

Metastability and Avalanches in a Nonequilibrium Ferromagnetic System

Pablo I. Hurtado, J. Marro and P.L. Garrido

Institute 'Carlos I' for Theoretical and Computational Physics, and Departamento de Electromagnetismo y Física de la Materia, University of Granada, 18071, Granada, Spain.

Abstract. We present preliminary results on the metastable behavior of a nonequilibrium ferromagnetic system. The metastable state mean lifetime is a non-monotonous function of temperature; it shows a maximum at certain non-zero temperature which depends on the strength of the nonequilibrium perturbation. This is in contrast with the equilibrium case in which lifetime increases monotonously as the temperature is decreased. We also report on avalanches during the decay from the metastable state. Assuming both free boundaries and nonequilibrium impurities, the avalanches exhibit power-law size and lifetime distributions. Such scale free behavior is very sensible. The chances are that our observations may be observable in real (i.e. impure) ferromagnetic nanoparticles.

Metastability is ubiquitous in Nature, and it often determines the system behavior. The microscopic understanding of this phenomenon, which is mathematically challenging,[1] is therefore of great practical and theoretical interest. In particular, metastability is relevant to the behavior of magnetic storage devices. A magnetic material may consist of magnetic monodomains. In order to store information on such material, one magnetizes each individual domain using a strong magnetic field, defining in this way a bit of information. A main concern is to retain the individual orientations of the domains for as long as possible in the presence of weak arbitrarily-oriented external magnetic fields. The interaction with these external fields often produces metastable states in the domains, and the resistance of stored information thus depends on the properties of these metastable states, including the details of their decay. On the other hand, real magnetic domains are usually *impure*. The microscopic nature of impurities, which shows up in actual specimens as spin, bond and/or lattice disorder and other inhomogeneities, quantum tunneling,[2] etc., suggests they might dominate the behavior of near-microscopic particles; in fact, they are known to influence even macroscopic systems. An interesting issue is therefore understanding the formation of a new phase inside a metastable domain which contains impurities.

Following recent efforts,[3] we study in this paper the simplest possible model of this situation, namely a $2d$ Ising ferromagnet with periodic boundary conditions that we endow with a weak dynamic perturbation competing with the thermal spin-flip process, which impedes equilibrium. Consider the Hamiltonian function $\mathcal{H}(\vec{s}) = -J \sum_{\langle i,j \rangle} s_i s_j - h \sum_{i=1}^N s_i$, where $J > 0$ (ferromagnetic interactions), $s_i = \pm 1$ stands for the two possible states of the spin at site i of the square lattice, $i = 1, \dots, N$, and the first sum is over any pair $\langle i, j \rangle$ of nearest-neighbor sites. The lattice side is L ; $N = L^2$. The system

CP661, *Modeling of Complex Systems: Seventh Granada Lectures*,
edited by P. L. Garrido and J. Marro

© 2003 American Institute of Physics 0-7354-0121-7/03/\$20.00

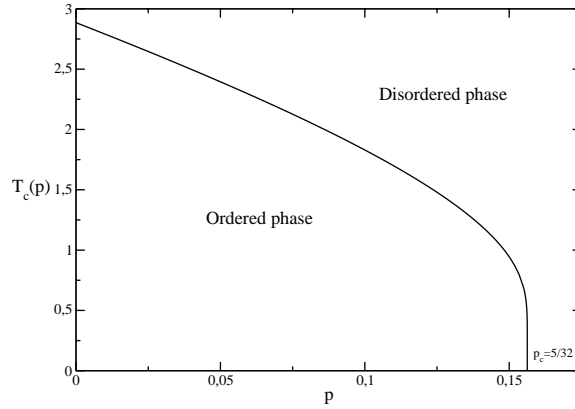


FIGURE 1. Critical temperature for our nonequilibrium ferromagnetic system as a function of p in a first-order mean-field approximation.

configuration, $\vec{s} \equiv \{s_i\}$, is let to evolve in time due to superposition of two canonical drives. That is, we chose the transition probability per unit time for a change of \vec{s} to be

$$\omega(\vec{s} \rightarrow \vec{s}^i) = p + (1-p) \frac{e^{-\frac{1}{T}\Delta\mathcal{H}_i}}{1 + e^{-\frac{1}{T}\Delta\mathcal{H}_i}} \quad (1)$$

Here \vec{s}^i stands for \vec{s} after flipping the spin at i , and $\Delta\mathcal{H}_i \equiv \mathcal{H}(\vec{s}^i) - \mathcal{H}(\vec{s})$. The Boltzmann constant is set $k_B \equiv 1$ in this paper. One may interpret that rule (1) describes a spin-flip mechanism under the action of two competing heat baths: with probability $2p$, the flip is performed completely at random (as if \vec{s} were in contact with a heat bath at ‘infinite’ temperature), while the change is performed at temperature T with probability $1-p$. For $p=0$, eq. (1) corresponds to the equilibrium Ising case, which exhibits for $h=0$ a critical point at $T = T_C \approx 2.2691J$. Otherwise, the conflict in (1) impedes canonical equilibrium, and the system evolves with time towards a non-equilibrium steady state that essentially differs from the Gibbs state for T . It is assumed that this kind of stochastic, non-canonical perturbation for $p > 0$ may actually occur in Nature due to microscopic disorder or impurities, etc.[2] The system shows for $p \neq 0$ and $h=0$ a continuous order-disorder phase transition at a critical temperature $T_c(p)$. This critical temperature can be calculated in first order mean field approximation as[4]

$$\frac{T_c(p)}{J} = \frac{-4}{\ln[-\frac{1}{2} + \frac{3}{4}\sqrt{\frac{1-4p}{1-p}}]} \quad (2)$$

Fig. 1 shows $T_c(p)$ as a function of p . $T_c(p=0)$ is the Bethe temperature, $T_{Bethe} \approx 2.8854J$, to be compared with the exact critical value for $p=0$, T_C . There is a critical value of p , p_c , such that for larger values of p there is no ordered phase for any temperature. This can be obtained from the condition $T_c(p_c) = 0$, yielding $p_c = \frac{5}{32} = 0.15625$. In this paper we pay attention to the ordered phase.

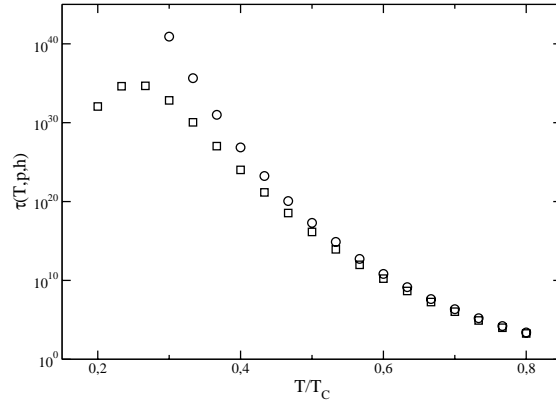


FIGURE 2. Semilog plot of the metastable-state mean lifetime as a function of temperature for a system with side $L = 53$ and, from top to bottom, $p = 0$ (equilibrium) and $p = 0.001$ (nonequilibrium). The data come from an average over 1000 independent runs. Notice the non-monotonous behavior of τ for the nonequilibrium case.

In order to characterize the metastable behavior of the model, we measured the mean lifetime for values of T and p such that $T < T_c(p)$ and a magnetic field $h = -0.1$. We take the initial state to be $s_i = +1$ for $i = 1, \dots, N$. Under the negative field, this ordered state is metastable, and it eventually decays to the stable state. In fact, the system rapidly evolves from the initial state to a state in the metastable region, with magnetization close to $+1$. After this fast relaxation, the system spends a long time wandering around the metastable state, eventually nucleating one or several critical droplets of the stable phase, which rapidly grow thus making the system to evolve from the metastable state to the stable one. We define the mean lifetime of the metastable state, $\tau(T, p, h)$, as the mean first-passage time (in Monte Carlo steps per spin, MCSS) to $m = 0$. The simulations reported here required in practice using the *s-1 Monte Carlo with absorbing Markov chains* (MCAMC) algorithm, and the *slow forcing approximation*. [5] Fig. 2 shows $\tau(T, p, h)$ as a function of T for a $L = 53$ and $p = 0$ (the equilibrium case) and $p = 0.001$. Rather amazing, we observe in the figure how the nonequilibrium lifetime exhibits a maximum and then decreases as the temperature drops. The maximum occurs at $T_{max}(p)$ which depends on the nonequilibrium parameter p . On the contrary, the lifetime in the equilibrium case grows exponentially fast as a function of $1/T$, as predicted by nucleation theory [3]. We do not have a simple explanation of this nonequilibrium effect. However, it may be emphasized that it has some practical implications. That is, one should not blindly decrease temperature when trying to maximize the lifetime of real (i.e., impure) metastable magnetic particles, but look for the temperature $T_{max}(p)$ which maximizes the lifetime for the typical impurity concentration of the particle.

The previous observations concern the *bulk* metastability. However, in real magnets, one needs in practice to create and to control fine grains, i.e., magnetic particles with *borders* whose size ranges from mesoscopic to atomic levels, namely, clusters of 10^4

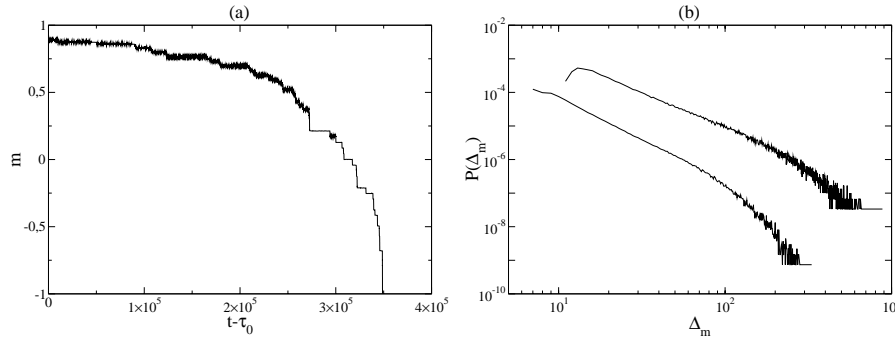


FIGURE 3. (a) Time variation of the magnetization showing the decay from a metastable state for a $R = 30$ particle; avalanches are seen here by direct inspection. Time is in Monte Carlo Steps per Spin (MCSS), and $\tau_0 \sim 10^{30}$ MCSS. (b) Large avalanche size distribution $P(\Delta_m)$ for the circular magnetic nanoparticle and $R = 30$ (bottom) and $R = 60$ (top). The second curve has been shifted in the vertical direction for visual convenience.

to 10^2 spins, and even smaller ones. Though experimental techniques are already accurate for the purpose,[6] the underlying physics is much less understood than for bulk properties. In particular, one cannot assume that such particles are neither *infinite* nor *pure*. That is, they have free borders, which results in a large surface/volume ratio inducing strong border effects, and impurities. Motivated by the experimental situation, we studied a finite, relatively small two-dimensional system subject to open circular boundary conditions. The system is defined on a square lattice, where we inscribe a circle of radius R ; sites outside this circle do not belong to the system and are set $s_i = 0$. We mainly report in the following on a set of fixed values for the model parameters, namely, $h = -0.6$, $T = 0.11T_C$ and $p = 10^{-9}$. This choice is dictated by simplicity and also because (after exploring the behavior for other cases) we came to the conclusion that this corresponds to an interesting region of the system parameter space, where the effects of p and T are comparable and clusters are compact and hence easy to analyze. We believe that we are describing here typical behavior of our model, and the chances are that it can be observed in actual materials.

The effects of free borders on the metastable-stable transition have already been studied for equilibrium systems.[7] In this case, the system evolves to the stable state through the *heterogeneous* nucleation of one or several critical droplets which always appear at the system's border. That is, the free border acts as a droplet condenser. This is so because it is energetically favorable for the droplet to nucleate at the border. Apart from the observed heterogeneous nucleation, the properties of the metastable-stable transition in equilibrium ferromagnetic nanoparticles do not change qualitatively as compared to the periodic boundary conditions case.[7] In our nonequilibrium system we observe a similar behavior, namely heterogeneous nucleation and the same qualitative nucleation properties. However, the fluctuations or noise that the nonequilibrium metastable system shows as it evolves towards the stable state subject to the combined action of free borders and the nonequilibrium perturbation are quite unexpected.

As illustrated in Fig. 3.a, the relaxation of magnetization occurs via a sequence of well-defined abrupt jumps. That is, when the system relaxation is observed after each MCSS, which corresponds to a 'macroscopic' time scale, *strictly monotonic changes of $m(t)$* can be identified that we shall call *avalanches* in the following. To be precise, consider the avalanche beginning at time t_a , when the system magnetization is $m(t_a)$, and finishing at t_b . We define its *size* and *lifetime* or *duration*, respectively, as $\Delta_m = |m(t_b) - m(t_a)|$ and $\Delta_t = |t_b - t_a|$. Our interest is on the histograms $P(\Delta_m)$ and $P(\Delta_t)$. Fig. 3.b shows the large avalanche size distribution $P(\Delta_m)$ for particle sizes $R = 30$ and $R = 60$, after the extrinsic noise[8] (i.e. the trivial, exponentially distributed small avalanches) has been subtracted. A power law behavior, followed by a cutoff is clearly observed. The measured power law exponents, $P(\Delta_m) \sim \Delta_m^{-\tau(R)}$, show size-dependent corrections to scaling. Similar corrections have been also found in real experimental systems.[9] In particular we find $\tau(R = 30) = 2.76(2)$ and $\tau(R = 60) = 2.06(2)$. The lifetime distribution also shows power law behavior, $P(\Delta_t) \sim \Delta_t^{-\alpha(R)}$ with a cutoff. Here we measure $\alpha(R = 30) = 3.70(2)$ and $\alpha(R = 60) = 2.85(2)$. This power-law behavior implies that avalanches are scale-free (up to certain maximum size and lifetime) in our nonequilibrium ferromagnet subjected to open boundary conditions. We also measured avalanches for $p = 0$ in the circular magnetic particle case, and for $p \neq 0$ in the periodic boundary conditions system. In both cases only small avalanches occur and the distributions are exponential-like, thus indicating the absence of scale invariance. That is, the combined action of free boundaries and impurities is behind the large, scale-free avalanches and essentially differs from the standard bulk noise driving the system and causing small, exponentially distributed avalanches only. The physical origin of this scale invariant behavior will be studied in a forthcoming paper.

Summing up, in this paper we present preliminary results on the metastable behavior of a nonequilibrium ferromagnetic system. The presence of nonequilibrium conditions considerably enriches the observed phenomenology. In particular, we study the metastable-state mean lifetime for a lattice with periodic boundary conditions. Under the action of the nonequilibrium perturbation parameterized by p , the lifetime $\tau(T, p, h)$ shows a maximum as a function of T for certain nonzero temperature $T_{max}(p)$, then decreasing for lower temperatures. This counter-intuitive behavior, not observed in equilibrium, has some practical implications for real magnetic systems with *impure* ferromagnetic domains. We also observe that, under the action of both the nonequilibrium impurity and free borders, the metastable-stable transition proceeds by avalanches. These are power-law distributed, thus showing scale invariance (up to certain cutoffs). The chances are that our observations about the effect of the nonequilibrium conditions on the properties of metastable states and their decay, which we can prove in our model cases, are a general feature of similar phenomena in real magnetic domains.

ACKNOWLEDGMENTS

We acknowledge M.A. Muñoz and S. Zapperi for very useful comments, and M.A. Novotny for sharing unpublished information about MCAMC algorithms. We also acknowledge financial support from Ministerio de Ciencia y Tecnología–FEDER, project

BFM2001-2841.

REFERENCES

1. O. Penrose and J.L. Lebowitz, in *Fluctuation Phenomena* (2nd edition), edited by E. Montroll and J.L. Lebowitz, North-Holland, Amsterdam 1987.
2. J. Marro and J.A. Vacas, *Phys. Rev. B* **56**, 8863 (1997).
3. See, for instance, P.A. Rikvold et al, *Phys. Rev. B* **49**, 5080 (1994); R. Mélin, *J. Mag. Mag. Mat.* **162**, 211 (1996).
4. Pablo I. Hurtado, *Dynamics of Nonequilibrium Systems: Metastability, Avalanches, Phase Separation, Absorbing States and Heat Conduction*, Ph.D. thesis (2002).
5. M.A. Novotny, *Phys. Rev. Lett.* **74**, 1 (1994); A.B. Bortz et al, *J. Compt. Phys.* **17**, 10 (1975); M. Kolesik, M.A. Novotny and P.A. Rikvold, *Phys. Rev. Lett.* **80**, 3384 (1998).
6. J. Shi et al, *Science* **271**, 937 (1996).
7. E.N.M. Cirillo and J.L. Lebowitz, *J. Stat. Phys.* **90**, 211 (1998); H.L. Richards et al, *Phys. Rev. B.* **55**, 11521 (1997); D. García-Pablos et al, *J. Appl. Phys.* **79**, 6019 (1996).
8. D. Spasojević et al, *Phys. Rev. E* **54**, 2531 (1996).
9. V. Frette et al, *Nature* **379**, 49 (1996).