrm{DSP}}(T,p)$, which divides the metastable region of parameter space (T,p,h) for finite systems in two subregions characterized by different relaxation morphologies [44]. In particular, for $|h| < h_{\mathrm{DSP}}(T,p)$ the metastable state relaxes through the nucleation of a single droplet of the stable phase, while for $h_{\mathrm{DSP}}(T,p) < |h| < h^*(T,p)$ the relaxation proceeds via the nucleation of multiple stable-phase droplets.

VI. CONCLUSION

This paper deals with some of the static properties of metastable states in a nonequilibrium Ising-type ferromagnetic model system, as obtained from first-order mean-field theory and computer (Monte Carlo) simulations. We studied, in particular, the spinodal or intrinsic coercive field, $h^*(T,p)$, defined as the magnetic field strength for which the metastable state becomes unstable. Our theoretical approximation predicts reentrace phenomena as a function of T in the strong nonequilibrium regime, $p > \pi_c \approx 0.0315$, where p controls the dynamic, nonequilibrium perturbation. More specifically, metastability is not observed at low temperatures for $p > \pi_c$, but it occurs as one increases the temperature. This noise-enhanced metastability reveals the existence of a complex interplay between the thermal and nonequilibrium noises. That is, adding the two effects—which, independently, tend to increase disorder—not always results in decreasing the system ordering. The above is fully confirmed in computer simulations, in which h^* may accurately be measured from the stable phase growth and shrinkage rates.

The physical origin of the observed noise-driven metastability is intriguing. A clue to understand this behavior is to notice that certain systems under the action of a multiplicative noise exhibit a similar reentrant behavior, namely, disorder is dominant at the low and high temperature regimes while well-defined order sets in at intermediate temperatures. That the competition between thermal and nonequilibrium fluctuations in (3) may induce an effective multiplicative noise can be understood on simple grounds. The effect of the nonequilibrium perturbation in our model may be described by means of an effective temperature T_{eff}, which is inhomogeneous throughout the system for any $p > 0$ [19]. In fact, for any $\Delta\mathcal{H} \neq 0$ we may write (3) as an equilibrium Glauber rate with effective parameters, $\omega \equiv \exp(-\beta_{\mathrm{eff}}\Delta\mathcal{H})/[1+\exp \times(-\beta_{\mathrm{eff}}\Delta\mathcal{H})]$, and this defines an effective temperature [45]

$$T_{\mathrm{eff}}(s,n) \equiv \frac{\Delta\mathcal{H}(s,n)}{\ln\left[\dfrac{1}{p + (1-p)\dfrac{\mathrm{e}^{-\beta\Delta\mathcal{H}(s,n)}}{1+\mathrm{e}^{-\beta\Delta\mathcal{H}(s,n)}}} - 1\right]}. \qquad (20)$$

As a matter of fact, the temperature a spin effectively feels depends on the local order around it, i.e., on the number of nearest neighbors pointing in the same direction; T_{eff} is an increasing function of local order and, consequently, the amplitude of the fluctuations depends on the local order parameter. This is a main feature of Langevin-type models with a multiplicative noise [25]. Developing further this possibility to treat the limit of metastability is an open question. This work, which seems most interesting, is outside the scope of the present paper.

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